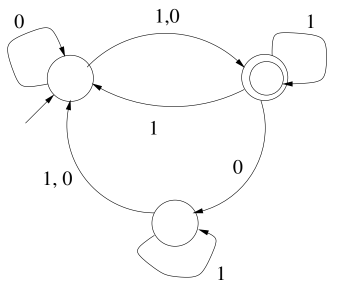
CS331 Assignment #1

Due Date: Sept 2, 2002

Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.

<u>Do not copy solutions from others.</u>

1. *[10+10 pts]* For purposes of this question, we'll assume all NFAs are free of ϵ-transitions. In class, we saw that a string w is accepted by an NFA $N = (Q, \Sigma, \delta, q_0, F)$ if and only if $\hat{\delta}(q_0, w) \cap F \neq \phi$. Suppose, we modify the definition of acceptance as follows:
String w is accepted by N if and only if $\hat{\delta}(q_0, w) \subseteq F$ and $\hat{\delta}(q_0, w) \neq \phi$.
Let the set of strings thus accepted by N be denoted by $L^{\#}(N)$.



Non-deterministic Automata

Figure 1:

   (a) Construct a DFA for $L^{\#}(N)$ for the NFA N shown in Fig. 1.

   (b) Given an arbitrary NFA N, is $L^{\#}(N)$ always regular? If your answer is in the negative, you must give an N and argue why $L^{\#}(N)$ is not regular. In case of an affirmative answer, you must outline a proof of regularity of $L^{\#}(N)$.

2. *[5+5+10+15 pts]* We have seen that regular languages are closed under *substitution*. In this question, we will attempt to see what happens when we subject regular languages to *inverse substitution*. Of course, we need to define *inverse substitution* precisely. For purposes of this question, consider the following definition:

   Let Σ_1 and Σ_2 be disjoint alphabets. Let $s : \Sigma_1 \rightarrow 2^{\Sigma_2^*}$ be a substitution function. As seen in class, this naturally gives rise to $\hat{s} : \Sigma_1^* \rightarrow 2^{\Sigma_2^*}$ mapping strings in Σ_1 to languages in Σ_2. Finally, this gives rise to $\tilde{s} : 2^{\Sigma_1^*} \rightarrow 2^{\Sigma_2^*}$ mapping languages in Σ_1 to those in Σ_2. In particular, $\tilde{s}(L) = \bigcup_{w \in L} \hat{s}(w)$ for all $L \subseteq \Sigma_1^*$.

   Now define the inverse substitution function $\tilde{s}^{-1} : 2^{\Sigma_2^*} \rightarrow 2^{\Sigma_1^*}$ as follows:
   $\tilde{s}^{-1}(L) = \{x \mid x \in \Sigma_1^* \text{ and } \hat{s}(x) \subseteq L\}$ for all $L \subseteq \Sigma_2^*$.

   (a) Show that $\tilde{s}(\tilde{s}^{-1}(L)) \subseteq L$.

   (b) Give an example (i.e., specify Σ_1, Σ_2, s and L) where $\tilde{s}(\tilde{s}^{-1}(L)) \neq L$.

   (c) Let $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c\}$. Let $L = (\mathbf{a} + \mathbf{b} + \mathbf{c})^*\mathbf{c}^+$, and the substitution function be defined as $s(0) = (\mathbf{a}^*\mathbf{b}^*)^*\mathbf{c}^+$ and $s(1) = \mathbf{c}^+(\mathbf{a} + \mathbf{b})^*$. Show that $\tilde{s}^{-1}(L)$ is regular.

   (d) Generalizing the result of the previous subquestion, show that if L is regular and $s(a)$ is regular for each $a \in \Sigma_1$, $\tilde{s}^{-1}(L)$ is also regular.
   [Hints: Recall how we proved closure of regular languages under inverse homomorphism. Try to construct an NFA for $\tilde{s}^{-1}(L)$ with appropriate acceptance condition.]

3. *[10 + 10 pts]* Show that the following languages are not regular

   (a) $\{0^i 1^j \mid gcd(i, j) = 1\}$

   (b) $\{0^i 1^j 2^k \mid i > j \text{ or } j > k\}$

4. *[10 + 10 pts]* Show that the following languages are regular

   (a) The set of all strings w in $(\mathbf{0} + \mathbf{1})^*$ that satisfy one of the following conditions:

- If w contains a sequence of three or more 0s, then every such sequence of 0s is followed (not necessarily immediately) by a sequence of three or more $1's$.
- There doesn't exist any sequence of three or more 0s in w

(b) $\{0^{mk}1^{nk} \mid 0 \leq k \leq 5;\ m, n, k \in \mathbf{N}\}$.