Residual White Space Distribution Based
Opportunistic Channel Access Scheme for
Cognitive Radio Systems

Technical Report: TR-CSE-2010-26
March 2010

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March 15, 2010
Abstract

We propose an opportunistic channel access scheme for cognitive radio-enabled secondary networks. In our work, we model the channel occupancy due to Primary User (PU) activity as a 2-state Alternating Renewal Process, with alternating busy and idle periods. Once a Secondary Node (SN) senses the channel idle, the proposed scheme uses the residual idle time distribution to estimate the transmission duration in the remaining idle time. The SN transmits the frames within the transmission duration without further sensing the channel, thereby reducing average sensing overhead per transmitted frame. The analytical formulations used by the scheme does not require the SN to know the start of the idle period. We validate the analytical formulations using simulations, and compare the performance of the proposed scheme with a Listen-Before-Talk (LBT) scheme.
1 Introduction

Opportunistic Spectrum Access (OSA) has emerged as a promising approach to efficiently utilize the electromagnetic spectrum. In this approach, the secondary users use those parts of the spectrum band that are not currently utilized in space or time by any primary user. Channel occupancy distribution (channel idle and busy time distribution) due to Primary User (PU) activity has recently been used to devise opportunistic channel access schemes for secondary networks. Analytical formulation of most of these schemes assume that the start of idle period (white space\(^1\)) is known to the Secondary Node (SN). In this report, we describe an opportunistic channel access scheme in which a secondary node senses the channel only when it has one or more frames to transmit. If the channel is sensed idle, it uses the residual (remaining) idle time distribution to estimate its transmission duration within the remaining idle time, without requiring the knowledge of start of the idle period. Within the estimated transmission duration, the SN transmits the frames without further sensing the channel. If the residual idle time ends (due to appearance of PU on the channel) before the SN transmission is over, the SN transmission collides with the PU transmission, thereby interfering in the PU transmission. Such a scheme is useful for energy constrained and mobile devices, which cannot be recharged frequently. On the other hand, the schemes, which assume knowledge of the start of the idle period, require the SN to sense the channel continuously to keep track of start of each idle period, which is costly for energy-constrained and mobile SNs. As an example, in Figure 1, if an SN transmits the first burst of frames between time instants \(t_1\) and \(t_2\) in idle period \(I_1\), and the second burst arrives at time \(t_3\) in idle period \(I_3\), then these scheme require the SN to continuously sense the channel between instants \(t_2\) and \(t_3\) so as to keep track of the start of idle period \(I_3\). But their additional knowledge of the start of idle period (along with the known idle time distribution) may lead to better channel utilization and lower PU interference probability. Our proposed scheme described in this report does not require continuous sensing of the channel and performs channel sensing only at time instants \(t_1\) and \(t_3\) when frames arrive at the MAC layer of SN. However, in absence of any knowledge of the idle cycle start time, it has to estimate its transmission duration solely based on the remaining idle time distribution of the channel. Both the approaches usually assume the knowledge of the channel occupancy distribution, obtained by some appropriate means.

This report is organized as follows. In Section 2, we present some related works reported in literature. Section 3 presents the system model used in our work.

\(^{1}\)In this report, we use the terms channel idle time/white space and channel residual idle time/residual white space synonymously.
In Section 4, we give an overview of the proposed opportunistic channel access scheme using residual white space distribution. In Section 5, we present the analytical formulations for the proposed channel access scheme for general channel idle and busy time distributions. In Section 6, we derive analytical formulations for a configuration in which primary channel occupancy is 2-Erlang distributed. Section 7 presents these derivations for Uniformly distributed primary channel occupancy. In Section 8, we present the simulation results for the above-mentioned two specific channel idle and busy time distributions (2-Erlang and Uniform). Section 9 presents conclusions and future work.

