CS331 Autumn 2001 Mid-term Examination

Time: 2 hrs 30 mins | Total marks: 65

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/assignments may simply be cited.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***
- ***Do not copy solutions from others.***

1. *[8 + 5 + 7 marks]*

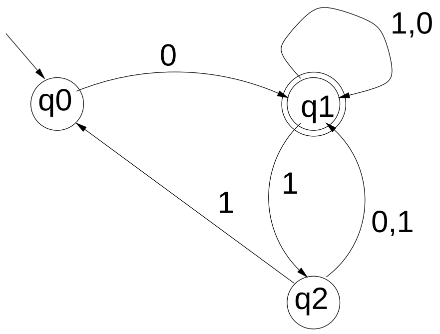

Figure 1:

Consider the NFA M in Fig. 1. Define $\Lambda^{(i)}(M)$ as the set of all strings $w \in \Sigma^*$ such that there exist *exactly i paths* labeled w that lead from q_0 to some state in F (different paths need not lead to the same state in F). Similarly, define $\Lambda(M)$ as the set of all strings $w \in \Sigma^*$ such that *all paths* from q_0 labeled w lead to some state in F. Clearly, $\Lambda^{(i)}(M) \subseteq L(M)$ for all $i > 0$. Similarly, $\Lambda(M) \subseteq L(M)$.

   (a) Show that $\Lambda^{(2)}(M)$ is regular by constructing an NFA/DFA.

   (b) Is $\Lambda(M)$ regular? If so, give a DFA for $\Lambda(M)$. Otherwise, give at least some intuitive reason for the non-regularlity of $\Lambda(M)$.

   (c) We wish to determine if $\Lambda(M) = L(M)$ for an arbitrary NFA M, possibly with ϵ-transitions. Suggest an algorithm for this without intersecting $\Lambda^c(M)$ and $L(M)$. Clearly explain why you think your algorithm gives the right answer for an arbitrary NFA.

2. *[5 + 5 marks]* Consider a PDA $A = (Q, \Sigma, \Gamma, \delta, q_0, X_0, F)$. Let $L(A)$ be the language accepted by the PDA by final states in F, and $N(A)$ be the language accepted by the empty stack. Suppose we further know that $\Gamma = \{X_0\}$.

   (a) Is $L(A)$ necessarily regular?

   (b) Is $N(A)$ necessarily regular?

   In each case, you must either

   - Give a procedure to construct a DFA/NFA/regular expression for the language, if you think the language is necessarily regular, or
   - Give a counterexample to show that the language can be non-regular.

3. *[10 marks]* Let P and Q be finite sets of non-negative integers. Define $F(P,Q) = \{n \mid n = p + q_1 + \dots q_k;\ p \in P;\ k \geq 1,\ q_1, \dots q_k \in Q\}$. In other words, $F(P,Q)$ is the set of integers formed by adding one or more (possibly repeated) elements of Q to an element of P.

   Now consider the DFA M in Fig. 2. We wish to show that there exists an integer $r > 0$ and sets $P_1, P_2, \dots P_r$ and $Q_1, Q_2 \dots Q_r$ (each a finite set of non-negative integers) such that the following conditions hold.

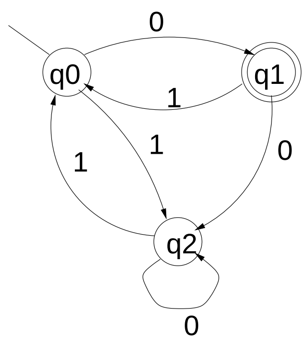

Figure 2:

- For every $w \in L(M)$, $|w| \in \bigcup_{i=1}^{r} F(P_i, Q_i)$.
- For every $n \in \bigcup_{i=1}^{r} F(P_i, Q_i)$, there exists a string $w \in L(M)$ such that $|w| = n$.

Determine $P_1 \dots P_r$ and $Q_1 \dots Q_r$ for M in Fig. 1. Give an informal argument why both the above conditions hold for your choice of sets $P_1, \dots P_r$ and $Q_1 \dots Q_r$.

4. *[5 + 5 + 5 marks]* Which of the following are true (give solutions to any three)? Give justifications in each case. A simple "True" or "False" answer will fetch no marks.

   (a) $\{xy \mid x, y \in (\mathbf{0} + \mathbf{1})^*,\ |x| = |y|,\ x \neq y\}$ is not a CFL.

   (b) $\{w \mid w \in (\mathbf{0} + \mathbf{1} + \mathbf{2})^*,\ N_0(w) = N_1(w) = N_2(w)\}$, where $N_i(w)$ is the number of i's in w, is a CFL.

   (c) $\{0^m 1^n \mid m \geq n \geq 0\}$ is a regular language.

   (d) $\{0^m 1^n 2^r 0^t \mid m + n + t = r,\ m, n, r, t\ \geq\ 0\}$ is not a CFL.

5. *[5 + 5 marks]* Consider the PDA shown in Fig. 3. We know that $N(M)$, the language recognized by the

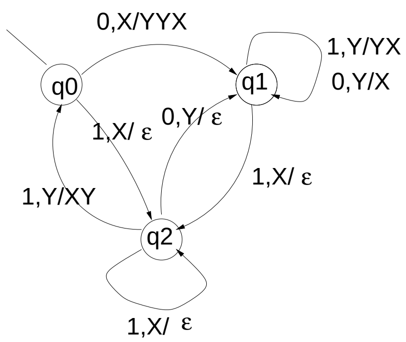

Figure 3:

empty stack, is a CFL. Now suppose that the stack is bounded to, say, size k. Thus, whenever the total no. of symbols pushed into the stack becomes $n > k$, the top $n - k$ symbols are lost, and the PDA operates from the remaining symbol on top of the stack. For example, in Fig. 3, if the size of the stack is 2, we have: $(q_0, 0111, X) \vdash (q_1, 111, YX) \vdash (q_1, 11, XX) \vdash (q_2, 1, X) \vdash (q_2, \epsilon, \epsilon)$. Thus, 0111 is accepted by the PDA with the bounded stack. Let $N^{(k)}(M)$ be the language accepted by the PDA by emptying a stack of size k.

   (a) Modify Fig. 3 (with justifications) to recognize $N^{(2)}(M)$ by the empty stack. The PDA you construct must not have any restrictions on the stack size.

   (b) It turns out that for any $k > 0$, $N^{(k)}(M)$ is a regular language. Give an upper bound on the no. of states of a DFA for $N^{(k)}(M)$. Justify your bound.