CS331 : THEORY OF COMPUTATION

Instructor : Prof. Supratik Chakraborty
C.S.E. Department, I.I.T. Bombay

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1 Notations used

• $\sum$: finite set of symbols. Example : {a , b , c}.
• w : string formed from symbols of $\sum$. Example : aabcab,0100013
• $\sum^*$: { w | w is a string on $\sum$ and $|w| \in$ N }
In other words $\sum^*$ is a set of strings formed from symbols of $\sum$ and length of each string maps to natural numbers
• $\sum^i$: { w | $w \in \sum^*$ and $|w|$ = i } i.e set of all strings of length i formed from symbols of $\sum$

So we have
$\sum^* = \sum^0 \bigcup \sum^1 \bigcup \sum^2 \bigcup \sum^3$ ....

2 Languages

L $\subseteq \sum^*$. We call L as a language.
Ex:- $\sum$ = {a,b,c,.....,z,ξ } "It ξ is ξ good" is a string of language L

A particular language is a certain subset of $\sum^*$.

$|\sum| \geq 1$
$|\sum^*|$ = |N|

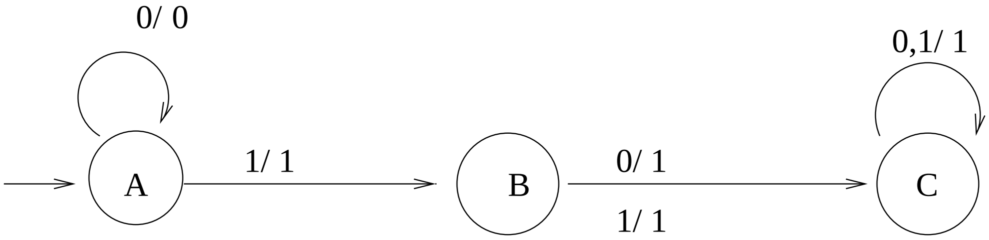
Number of languages on $\sum$ = all subsets of $\sum^*$ = $|2^S|$ =R,where S=$\sum^*$

Question: Find list of all languages formed over $\sum = \{0\}$
Solution: Since we can enumerate all the strings formed over $\{0\}$, thus $|\sum^*| = |N|$. The list of all languages over $\sum$ will have same cardinality as $|2^{\sum}| = |R|$, and hence cannot be enumerated (since we know that the set of real number is not enumerable).
We study language problem because we can cast many problems into the pattern of language problems.

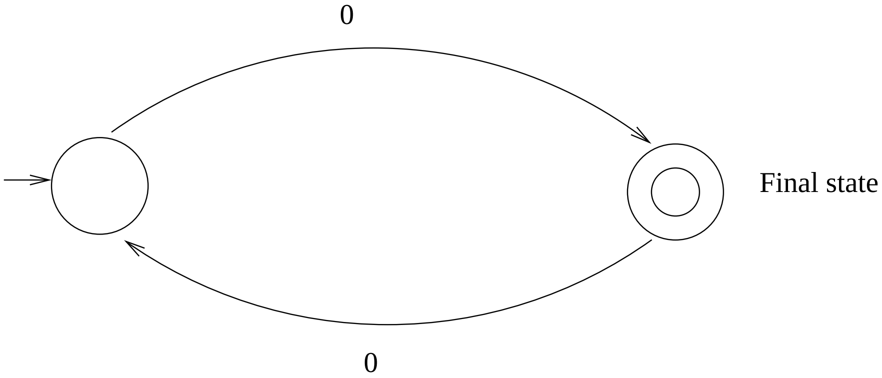
Example:Given n,find n^2+2n+3.
Solution:We cast this problem in language and string.
$\sum = \{0, 1\}$
L = { 0^n10^p | n≥1 where p=n^2+2n+3.}
Then start examining strings of the form : 0^n10^m for m≥0
If we find an m such that m=n^2+2n +3, then that string belongs to L.
Thus strings in L represent solutions to the problem according to number of 0's on the left side of symbol 1 and on its right side.
So we have encoded the problem and solution in the language and compare parts of the strings to get solution.
We can encode the problem and solution in many different ways but then the comparison to be performed can be of varying complexity.

Examples:

1:



L = { w | $w \in$ {0,1} , w has at least one 1 }
Here the number to the right of the '/' symbol represents the output. The output 1 signifies that the the string input so far is a member of L.
2:
To find strings over {0} of odd length



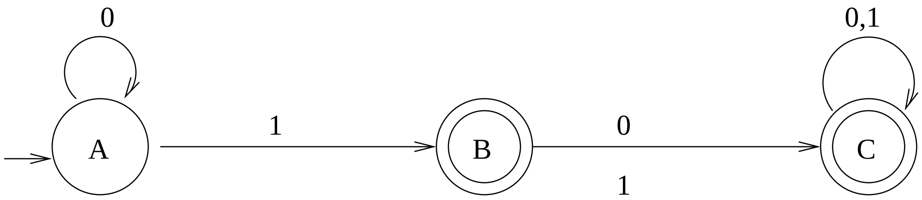
Here we have dropped the output and used the state achieved by machine at the end of input to decide whether string is of even or odd length. The state marked with a double circle is called a final state or an accepting state. The string is said to be in L if on inputting that string completely, the automaton is in a final state.

3 DETERMINISTIC FINITE AUTOMATON

NOTATION USED:
Q : finite set of states
$\sum$: alphabet
$s_0 \in$ Q : starting state.
F$\subseteq$ Q: set of accepting states.
δ:Q$\times$ $\sum$–>Q : transition function

In words, a deterministic finite automaton M consists of a finite set of states and a set of transitions from state to state that occur on input symbols chosen from an alphabet $\sum$

Example:



Q = {A,B,C}
$\sum$ = {0,1}

s_0 = A
F = {B,C}
The transition function δ maps a state and a symbol to some state.
Example:
δ(A,0)=A
δ(A,1)=B,etc

We denote a finite automaton by a 5-tuple as follows : M = $\{Q, \sum, \delta\, s_0, F\}$ defining a language denoted as L(M).

If F is complemented
M$'$ = $\{Q, \sum, \delta\, s_0, F'\}$
then $L(M') = L(M)'$ i.e. complement of L(M)

We extend the transition function so that it applies on a state and a string rather than state and symbol. We define a function $\hat{\delta}$ from Q$\times \sum$ to Q. This function will return the state after reading the given string from specified initial state. In other words, $\hat{\delta}$(q,w) is the unique state p such that there is a path in the transition diagram from q to p labelled w and q is starting state. Hence we have.
1:$\hat{\delta}$(q,$\in$) = q
2:$\hat{\delta}$(q,wa) =$\delta(\hat{\delta}(q, w), a)$
where:
a = input symbol
q = starting state
w = applied string

Language recognized by M is expressed as
L(M)=$\{w | w \in \sum^*$ and $\hat{\delta}(s_0, w) \in F\}$