CS628 Homework #3

Max marks: 55 **Due April 17, 2004**

- *You must take a printout of this homework, and fill in your answers in the spaces provided. You must turn in the sheets filled with your answers.*
- *You may work out the answers separately and then fill in* ***all*** *the steps and the final solution in the spaces provided.*
- **Please use both sides of your printout sheets for filling in your answers.**
- *Be brief, complete and stick to what has been asked.*
- *If needed, you may cite results/proofs covered in class without reproducing them.*
- ***Do not copy solutions from others. Penalty for suspected copying: FR grade***

In this homework, we will deal with automata describing the behaviour of systems. For our purposes, an automaton $\mathcal{A}$ consists of a set of states, Q, a set of edges $E \subseteq Q \times Q$, a starting state $s_0 \in Q$, a set of error states $S_e \subseteq Q$, an alphabet Σ of actions, and an edge labeling function, $\lambda : E \to 2^{\Sigma}$, that labels each state with one or more actions. An infinite sequence of actions, or a *trace*, is accepted by the automaton if it causes the automaton to run through an infinite sequence of states without visiting any error state. The language of the automaton is the set of traces accepted by the automaton. An automaton is said to describe the behaviour of a system if its language is the same as the sequences of events describing the interaction of the system with its environment.

1. *[5 marks]* Draw an automaton describing the behaviour of a C-element with inputs b and c and output d. You may assume that the values of b, c and d are identical (all 0 or all 1) in the start state. You **must not** use edge labels $b+$, $c-$, etc in your automaton. Instead, you must use edge labels b, c and d, denoting events on these signals. Label the states of your automaton using numbers 1, 2, 3, etc.

   Your automaton must accept all traces except those that cause *glitches* on the output of the C-element. A glitch is an output change that is enabled and then disabled before the output actually changes. For example, if the values of b,c and d are all identical in the initial state, your automaton must accept the sequence of events $bbbbbb(cbd)^{\omega}$, where $(cbd)^{\omega}$ represents an infinite repetition of cbd. Similarly, it must not accept the sequence $bccd(bcd)^{w}$ since an event on d is enabled after the first event on c but disabled after the second event on c, resulting in a glitch.

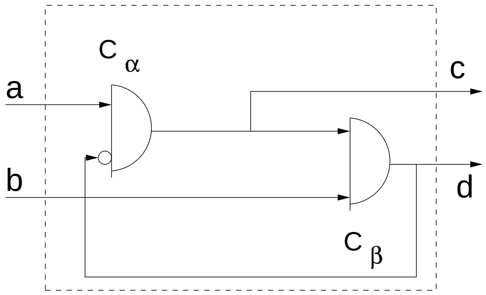

Figure 1: A circuit built with C elements

2. We wish to draw an automaton describing the behaviour of the circuit shown in Fig. 1. This automaton must be derived by composing automata for the individual C-elements. Initially, all of a, b, c and d have the same value (all 0 or all 1). Clearly, the automaton drawn in Question 1 above describes the behaviour of C-element C_β in Fig. 1.

   (a) *[3 marks]* Noting that one of the inputs of C-element C_α in Fig. 1 is inverted, and keeping in mind that all of a, b, c and d have the same value to begin with, draw an automaton describing the behaviour of C_α. As in Question 1, you must label the edges in your automaton only by a, d and c, and not by $a+$, $d-$, etc. Label the states of this automaton using upper case letters A, B, C, etc.

(b) *[12 marks]* Compose the automata in Question 1 and Question 2a to get the automaton of the circuit shown in Fig. 1. Each state of your automaton, except the error states, **must be** labeled with a pair (i, j) where i denotes the state in which C_α is, and j denotes the state in which C_β is. Error states of the composed automaton may simply be labeled Err. As in Question 1, the language of your resulting automaton must be the set of event sequences describing the interaction of the circuit in Fig. 1 with its environment, without producing a glitch at an output of the circuit.

3. Suppose the circuit in Fig. 1 was designed to implement the following process:
   $P \;=\; (a? \,||\, b?) \;\rightarrow\; c! \;\rightarrow\; d! \;\rightarrow\; P$

   (a) *[3 marks]* Draw an automaton representing the behaviour of the environment of process P. Label the states of this automaton using α, β, etc.

   (b) *[7 marks]* Compose the automaton of the above subquestion with the automaton derived in Question 2b describing the behaviour of the circuit in Fig. 1. Each state in your resulting automaton, other than the error states, must be labeled by a triple (i, j, k), where i denotes the state in which C_α is, j denotes the state in which C_β is and k denotes the state in which the environment is. Error states may be labeled by $(err,\ -)$ if the circuit enters an error state while the environment does not. Similarly, if the environment enters an error state but not the circuit, the corresponding error state may be labeled $(-,\ err)$. Finally, if both the circuit and the environment are driven to error states, the corresponding state in the composed automaton may be labeled $(err,\ err)$.

(c) From the automaton obtained in the previous subquestion, indicate whether

i. *[2.5 marks]* The circuit in Fig. 1 is receptive to all input sequences that may be provided by the environment.

ii. *[2.5 marks]* The circuit in Fig. 1 can lead to a violation of the protocol between the circuit and its environment in response to a valid sequence of inputs from the environment.

In both cases, answers without justifications will fetch no marks.

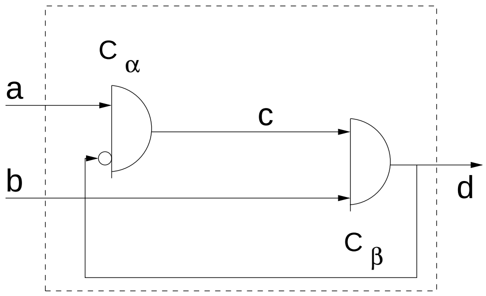

Figure 2: Previous circuit with output c hidden

4. Now consider the circuit shown in Fig. 2, which is the same as the circuit in Fig. 1, except that c is no longer an output visible to the environment.

(a) *[5 marks]* Draw an automaton describing the behaviour of this circuit as it interacts with its environment. As before, you may assume that all signals have the same value (all 0 or all 1) in the start state, and transitions in your automaton must be labeled using a, b, d and not using $a+$, $d-$ etc. Note that there must be no transition labeled c in the resulting automaton, since c is not visible to the environment and cannot participate in the interactions of the circuit with its environment. Also, every transition in your automaton must be labeled with a, b or d (no ϵ labels allowed).

Your resulting automaton must be such that the set of traces accepted by this automaton is exactly the set of traces accepted by the automaton of Question 2b with every occurrence of c replaced by ϵ.

(b) *[3 + 7 + 2.5 + 2.5 marks]* Suppose the circuit in Fig. 2 was designed to implement the process

$$Q = a? \rightarrow Q1$$

$$Q1 = ((b? \rightarrow d!) \mid\mid a?) \rightarrow Q1$$

Repeat Questions 3a, 3b and 3c for the circuit in Fig. 2 and for the process Q described above.