CS331 Autumn 2002 Mid-term Examination

Time: 2 hrs 30 mins | Total marks: 55

- ***The exam is open book and notes.***
- ***Results/proofs covered in class/problem sessions/assignments may simply be cited.***
- ***Unnecessarily lengthy solutions will be penalized.***
- ***If you need to make any assumptions, state them clearly.***

Do not copy solutions from others.

1. *[3+5+7 marks]* Let $A = (Q, \Sigma, \delta_A, q_A, F_A)$ and $B = (Q, \Sigma, \delta_B, q_B, F_B)$ be DFAs with the same set of states over the same alphabet. Define $L(A\&B) = \{w \mid w = x.y,\ w, x, y \in \Sigma^*,\ |x| = |y|,\ \hat{\delta_B}(\hat{\delta_A}(q_A, x), y) \in F_B\}$. In other words, we start from state q_A in A and apply the first half, x, of w. Suppose this takes us to state q. We now apply the second half, y, of w, starting from q in B. If this takes us to a final state in B, we say that $w = xy$ is in $L(A\&B)$.

   (a) Prove that $L(A\&A)$ is regular for all DFAs A.

   (b) For arbitrary DFAs A and B with the same set of states and same alphabet, is $L(A\&B)$ necessarily regular? An affirmative answer must be accompanied by a proof of regularity. A negative answer must be accompanied by a counterexample, i.e., DFAs A and B and a proof (unless already covered in class) that $L(A\&B)$ is not regular.

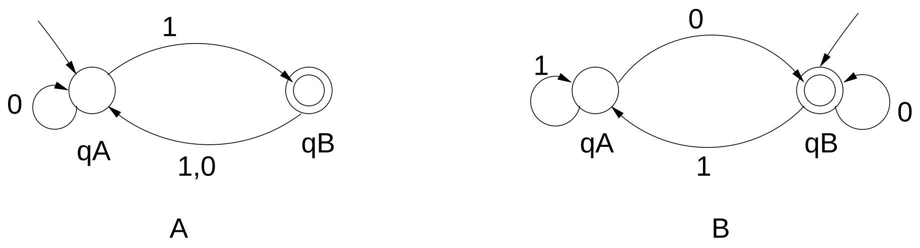

Figure 1:

   (c) Consider DFAs A and B in Fig. 1. The correspondence between states in A and B is through their labels. Construct a CFG for $L(A\&B)$.

2. *[7 marks]* Let $L_1 = \{0^m \mid m \text{ is a prime number } \geq 3\}$. It is known that L_1 is not regular, although L_1^* is. Construct a DFA M for L_1^*. You must prove that $L(M) = L_1^*$.

3. *[10+3 marks]* Consider the language $L = \{0^m 1^n 0^n 1^m \mid m, n \geq 1\}$.

   (a) Show that there does not exist any DPDA P with only one stack symbol that accepts L by emptying its stack.
   [Remembering pigeons fighting over holes may be of help here!]

   (b) Construct a DPDA P with only two stack symbols such that $N(P)$, the language accepted by P by emptying its stack, is L.

4. *[5+5 marks]*

   (a) Show that the language $L = \{0^i 1^j \mid j \neq i^n \text{ for any natural no. } n\}$ is not regular.

   (b) Give a CFG or PDA (deterministic or non-deterministic) for the language $L = \{0^i 1^j 2^k 3 \mid i, j, k \geq 1,\ j + k - i \text{ is a positive even number}\}$. If you are giving a PDA as a solution, indicate whether it accepts by final state or by empty stack.

5. *[5+5 marks]* Consider the grammar $S \to 0SA \mid 0A$, $A \to AA \mid AS \mid 1$, where the set of terminals is $\{0, 1\}$, the set of non-terminals is $\{S, A\}$ and S is the starting symbol.

(a) Show that the above grammar is ambiguous by giving a string which has two leftmost derivations. You must show the two derivations.

(b) Can you convert this to a non-ambiguous grammar? Give at least an informal argument justifying your answer.