2 Related Work

A number of approaches have been proposed in literature to devise opportunistic channel access schemes for secondary networks. The authors in [1] and [2] propose channel sensing and transmission strategies under the Partially-Observed Markov Decision Process (POMDP) framework, but assumes the slotted Primary as well as slotted Secondary networks. This requires synchronization between the Primary and Secondary transmission slot structures. In [3] and [4], the authors consider an unslotted Primary Network with multiple channels and a slotted Secondary Network, which sense the channels at the beginning of each slot. In [3], the authors model the channels occupancy using continuous-time Markov chain. The Secondary Network is assumed to have a slotted transmission structure in which, at the beginning of each slot, the secondary node decides which channel to sense and potentially transmit over. The scheme essentially adopts a single secondary frame transmission per sensing operation, but the decision to transmit in a slot is based on an optimal joint sensing and access strategy of the secondary networks. This strategy maximizes the expected total number of bits that can be delivered by the secondary node in T slots under a given collision constraint with the PU. In [4], the authors assume that channel idle and busy periods are exponentially distributed, and have proposed a learning-based approach in which each secondary user maintains an estimate of its own belief vector as well as that of other secondary nodes by learning collision events. Based on reward accrued (for successful transmissions)
and the channel idle probabilities (computed using belief vectors), the secondary node decides with what probability to sense each channel so that the collision with other users are minimized. It then maps these channel sensing probabilities to a concrete decision in determining which channel to access, so that the total throughput of the secondary users in T slots is maximized. In [5], the authors have proposed a cost and reward-based access policy to maximize the secondary network utility. The authors consider an unslotted Primary network, but their analytical formulation of the access scheme is based on the assumption that the secondary node can detect the beginning of the idle period. A proactive spectrum access approach is proposed in [6] where secondary nodes take input from spectrum sensing modules, and build a three-tier predictive statistical models of spectrum availability on each channel. A Semi-Markov model for WLAN channels was proposed in [7]. The authors (in [8]) approximated the Semi-Markov model with continuous-time Markov chain and proposed the Cognitive Medium Access (CMA) protocol. CMA assumes a slotted secondary network and WLAN primary network. The SN senses the state of the channel(s) at the beginning of every slot, and cast the channel access problem as constrained Markov decision process (CMDP) for fully observable system. Based on the previous sensing history, the CMA controller decides whether to transmit in next slot, and if yes, on which channel. In [9], alternating renewal theory is used to analyse how often to sense the availability of licensed channel and in which order to sense those channels. In [10], the authors have used PDF and CDF of a whitespace trace (idle periods), obtained using simulation of a linear array of five primary WLAN nodes, to compute the sensing duration and the number of frames that an SN should transmit on sensing a channel idle, subject to a PU interference bound. The paper computes, both analytically and using simulations, the Effective Secondary Throughput, and Primary User Interference, but it assumes that the start of idle period is known.

Contrary to the above mentioned schemes, our scheme does not make any assumption regarding the transmission structure of primary and secondary networks and is based on the residual idle time distribution of the channel. Once a secondary node senses the channel idle, it estimates, using the theory described in this report, the transmission duration in the remaining white space so that the PU interference bound is not violated due to secondary node’s transmissions. It then transmits appropriate number of frames within the estimated duration.

3 System Model

We consider a spectrum sharing model in a wireless network. The network communicates using a single channel. The designated users of the single channel are set
of users termed as Primary Users (PU) of the channel. There are other set of nodes known as Secondary Nodes (SN) which share the channel with PUs. However, the SNs can only access the channel when no PU is using the channel. Thus, the SNs have to look for the so called white spaces (i.e., idle periods in the channel) and opportunistically transmit their packets. The PUs are not aware of secondary node’s transmission and can initiate their transmission whenever they require. We assume that an SN can distinguish between the primary and secondary frame transmissions using physical layer detection techniques such as matched filter and cyclostationary feature detection methods. It is secondary node’s responsibility to detect the PU transmission and evacuate the channel. So, the SNs must ensure that their transmissions do not cause interference to any PU beyond a certain limit. This limit is specified by the PU which we denote as \( \eta \). Thus, \( \eta \) is the upper bound of probability of interference by SNs which PUs can tolerate. The value of \( \eta \) lies between 0.0 and 1.0. A higher value of \( \eta \) indicates that PU can tolerate more interference due to SN transmissions. Lesser values indicate less tolerance of PU to SN-generated interference. For network systems where PU owns a license to use the channel, its tolerance to the secondary network-generated interference is usually quite low (less value of \( \eta \)). Our model does not assume any specific MAC protocol or transmission structure (slotted/unslotted) for the PU.

From a secondary node’s perspective, the channel is considered idle when it is not used by a PU and is considered busy when it is used by a PU. We model the channel occupancy due to PU activity as an Alternating Renewal Process (See Figure 1), in which a cycle consisting of channel busy duration (denoted as \( B \)) followed by channel idle duration (denoted as \( I \)), repeats (renews) in time. It is important to note that for channel occupancy modeling, the term white space (or channel idle duration) refers to the duration when no PU uses the channel. Any transmission by an SN during such a white space does not change the white space duration as perceived by other secondary nodes. In this work we assume that the SN knows the channel idle and busy time distributions. Some of the earlier research work have also used this assumption (see, for e.g., [1], [2], [3], [4], and [8]). This information can be made available to a secondary node by some designated central server, which measures the channel occupancy due to PU activity, fits the appropriate distribution, and estimates the distribution parameters\(^2\).

