Efficiency and Budget Balance

Uniqueness of Groves for Efficiency

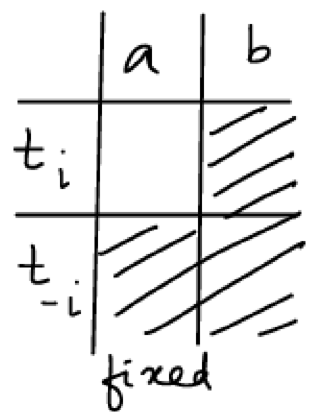
$$f^{eff}(t) \in \arg\max_{a \in A} \sum_{i \in N} t_i(a)$$

Theorem (Green and Laffont (1979), Holmström (1979))
If the type space is 'sufficiently' rich, every efficient and DSIC mechanism is a Groves mechanism.

Proof sketch: two alternatives $A = \{a, b\}$

welfares $\sum t_i(a)$ and $\sum t_i(b)$

$\sum t_i(a) \geqslant \sum t_i(b)$ then a is chosen

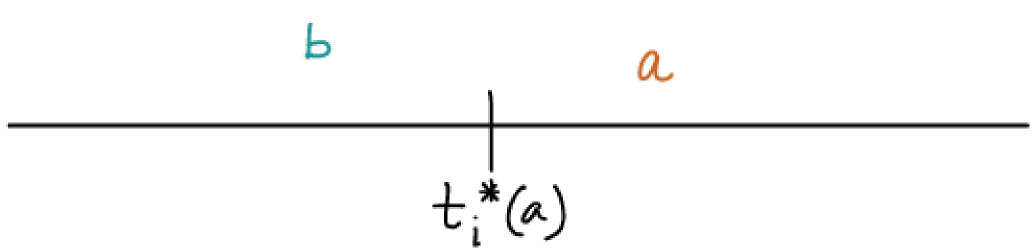


- fix the valuations of the other agents to t_{-i}
- fix value of i at alternative b at $t_i(b)$

$\exists$ some threshold $t_i^*(a)$ s.t.

$\forall\ t_i(a) \geqslant t_i^*(a)$ a is the outcome

$t_i(a) < t_i^*(a)$ b is the outcome



Use DSIC for $t_i^*(a) + \epsilon = t_i(a)$, $\epsilon > 0$

$t_i^*(a) + \epsilon - p_{ia} \geqslant t_i(b) - p_{ib}$

note: payment for a player has to be same for an allocation.

similarly, $t_i'(a) = t_i^*(a) - \delta$, $\delta > 0$

$t_i(b) - p_{ib} \geqslant t_i^*(a) - \delta - p_{ia}$

since ϵ, δ are arbitrary, then

$$t_i^*(a) - p_{ia} = t_i(b) - p_{ib} \quad \text{(1)}$$

But $t_i^*(a)$ is the threshold of the efficient outcome

$$t_i^*(a) + \sum_{j \neq i} t_j(a) = t_i(b) + \sum_{j \neq i} t_j(b) \quad \text{(2)}$$

from (1) and (2)

$$p_{ia} - p_{ib} = \sum_{j \neq i} t_j(b) - \sum_{j \neq i} t_j(a)$$

hence the payment has to be of the form $p_{ix} = h_i(t_{-i}) - \sum_{j \neq i} t_j(x)$

Theorem (Green and Laffont (1979))

No Groves mechanism is budget balanced, i.e., $\nexists p_i^G$ s.t.,

$$\sum_{i \in N} p_i^G(t) = 0 \quad \forall t \in T.$$

Proof sketch: Two alternatives $\{0,1\}$ 0: a project is undertaken
1: project is not undertaken

in outcome 0, every agent has zero value.

Suppose, $\exists h_i$ s.t. $\sum_{i \in N} p_i(t) = 0$

Consider two types w_1^+, w_1^- for player 1, and one type w_2 for player 2, such that

$w_1^+ + w_2 > 0$ hence project is built

$w_1^- + w_2 < 0$ and project is not built

Budget balance at type profile (w_1^+, w_2)

$$h_1(w_2) - w_2 + h_2(w_1^+) - w_1^+ = 0$$

at (w_1^-, w_2), $h_1(w_2) + h_2(w_1^-) = 0$

eliminating $h_1(w_2)$, $w_2 = h_2(w_1^+) - h_2(w_1^-) - w_1^+$

The RHS depends only on w_1, hence it is possible to alter w_2 slightly to retain the inequalities, but then the above equality cannot hold.

Corollary: If the valuation space is sufficiently rich, no efficient mechanism can be both DSIC and BB.

Weakening DSIC for positive results

Allocation is still the efficient one.

