Description Logics

An Introduction

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Description Logics

- A family of languages for formal knowledge representation
- Represents knowledge by
  1. Defining relevant concepts in the domain - Terminology
  2. Using terminology to specify properties of individuals and objects - World Description
- Reasoning - a central service

Basics

Concepts Unary predicates *Eg. Person, Female*

Roles Binary predicates *Eg. hasChild*

Individuals Constants *Eg. Mary, John*

Constructors

- Union $\sqcup$: Eg. Man $\sqcup$ Woman
- Intersection $\sqcap$: Eg. Person $\sqcap$ Female
- Restriction Exists $\exists$: Eg. $\exists$hasChild.Female
- Restriction ForAll $\forall$: Eg. $\forall$hasChild.Engineer
- Negation $\neg$: Eg. $\neg$Man
- Number restriction: $\leq k, \geq m$

Axioms Mother $\sqsubseteq$ Parent

Example

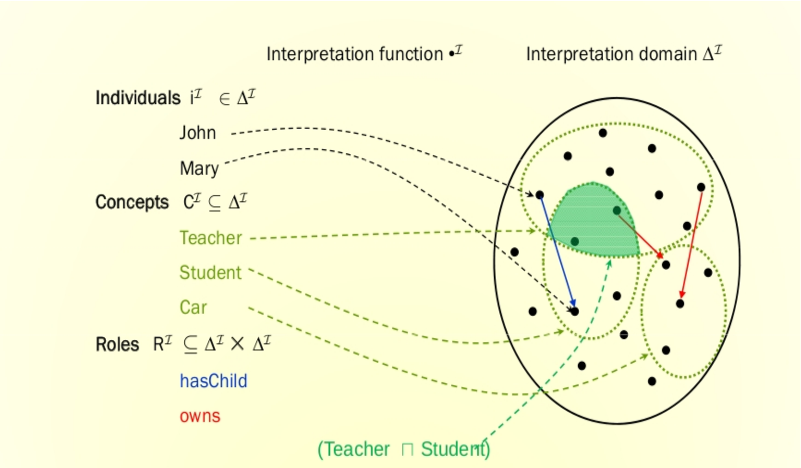
"A man that is married to a doctor and has at least five children, all of whom are professors"

$$Human \sqcap \neg Female \sqcap \exists married.Doctor \sqcap (\geq 5 child) \sqcap \forall child.Professor$$

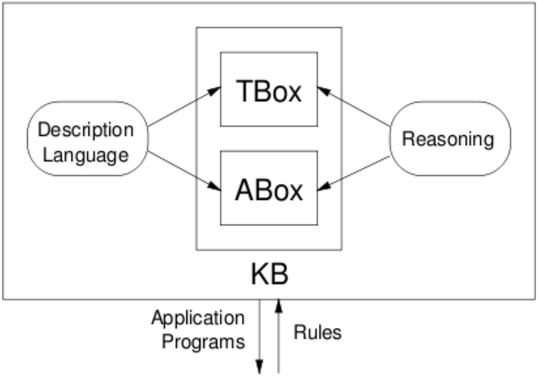
DL Semantics

- A terminology consists of concepts and roles
- An interpretation $\mathcal{I}$ is a tuple$(\triangle^I, \cdot^I)$ where
  - $\triangle^I$ is the domain
  - $\cdot^I$ is a mapping which maps
    - Names of individuals to elements of $\triangle^I$
    - Names of concepts to subsets of $\triangle^I$
    - Names of roles to subsets of $\triangle^I \times \triangle^I$

DL Semantics



Knowledge Representation System Architecture



TBox and ABox

- Knowledge Base = TBox + ABox
- TBox: Terminology - Inclusion axioms and Equivalence axioms

$$
\begin{aligned}
Woman &\equiv Person \sqcap Female \\
Man &\equiv Person \sqcap \neg Woman \\
Mother &\equiv Woman \sqcap \exists hasChild.Person \\
Father &\equiv Man \sqcap \exists hasChild.Person \\
Parent &\sqsubseteq Person
\end{aligned}
$$

- ABox: Assertions

$$
\begin{aligned}
&Father(PETER) \\
&Mother(MARY) \\
&hasChild(MARY, PETER) \\
&hasChild(PETER, HARRY)
\end{aligned}
$$

Inferencing

1. Satisfiability: Is there some interpretation that satisfies axioms in TBox?
2. Subsumption: Is concept A more general than concept B?
3. Equivalence: Same?
4. Instance check: Assertion α entailed by ABox?
5. Retrieval: Which individuals satisfy concept C?