\(^2\)Channel idle and busy time distribution information can be gathered by the SN itself if it uses a dedicated sensing module to sense the PU activity on the channel, but it would be prohibitively costly for mobile and energy constrained secondary devices.
4 Opportunistic Spectrum Access using Residual White Space Distribution: An Overview

Whenever a Secondary Node’s MAC layer receives one or more packets for transmission, the node senses the channel. If the channel is sensed idle, the node concludes the presence of white space which can be used for secondary frames transmission. But in absence of continuous sensing, the node cannot keep track of the start of the white space (idle period). This sensing operation can be seen as random incidence in the idle cycle (such as at time instants $t_1$ and $t_3$ in Figure 1 and instants $t$ in Figure 2 and 3). In our work, we derive the residual white space distribution (or, residual idle time distribution) based on random incidence in a renewal cycle (see [11, pp. 328–331]), as explained in Section 5. The SN uses the residual white space distribution to estimate the maximum duration for which it can transmit (say, $y_{max}$) within the remaining white space so as to satisfy the acceptable interference constraint set by the PU. The SN transmits the frames within the estimated transmission duration without any further sensing of the channel. It receives an acknowledgment from the receiver for each successfully received frame. Absence of acknowledgment for a transmitted frame indicates collision of the frame or its acknowledgment with PU transmission due to appearance of the PU on the channel. We assume that SU data and acknowledgment frame sizes are considerably smaller than the PU frame size, so that SU detects the collision within a single
PU frame transmission time. The SU successfully transmit the frames within the estimated transmission duration $y_{\text{max}}$ if the remaining white space is greater than $y_{\text{max}}$; otherwise, SN transmission collides with the first PU frame due to appearance of PU on the channel and causes interference to the PU. Figure 2 and Figure 3 shows these two cases respectively. In both the figures, $T_A$, $T_B$, and $T_c$ denote respectively the start of an idle cycle (white space), end of the idle cycle (or, equivalently, start of the next busy cycle), and end of the busy cycle. On receiving one or more frames for transmission at time instant $t$, the SN senses the channel from time $t$ to $t_1$, and since the channel is sensed idle, it transmit frames for a maximum duration of $y_{\text{max}}$ (which is equal to $t_2 - t_1$). The transmission duration $y_{\text{max}}$ is computed by the SN using analytical formulations explained in Sections 5.1, 6.2, and 7.2. The duration $(T_B - t_1)$ (represented as $RI$) denotes the residual white space (or residual idle time). In Figure 2, since $RI$ is more than $y_{\text{max}}$, the SN successfully transmits for complete $y_{\text{max}}$ duration. On the other hand, in Figure 3, since $RI$ is less than $y_{\text{max}}$, the SN transmits only for $RI$ duration and collides with the next busy cycle due to PU transmission, thereby interfering with the PU transmission.

After transmission (for $y_{\text{max}}$ duration in Figure 2 and for $RI$ duration in Figure 3), if the SN has more frames to transmit, it backs off randomly for a duration, which is exponentially distributed with mean channel busy time period and senses the channel again. This backoff and sensing operation is repeated until the SN again (randomly) lands in an idle cycle (white space) and transmit additional frames for $y_{\text{max}}$ duration.

5 Channel Access Scheme

In this section, we derive the analytical formulations for computing the maximum transmission duration ($y_{\text{max}}$) within the remaining white space, as well as formulations for average channel utilization per white space by the SN, and the PU interference probability. These formulations use residual idle time distribution, which are derived using the known channel idle time distributions. We first present the formulations for general channel idle time distribution, followed by formulations for two specific channel idle time distributions presented in subsequent sections.

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3If SN does not have enough frames to consume the duration of $y_{\text{max}}$, it stops after sending all the frames.
5.1 Access Strategy

On detecting a white space, the SU needs to compute the duration for which it can transmit within the remaining (residual) white space so as to satisfy the acceptable interference constraint specified by the PU. The SU uses the residual white space distribution to compute this transmission duration. The residual white space distribution is obtained using the known idle and busy time distributions for the channel.

