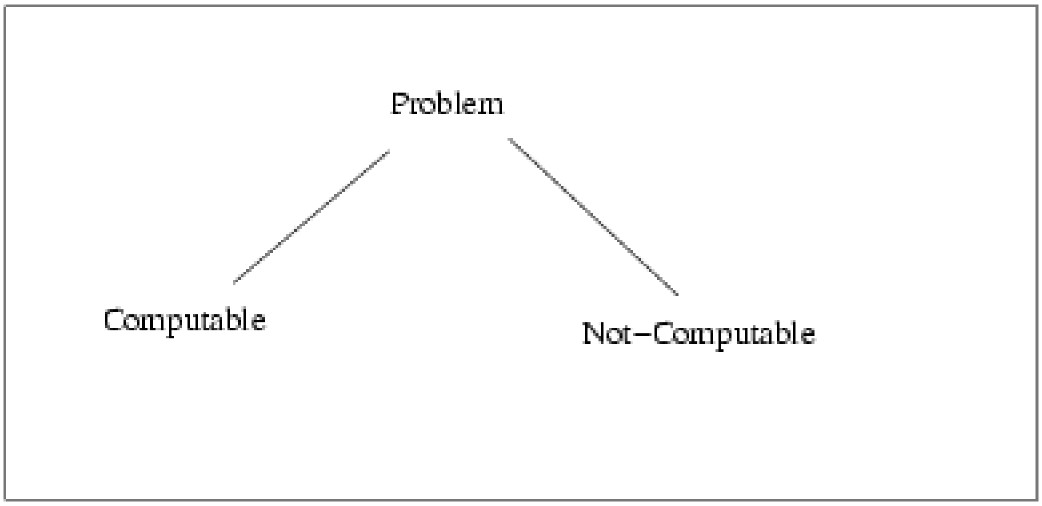
CS331 : THEORY OF COMPUTATION

Lecture 1

July 22, 2003

INTRODUCTION

Computations are designed to solve problems. Programs are descriptions of computations written for execution on computers. The field of computer science is concerned with the development of methodologies for designing programs, and with the development of computers for executing programs. It is therefore of central importance for those involved in the field that the characteristics of programs, computers, problems, and computation be fully understood.
Every problem that we face in the real world is either computable or not-computable.



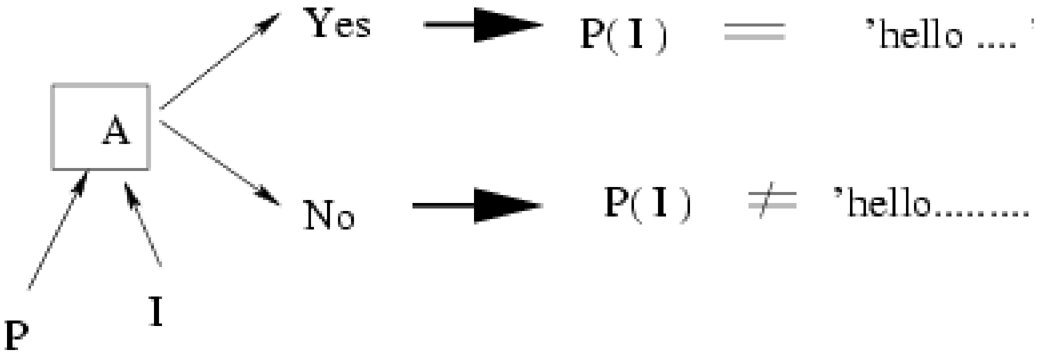
Example

Problem :- Is there any algorithm to find out whether a program outputs 'hello' as its first five characters ?

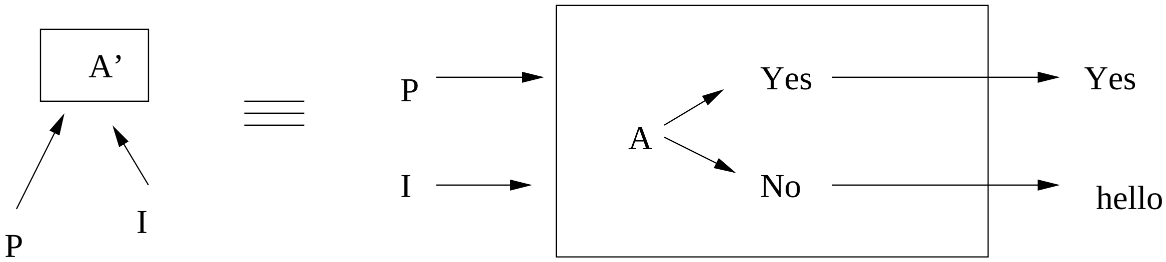
Solution :- NO

Proof :-

Let such an algorithm **A** exist. It takes two inputs, a program **P** and input given to **P** i.e. **I** and outputs 'yes' if **P** outputs 'hello' as its first five characters and 'no' otherwise. So **A** can be represented as :-



Let **A'** be an algorithm such that :-

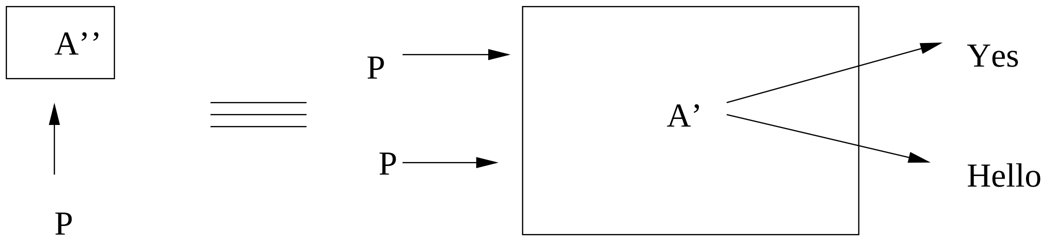


If **A** exist then **A'** must exist.

Notice that :

$$A'(P,P) = \text{Yes} \Leftrightarrow A(P,P) = \text{Yes} \Leftrightarrow P(P) = \text{'hello....'}$$
$$A'(P,P) = \text{'hello......'} \Leftrightarrow A(P,P) = \text{No} \Leftrightarrow P(P) \neq \text{'hello....'}$$

Let **A"** is an algorithm such that :-



If **A'** exist then **A"** must exist.

A" is a valid input (argument) for **A"**. But it leads to a **contradiction**.

If

$$\begin{gathered} \text{A''}(\text{A''}) = \text{Yes} \\ \Leftrightarrow \text{A}(\text{A'',A''}) = \text{Yes} \\ \Leftrightarrow \text{A''}(\text{A''}) = \text{'hello.....'} \end{gathered}$$

Contradiction

If

$$\begin{gathered} \text{A''}(\text{A''}\ ) = \text{'hello.....'} \\ \Leftrightarrow \text{A}(\text{A'',A''}) = \text{No} \\ \Leftrightarrow \text{A''}(\text{A''}) \neq \text{'hello.....'} \end{gathered}$$

Contradiction

SO
A" can't exist. Hence **A'** can't exist. Therefore **A** can't exist.
Result :-
There doesn't exist any algorithm to find whether a program outputs 'hello' as its first five characters.