CS331 Practice Problem Set #1

1. *[10 pts]* When converting NFAs to DFAs using the subset construction, we noted that the number of states in a DFA can become exponentially larger than the number of states in the corresponding NFA. However, the DFA that results from the subset construction is not necessarily a minimal DFA (least no. of states) for the language; hence it can be subjected to the DFA minimization algorithm studied in class. The question that arises naturally is whether NFAs really help in representing languages exponentially more compactly than minimized DFAs. The answer is "Yes", and in this problem you are required to show this.

   Give a class of regular languages L_n (parameterized by $n \in \mathbf{N}$), such that for every $n > 0$:

   - There exists an NFA with n states recognizing L_n.
   - A minimal DFA for L_n has c^n states for some $c > 1$.

2. *[10 pts]* Let Σ be an alphabet. Let $L_1, L_2 \subseteq \Sigma^*$ be languages such that L_1 is regular and L_2 is not. In addition, we know that:

   - L_2 is not $\subseteq L_1$
   - L_1 is not $\subseteq L_2$
   - $L_1 \cap L_2$ is an infinite set.

   We are interested in the regularity of $L_1 \cap L_2$.

   If you think $L_1 \cap L_2$ is necessarily regular or necessarily non-regular, give a proof to support your claim. In case you believe that $L_1 \cap L_2$ may or may not be regular depending on the choice of L_1 and L_2, give two examples: one in which $L_1 \cap L_2$ is regular, and the other in which it is not.

3. *[5+5+5+5+5 pts]* Which of the following languages are regular? Justify your answer in each case.

(a) $\{x \mid x \in (\mathbf{0}+\mathbf{1})^*$, number of "01"s in x = number of "10"s in $x\}$.

(b) L^* where $L = \{0^{i^2} \mid i \geq 1\}$.

(c) $\{wxx^Ry \mid w, x, y \in (\mathbf{0}+\mathbf{1})^+\}$, where x^R is the string x reversed.

(d) $\{x \mid x \in (\mathbf{0}+\mathbf{1})^*, x \neq x^R\}$.

(e) $\{0^n 1^{n+2k} \mid n \geq 1, k \in \mathbf{Z}\}$, where $\mathbf{Z}$ is the set of integers.

4. *[10+10+10 pts]* Let L be the language accepted by the DFA in Fig. 1.

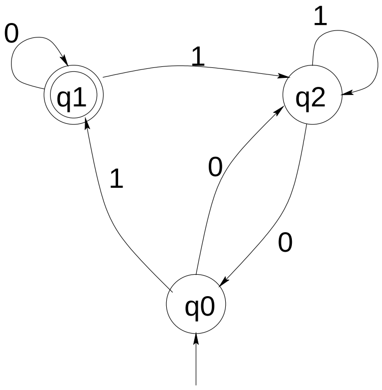

Figure 1:

Construct DFAs/NFAs for the following languages:

(a) $\{x \mid xx^R \in L\}$, where x^R is the string x reversed.

(b) $\{x \mid \exists y \in (\mathbf{0}+\mathbf{1})^*,\ |x| = |y|,\ yx \in L\}$, *i.e.,* the second halves of all strings in L.

(c) $\{x_1x_2 \mid x_1 \text{ and } x_2 \in (\mathbf{0}+\mathbf{1})^*,\ x_2x_1 \in L\}$

Give at least some informal argument in each case why you think your construction recognizes the desired set of strings and nothing more.