Payment in this setting is also defined via a prior

$$\delta_i(t_i) = \mathbb{E}_{\underline{t}_i | t_i} \sum_{j \neq i} t_j(a^*(t))$$

allocation, $a^*(t) \in \underset{a \in A}{\arg\max} \sum_{i \in N} t_i(a)$

payment, $p_i^{dAGVA}(t) = \frac{1}{n-1} \sum_{j \neq i} \delta_j(t_j) - \delta_i(t_i)$

(named after d'Aspremont, Gerard-Varet (1979), Arrow (1979))

This payment implements the efficient allocation rule in Bayes Nash equilibrium.

$$\mathbb{E}_{\underline{t}_i | t_i} \left[t_i(a^*(t)) - p_i^{dAGVA}(t) \right]$$

$$= \mathbb{E}_{\underline{t}_i | t_i} \sum_{j \in N} t_j(a^*(t)) - \mathbb{E}_{\underline{t}_i | t_i} \left[\frac{1}{n-1} \sum_{j \neq i} \delta_j(t_j) \right]$$

$$\geq \mathbb{E}_{t_{-i}|t_i} \sum_{j\in N} t_j\left(a^*(t_i', t_{-i})\right) - \mathbb{E}_{t_{-i}|t_i}\Big[\underbrace{\frac{1}{n-1}\sum_{j\neq i}\delta_j(t_j)}_{\text{not a function of } t_i}\Big]$$

$$= \mathbb{E}_{t_{-i}|t_i}\left[t_i\left(a^*(t_i', t_{-i})\right) - p_i^{dAGVA}(t_i', t_{-i})\right]$$

To show budget balance, consider

$$\sum_{i\in N} p_i^{dAGVA}(t) = \frac{1}{n-1}\sum_{i\in N}\sum_{j\neq i}\delta_j(t_j) - \sum_{i\in N}\delta_i(t_i)$$

$$= \frac{n-1}{n-1}\sum_{j\in N}\delta_j(t_j) - \sum_{i\in N}\delta_i(t_i) = 0$$

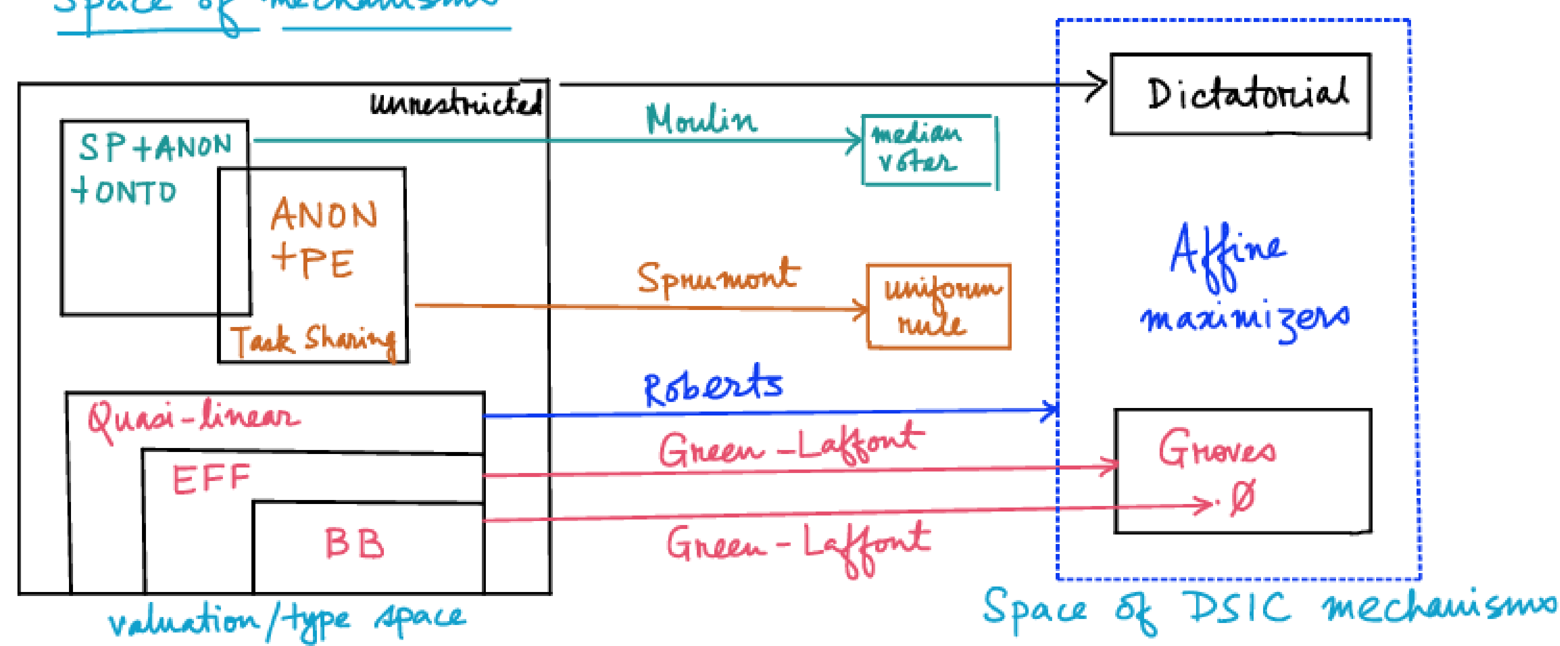
Theorem: The dAGVA mechanism is efficient, BIC, and budget balanced.

