CS331 Autumn 2000 Mid-term Examination

Time: 2 hrs 30 mins | Total marks: 100

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***

1. *[7+1+2 marks]* Consider the NFA in Fig. 1. Assume $\Sigma = \{0, 1\}$.

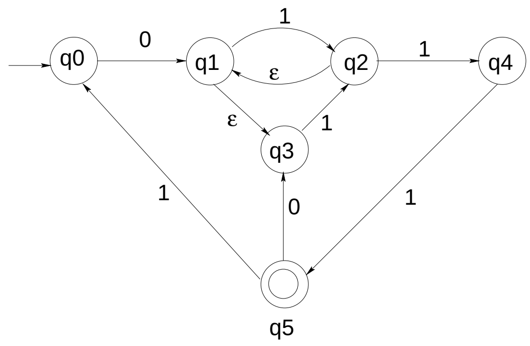

Figure 1:

   (a) Convert the NFA to a DFA. Show all steps clearly.

   (b) Describe as crisply as you can in English, the set L of strings accepted by the automaton.

   (c) Suppose we make states q_0, q_1, q_2, q_3 and q_4 in Fig. 1 as accepting states, and state q_5 as a non-accepting state. Is the language accepted by the resulting automaton $L^c = \Sigma^* - L$? Explain why.

2. *[15 marks]* Let Σ_1 and Σ_2 be disjoint alphabets. Let $L_1 \subseteq \Sigma_1^*$ and $L_2 \subseteq \Sigma_2^*$. Define Shuffle(L_1, L_2) as the set of strings $w \in (\Sigma_1 \cup \Sigma_2)^*$ with the following properties:

   - If all symbols in w that belong to Σ_1 are replaced by ϵ, the resulting string belongs L_2.
   - If all symbols in w that belong to Σ_2 are replaced by ϵ, the resulting string belongs to L_1.

   Intuitively, you take two strings, w_1 from L_1 and w_2 from L_2, and shuffle the symbols of w_1 and w_2 (like you shuffle two halves of a pack of cards) to get strings in Shuffle(L_1, L_2). As an example, if $L_1 = \{ab\}$ and $L_2 = \{c, cd\}$, Shuffle$(L_1, L_2) = \{cab, acb, abc, cdab, cadb, cabd, acdb, acbd, abcd\}$.

   Show that if L_1 and L_2 are regular, so is Shuffle(L_1, L_2).

3. *[5+5 marks]* Give CFGs for the following languages. You should also provide some informal argument (formal proofs not necessary) why you believe your CFG generates all strings in the desired language and nothing more.

   (a) $\{a^m b^n c^p d^q \mid m + n = p + q,\ m, n, p, q \geq 1\}$.

   (b) $\{a^m b^n \mid m, n \geq 1;\ n < m < 2n\}$.

4. *[7.5+7.5 marks]* Prove or disprove (a) and ((b) or (c)):

   (a) $\{0^{n+3k+2} 1^n \mid n \geq 0,\ k \text{ is any integer}\}$ is a regular language.

(b) $\{0^n 1^m \mid n \geq 1, m \geq n!\}$ is a CFL.

(c) $\{a^m b^n c^p d^q \mid m + q = n + p,\ m, n, p, q \geq 1\}$ is a CFL.

5. *[2+5+5+3] marks* Consider the grammar $G = (\{S, A\}, \{0, 1\}, P, S)$, where P consists of:

$$S \rightarrow A1S \mid SA1 \mid A$$
$$A \rightarrow 0 \mid 0A1$$

   (a) The grammar G is ambiguous. Find a shortest terminal string that has at least two derivation trees in G.

   (b) Convert the grammar to an unambiguous one.

   (c) Convert your unambiguous grammar to CNF. Minimize the number of additional variables introduced.

   (d) Starting from an unambiguous grammar, do you always get an unambiguous CNF grammar? Explain.

6. *[5 + 15 marks]* Fig. 2 shows a Mealy machine that converts an input stream of 0s and 1s to an output stream of as and bs. Note that the machine can output more than one symbol after reading a single input symbol.

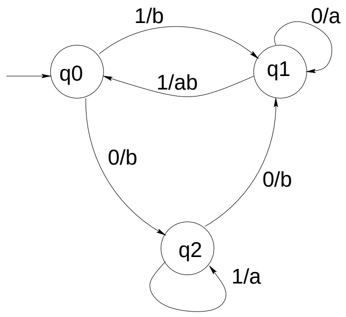

Figure 2:

   (a) Give a CFG for the set of strings that can be output by the Mealy Machine for all possible input streams.

   (b) Let L be the language generated by the CNF grammar $G = (\{S, A, B\}, \{0, 1\}, P, S)$ where P consists of the following: $S \rightarrow AS \mid 1$, $A \rightarrow 0 \mid BA$, $B \rightarrow 1$.
   For $w \in L$, define $F(w)$ be the output string generated by the Mealy Machine in Fig. 2 when w is fed as the input. Define $F(L)$ as $\bigcup_{w \in L} F(w)$. Show that $F(L)$ is context-free.

7. *[15 marks]* Define $\frac{1}{2}(L) = \{w \in \Sigma^* \mid \exists x \in \Sigma^*, |x| = |w| \text{ and } wx \in L\}$. In other words, $\frac{1}{2}(L)$

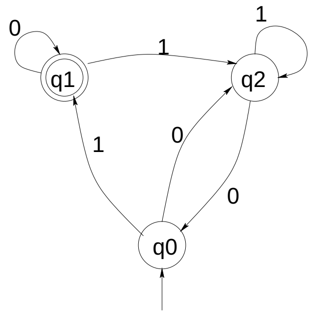

Figure 3:

is the set of first half of all strings in L. Consider the DFA in Fig. 3 representing a regular language L. Construct an NFA which accepts a string w iff only if $w \in \frac{1}{2}(L)$.