# Formal Concept Analysis for Analysing Classes

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# Outline

# An { Objects X Attributes } Table

| Name | Arts | Lit. | Sports | Env. |
|---|---|---|---|---|
| John | 1 | 1 | 0 | 1 |
| Tom | 1 | 1 | 0 | 1 |
| Stella | 1 | 1 | 0 | 1 |
| Harry | 0 | 1 | 1 | 1 |
| Milli | 0 | 1 | 1 | 0 |
| Stephan | 1 | 0 | 1 | 1 |
| Steffi | 1 | 0 | 1 | 1 |
| Sara | 1 | 0 | 0 | 1 |
| Maya | 0 | 1 | 0 | 1 |
| Hamid | 0 | 0 | 1 | 1 |
| Don | 1 | 1 | 1 | 1 |
| Rosy | 1 | 1 | 1 | 1 |
| Tim | 0 | 0 | 0 | 0 |
| Mary | 0 | 0 | 0 | 1 |

*Concepts are interesting pairings of some objects and some attributes.*
*Pair up such that no object is left out if it has all those attribute, and no attribute is left out if all those objects share it.*
**Find out all concepts from this context.**

# A Concept: Objects X Attributes

- Let's try to find a concept around attribute 'Arts'.
- **{ Sara X Arts }** is not a concept since there are more people with Arts interest who have been left out. Let's include them.
- Is **{Sara, Steffi, Stephan, Don, Rosy, Tom, Stella, John X Arts }** is still not a concept since there is one more common interest that has been left out. Let's include it.
- Now consider pairing **{Sara, Steffi, Stephan, Don, Rosy, Tom, Stella, John X Arts, Environment}**

# Exercise

1. We can see that 5 of these 8 people in the paring **{Sara, Steffi, Stephan, Don, Rosy, Tom, Stella, John X Arts, Environment}** have **Literature** in common. Can we include **Literature** too in the concept? Why?
2. We find that there are more people interested in **Environment**. So can we include **Mary, Hamid, Maya, Milli, Harry** in this concept?

# Answers to Exercise

1. Since not all of them have Literature in common, we don't include Literature in the concept.
2. The answer is 'No' because they don't have the attribute 'Arts' in common.

# A Concept

**{Sara, Steffi, Stephan, Don, Rosy, Tom, Stella, John**
**X**
**Arts, Environment}**

- all attributes that are common to all objects are listed.... and ...
- all objects having all these attributes are also listed

# Another Concept

Let's work on a concept around 'Environment'.

- starting point: list all objects having 'Environment' attribute:
  **{John, Tom, Stella, Harry, Stephan, Steffi, Sara, Maya, Hamid, Don, Rosy, Mary**
  **X**
  **Environment }**
- They do not have any other attribute in common. We are done with another concept.

# Third Concept

Let's work on a concept around 'Literature'.

- starting point: list all objects having 'Literature' attribute:
  **{John, Tom, Stella, Harry, Milli, Maya, Don, Rosy**
  **X**
  **Literature }**
- They do not have any other attribute in common. We are done with this concept.

# Two interesting concepts

– Let's see what concept do we get with all objects, and one with all attributes.
– Are there attributes common to all objects?
– Are there objects that share all attributes?

- **{John, Tom, Stella, Harry, Milli, Stephan, Steffi, Sara, Maya, Hamid, Don, Rosy, Tim, Mary X Φ }** is a concept.
- **{ Φ X Arts, Lit., Sports, Env. }** isa concept

# Formal Concepts

A formal context is a triple $(G, M, I)$ where $G$ and $M$ are two finite sets and $I$ is a binary relation between these sets such that $I \subseteq G \times M$.
All the elements of $G$ are called as *objects* and those of $M$ are called as *attributes* or *properties*.
The *derived operators* $A'$ and $B'$, for $A \subseteq G$ and $B \subseteq M$ are defined as

$$A' := \{m \in M | (g, m) \in I \text{ for all } g \in A\}$$
$$B' := \{g \in G | (g, m) \in I \text{ for all } m \in B\}$$

A formal concept is defined as a pair (A,B) if and only if

$$A \subseteq G, B \subseteq M, A' = B \text{ and } B' = A,$$

where $A$ is *extent* and $B$ is *intent* of the concept $(A, B)$.

