CS331 Assignment #1

Due Date: Oct 13, 2003

Be brief, complete and stick to what has been asked. If needed, you may cite results/proofs covered in class without reproducing them.

Do not copy solutions from others
Penalty for copying: FR grade

1. *[10+10 pts]* Consider the language $L = \{0^n 1^{n+2k} \mid n \geq 1,\ k \geq 0\}$ over the alphabet $\Sigma = \{0, 1\}$. Let $L_1 = \Sigma - L$ be the complement of L.
   (a) Show using Myhill-Nerode Theorem and closure properties of regular languages that L_1 is not regular.
   (b) Show that L_1 satisfies the Pumping Lemma for regular languages.
2. *[15 pts]* Let L be any infinite subset of $\mathbf{0}^*$. Show that L^* is regular.
3. *[5+5+5 pts]* Let L be a regular language over the alphabet Σ. Are the following languages necessarily regular?

   [In each case, you must give proof of regularity or non-regularity. For proof of regularity, you can either use Myhill-Nerode Theorem, or outline the construction of a (non)deterministic finite automaton or regular expression, and then show that the language represented by the automaton or expression is indeed the given language. For proof of non-regularity, you can use Myhill-Nerode Theorem, Pumping Lemma or provide a counter-example.]

   (a) chop(L) $= \{w \mid \exists x, y, z \in \Sigma^*,\ x.y.z \in L,\ w = x.z\}$.
   (b) pal(L) $= \{w \mid \exists x \in L,\ w = x.x^R\}$. Here x^R is the string x reversed.
   (c) log(L) $= \{w \mid \exists u \in \Sigma^*,\ |u| = 2^{|w|}, w.u \in L\}$.

4. *[10 pts]* Let $\Sigma = \{a, b, c, d, e, f\}$. For $w \in \Sigma^*$, let $N_i(w)$ denote the number of i's in w. Consider the language $L \subseteq \mathbf{a}^*.(\mathbf{b}+\mathbf{c})^*.(\mathbf{d}+\mathbf{e})^*.\mathbf{f}^*$ such that for each string $w \in L$, $N_a(w) + N_d(w) + N_e(w) = N_b(w) + N_c(w) + N_f(w)$, $N_a(w) \leq N_b(w) + N_c(w)$, and $N_f(w) \leq N_d(w) + N_e(w)\}$.

   Give a CFG G for L. You must also give justification why every string in $L(G)$ is in L and vice versa.

5. *[10+5+5 points]* Consider the PDA M depicted in Fig. 1. The inital symbol on the stack is Z and the initial state is A. Let $N(M)$ denote

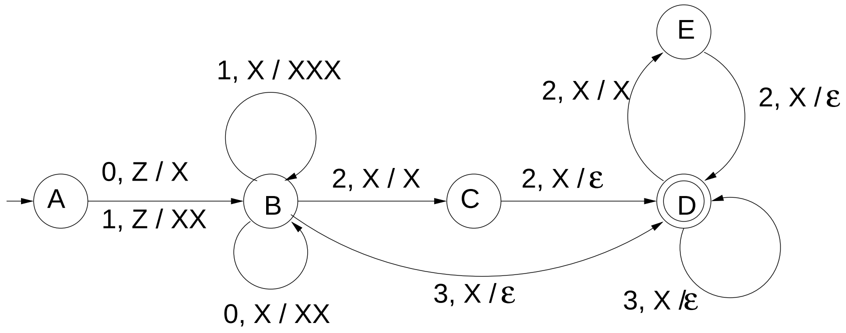

Figure 1:

the language accepted by M by an empty stack, and let $L(M)$ denote the language accepted by M by final state. It is easy to see that both $L(M)$ and $N(M)$ are subsets of $((\mathbf{0}+\mathbf{1})^+(\mathbf{2}+\mathbf{3})^+)$.

   (a) Give a CFG G_1 for $L(M)$. Show that every string in $L(G_1)$ is accepted by PDA M by final state and vice versa.

   (b) Give a CFG G_2 for $N(M)$. You need not show that $L(G_2) = N(M)$.

   (c) We wish to modify M *minimally* to obtain a PDA M' such that $N(M') = L(M') = L(M)$. Draw PDA M' and give justifications for your construction.

   You must not introduce more than 2 new states and 2 new transitions to PDA M. Similarly, you must not delete more than 1 state or 1 transition from M. You are however allowed to change the labels of existing transitions, if needed.