All are reducible to satisfiability check.

Tableau reasoning algorithm

Central Idea: Find an individual which satisfies the concept

- Convert description to Negation Normal form
- For any existential restriction, introduce a new individual as role filler such that it satisfies the constraints expressed by the restriction.
- Use value restrictions in interaction with already defined role relationships to impose new constraints on individuals
- For disjunctive constraints, try both possibilities in successive attempts. Backtrack if you reache an obvious contradiction
- If an at-most number restriction is violated then the algorithm must identify different role fillers

Completion rules

The $\rightarrow_{\sqcap}$-rule
Condition: $\mathcal{A}$ contains $(C_1 \sqcap C_2)(x)$, but it does not contain both $C_1(x)$ and $C_2(x)$.
Action: $\mathcal{A}' = \mathcal{A} \cup \{C_1(x), C_2(x)\}$.

The $\rightarrow_{\sqcup}$-rule
Condition: $\mathcal{A}$ contains $(C_1 \sqcup C_2)(x)$, but neither $C_1(x)$ nor $C_2(x)$.
Action: $\mathcal{A}' = \mathcal{A} \cup \{C_1(x)\}$, $\mathcal{A}'' = \mathcal{A} \cup \{C_2(x)\}$.

The $\rightarrow_{\exists}$-rule
Condition: $\mathcal{A}$ contains $(\exists R.C)(x)$, but there is no individual name z such that $C(z)$ and $R(x,z)$ are in $\mathcal{A}$.
Action: $\mathcal{A}' = \mathcal{A} \cup \{C(y), R(x,y)\}$ where y is an individual name not occurring in $\mathcal{A}$.

The $\rightarrow_{\forall}$-rule
Condition: $\mathcal{A}$ contains $(\forall R.C)(x)$ and $R(x,y)$, but it does not contain $C(y)$.
Action: $\mathcal{A}' = \mathcal{A} \cup \{C(y)\}$.

The $\rightarrow_{\geq}$-rule
Condition: $\mathcal{A}$ contains $(\geqslant n\, R)(x)$, and there are no individual names $z_1, \ldots, z_n$ such that $R(x, z_i)$ $(1 \leq i \leq n)$ and $z_i \neq z_j$ $(1 \leq i < j \leq n)$ are contained in $\mathcal{A}$.
Action: $\mathcal{A}' = \mathcal{A} \cup \{R(x, y_i) \mid 1 \leq i \leq n\} \cup \{y_i \neq y_j \mid 1 \leq i < j \leq n\}$, where $y_1, \ldots, y_n$ are distinct individual names not occurring in $\mathcal{A}$.

The $\rightarrow_{\leq}$-rule
Condition: $\mathcal{A}$ contains distinct individual names $y_1, \ldots, y_{n+1}$ such that $(\leqslant n\, R)(x)$ and $R(x, y_1), \ldots, R(x, y_{n+1})$ are in $\mathcal{A}$, and $y_i \neq y_j$ is not in $\mathcal{A}$ for some $i \neq j$.
Action: For each pair y_i, y_j such that $i > j$ and $y_i \neq y_j$ is not in $\mathcal{A}$, the ABox $\mathcal{A}_{i,j} = [y_i/y_j]\mathcal{A}$ is obtained from $\mathcal{A}$ by replacing each occurrence of y_i by y_j.

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- Jidi Zhao, 2008. *Introduction to Description Logic and Ontology Languages.* Institute of Computer Technology, TU Vienna.