# Concept Lattice

The *partial order* or the *subconcept-superconcept relation* in the lattice is defined as follows:
given concepts $(A_1, B_1)$ and $(A_2, B_2)$,

$$(A_1, B_1) \leq (A_2, B_2) \iff A_1 \subseteq A_2 \iff B_2 \subseteq B_1.$$

The ordered set of all the formal concepts represents a *formal concept lattice*.

# Top and Bottom Concepts

- Concept with all objects in the context.
- Concept with all attributes in the context.

# Access Graph

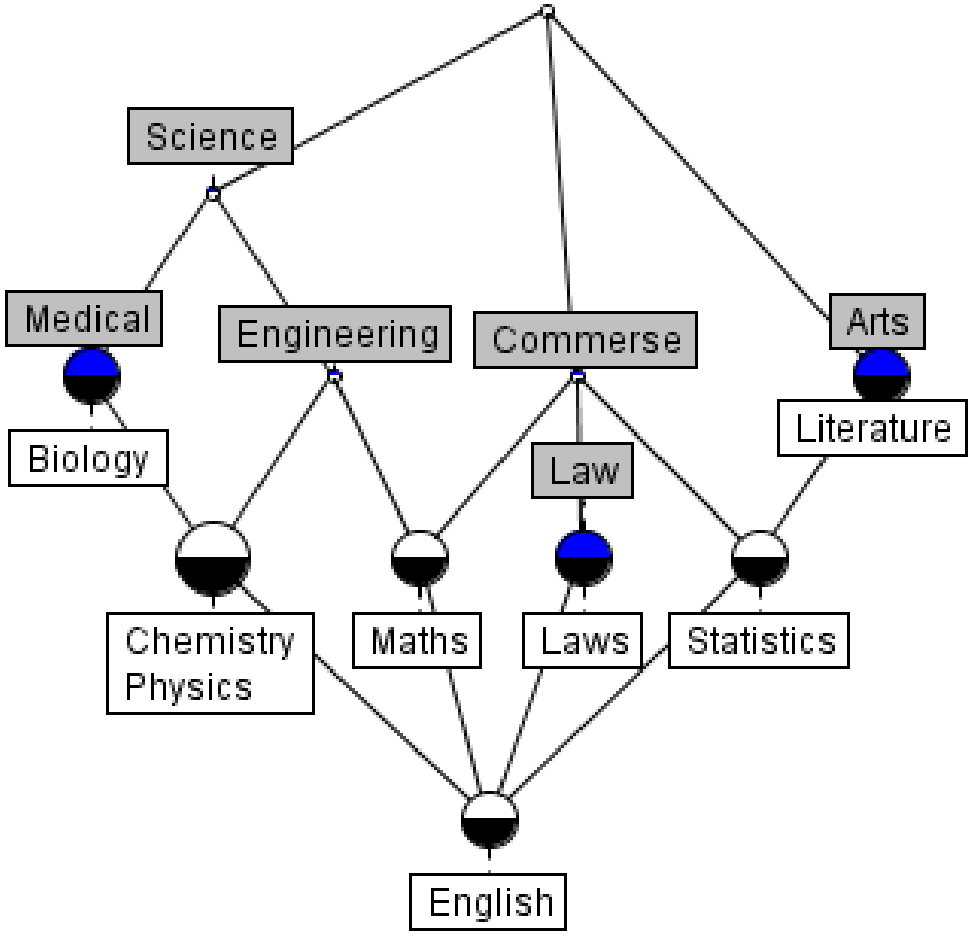
Context: topics of study applicable in different disciplines

