CS331 : THEORY OF COMPUTATION

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1 INTRODUCTION

Suppose, we have a Problem 'P', it can be either computable or non-computable. If we want to perform computation of P in a Sequential Machine, we can find out :

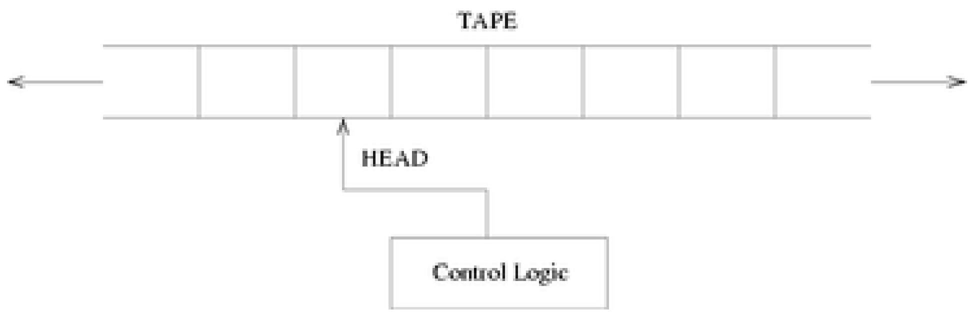
- Space : Memory required for computation of P.
- Time : Number of steps for performing computation of P.

2 An Example Of a Computing Machine

Consider a machine M which is coupled through a 'head' to an infinitely long storage register, called the 'tape'. The tape is divided into cells, so that each cell stores a single symbol.
The head is capable of performing 3 operations :-

1. Reading the symbol contained in the cell being scanned.
2. Writing a new (not necessarily distinct) symbol in the scanned cell.
3. Shifting the head one cell in either direction. For convenience, we shall assume that the tape is stationary while the head is moving.

Such a machine is called a *Turing Machine* after A.M. Turing. The machine receives its inputs by reading the pattern of symbols written on the tape. Its control logic has the dual function of providing the head with the new symbols to be written on the tape and shifting the head in either direction. At the end of the computation, a new pattern of symbols is written on the tape. This pattern is the final objective of the entire computation. Following figure shows what a Turing Machine looks like :



3 Alphabet and String

3.1 Alphabet

An alphabet is a finite set of symbols. An alphabet is denoted by $\sum$.
Example of an Alphabet : $\sum = \{$a , b , c$\}$.

3.2 String

A string is a finite sequence of symbols from $\sum$.
For example, for an alphabet $\sum = \{$a , b , c$\}$, *abca* is a string.

The Length of a string is the number of symbols composing ths string.
For a string say w , its length is denoted by $|w|$.
For example, *abcab* has length 5.

The empty string denoted by ϵ , is the string consisting of zero symbols.
Thus $|\epsilon| = 0$. It is independent of any $\sum$ as it doesn't use any symbol.
It is used in string operations just like any other string.

For an alphabet, number of strings is countably infinite. HOW ?
For example: We can map strings over $\{0, 1\}$ to Natural Numbers by considering the strings in the order of their lengths.

According to length :
1 —–> 0 , 1 (2 strings of length 1)
2 —–> 00 , 01 , 10 , 11 (4 strings of length 2)
—–
—–
—–
i —–> 2^i strings of length i

For proving that these strings are countably infinite, we have to form the bijection:
f_1 = strings on $\{0, 1\}$ —-> N

$$f_1(string) = 2^i + \beta - 1 \tag{1}$$

where :
i = length of the string.
β = Decimal value of the string.
It can be proved that f_1 is an injection.

f_2 = N —> strings on $\{0, 1\}$

$$f_2(D) = string \tag{2}$$

For f_2 select the maximum value of i such that
$(2^i - 1) \leq D$
and
$m = (D - 2^i - 1)$
Then , get binary value of m and add 0's before it to make it of length i.
It can be proved that f_2 is an injection.
So, we can define a Bijection from $\{strings\}$ to N.

4 Language

A Language is a set of strings over an alphabet $\sum$.
It is denoted by the symbol $\sum^*$.

For example, if $\sum = \{0, 1\}$; $\sum^* = \{\epsilon, 0, 1, 00, 01, 10, 11, ...\}$.
Note that $|\sum^*| = |N|$, meaning $\sum^*$ is countably infinite.

When $\sum = \{a, b, c, ..., space, *, +, -, 1, 2, 3, ...\}$ i.e. all characters that can be typed using the keyboard :
a typed $program$ = a string over $\sum$
So, we can see that JAVA $\subset \sum^*$