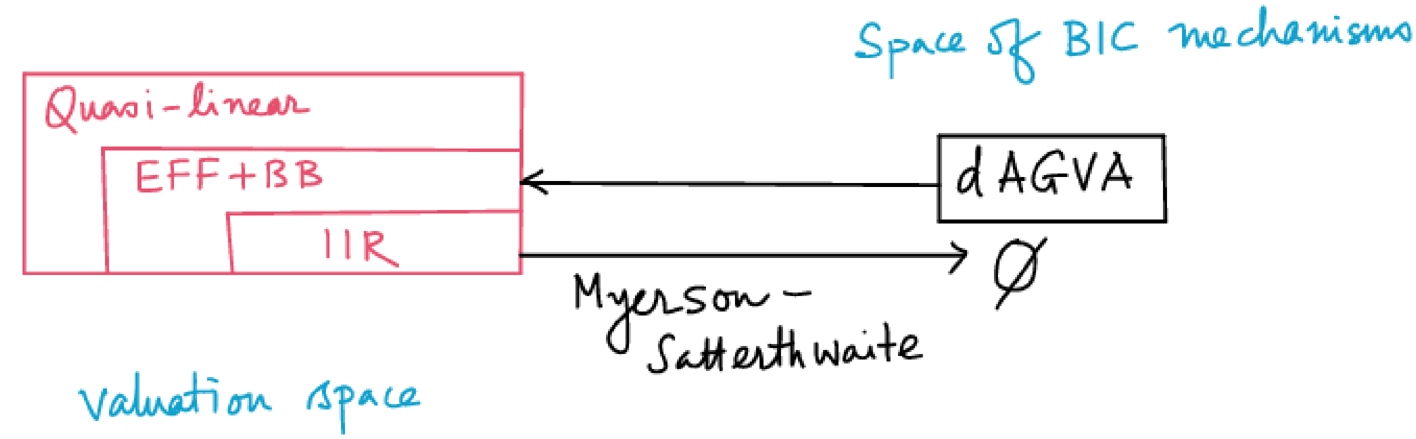
Q: participation guarantee?

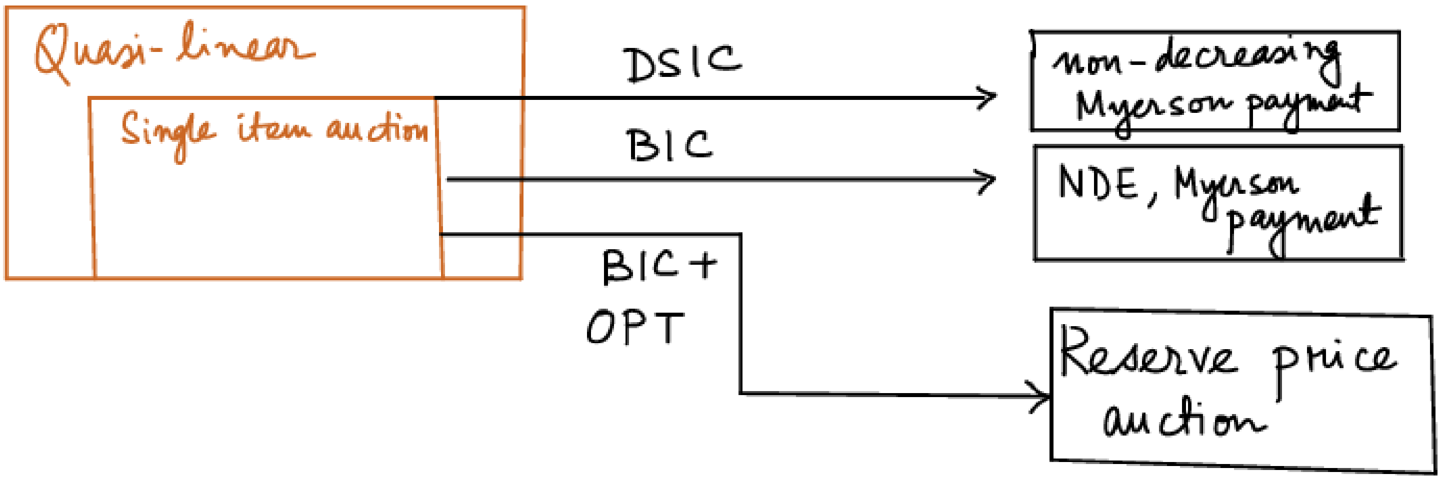
A: dAGVA is not IIR.

Theorem (Myerson, SatterThwaite (1983))
In a bilateral trade (that involves two types of agents: seller and buyer) no mechanism can be simultaneously BIC, efficient, IIR, and budget balanced.

Space of mechanisms




Space of BIC mechanisms
Quasi-linear
EFF+BB
IIR
dAGVA
Myerson–Satterthwaite
∅
Valuation space


Quasi-linear
Single item auction
DSIC
non-decreasing Myerson payment
BIC
NDE, Myerson payment
BIC+ OPT
Reserve price auction