Table: An Example for Concept Lattice

| | Engg. | Med. | Arts | Comm. | Law | Sci. |
|---|---|---|---|---|---|---|
| Physics | X | X | | | | X |
| Chemistry | X | X | | | | X |
| Biology | | X | | | | X |
| Maths | X | | | X | | X |
| English | X | X | X | X | X | X |
| Statistics | | | X | X | | |
| Laws | | | | X | X | |
| Literature | | | X | | | |

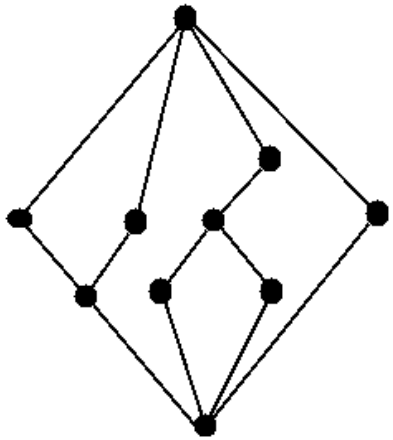
# The Concept Lattice

short-form notation: white boxes (objects) merge upwards, gray boxes (attributes) downwards

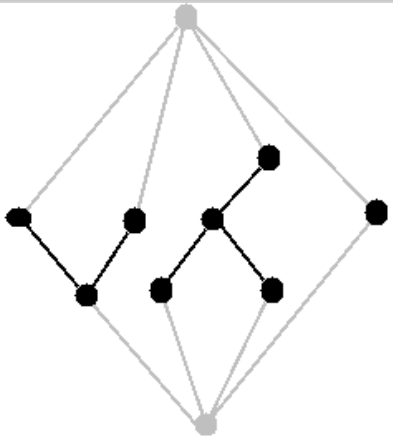


(a) Dot

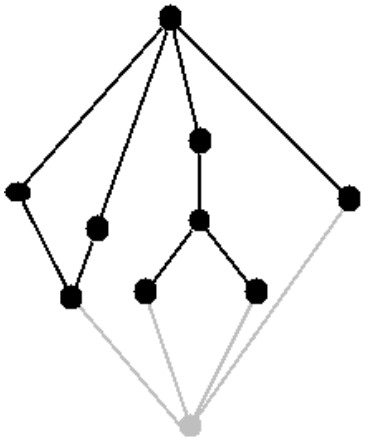
(b) Chain



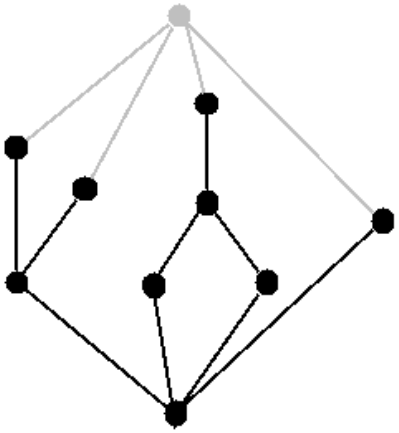
(c) Diamond



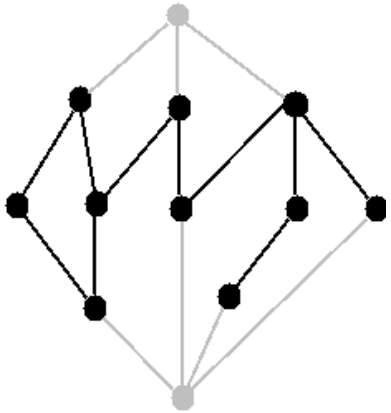
(d) Partitions



(e) Umbrella



(f) Inverted Umbrella



(g) Mountain Range

# Dot

A lattice which has $supremum_L = infimum_L = (A_c, M_c)$ is called *Dot* cohesion lattice.
Four types of *dot* structures can be observed
The dot lattice with a non-$\phi$ extent and a non-$\phi$ intent is highly cohesive as all methods share the same set of attributes.

# Thank you!