Let $I$ and $B$ represent the random variables denoting channel idle and busy time values, and $F_I$ and $F_B$ represent respectively the channel idle and busy time distributions with known parameters. Let $f_{RI}$ represent the residual idle time (or residual white space) density function, which is computed by the SN as follows [11, pp. 331]:

$$f_{RI}(y) = \frac{1 - F_I(y)}{E[I]}$$

The residual idle time distribution function can then be computed as follows:

$$F_{RI}(y) = \int_0^y f_{RI}(z)dz$$  \hspace{1cm} (2)

Here $E[I]$ denotes the mean channel idle time and $F_{RI}(y)$ denotes the probability that the residual white space is less than $y$. In other words, if SN transmits for duration $y$, then $F_{RI}(y)$ denotes the probability that the remaining white space will end (due to PU’s appearance on the channel) before SN transmission is over. In this case, SN transmission will interfere with PU transmission. So, on sensing the channel idle, the SN computes the transmission duration $y$ in the remaining white space so as to satisfy the following constraint:

$$F_{RI}(y) \leq \eta$$  \hspace{1cm} (3)

The maximum value of $y$ (which we denote as $y_{max}$) for which the above inequality is satisfied, is taken as the transmission duration by an SU in the remaining white space. The channel access algorithm used by SN is broadly summarized in Algorithm 1. For a given channel idle and busy time distribution and parameters, the value of $y_{max}$ (computed using (3)) is fixed. Therefore, it can be computed by the SN once in the beginning and need not be computed every time the channel access is made. Moreover, since the SN has already estimated the transmission duration $y_{max}$ in the remaining white space (step 1 in the Algorithm 1), the frame transmission (step 8) within the for loop is performed without sensing the channel.
Algorithm 1 Channel Access Algorithm

1: Compute $y_{max}$, which satisfy constraint (3).
2: while SN transmission queue not empty do
3:   $PU\_INTERFERENCE\_FLAG \leftarrow FALSE$
4:   Sense the channel.
5:   if channel is idle then
6:       $Z \leftarrow \min$(number of frames that can be sent in duration $y_{max}$, number of frames in the queue)
7:       for $i \leftarrow 1, Z$ do
8:           Transmit an SN frame and Wait for acknowledgment (ACK).
9:           if ACK not received then $PU\_INTERFERENCE\_FLAG \leftarrow TRUE$
10:              break
11:       end if
12:       end for
13:   end if
14:   if ($PU\_INTERFERENCE\_FLAG == TRUE$) OR (SN Tx Queue Not Empty) then
15:       Perform exponential random backoff with mean busy time parameter and sense again.
16:   end if
17: else if channel is busy then
18:       Perform exponential random backoff with mean busy time parameter and sense again.
19: end if
20: end while

5.2 Average Channel Utilization per White Space

Once an SN senses the channel idle and detects the presence of white space, it transmits frames for $y_{max}$ duration, which is computed using (3), within the remaining white space. There are two possible transmission scenarios, which are depicted in Figure 2 and Figure 3 respectively. In first scenario (see Figure 2), if the remaining white space duration ($RI$) is more than the transmission duration $y_{max}$, then the channel is utilized for complete $y_{max}$ duration and there is no interference to PU. In second scenario (see Figure 3), if the remaining white space duration ($RI$) is less than the transmission duration $y_{max}$, then the channel is utilized only for $RI$ duration. In this scenario, the SU transmission interferes with PU transmission because the white space ends (due to PU’s appearance on the channel) before the SN transmission is over. Therefore, for a given $y_{max}$ value, the analytical expression for average channel utilization per white space by the SN is given as:

$$AUPWS = \frac{y_{max}(1 - F_{RI}(y_{max})) + \int_{q=0}^{y_{max}} qf_{RI}(q)dq}{E[I]} \quad (4)$$

Here, integral variable $q$ denote the values that residual idle time random variable $RI$ takes, and $E[I]$ is the mean idle time value. The first term in the numerator
of the above formula represents first scenario and the second (integral) term in the numerator represents the second scenario. Note that \((1 - F_{RI}(y_{max}))\) is the probability of successful SN transmission for \(y_{max}\) duration without interfering with PU.

5.3 Primary User Interference Probability due to Secondary Transmissions

*Primary User Interference Probability* (PUIP) represents the probability that a secondary transmission interfere with PU transmission. The SN transmission interferes with the PU transmission if the actual residual (remaining) white space turns out to be less than the transmission duration \(y_{max}\) computed by the SN using (3). Therefore, for a given value of \(y_{max}\), the analytical expression for PU interference probability (PUIP) can be written as:

\[
PUIP = \int_{q=0}^{y_{max}} f_{RI}(q) dq
\]

where \(f_{RI}\) is given by (1), and integral variable \(q\) denote the values that residual idle time random variable \(RI\) takes.

In Section 8, we obtain the *average channel utilization per white space*, and the *PU interference probability* using simulation, and compare it with the analytical results obtained using the analytical expressions (4) and (5) for two different channel idle and busy time distributions.

6 Formulations for 2-Erlang Distributed Channel Idle and Busy Time Durations

In this section, we derive the equation for computing the transmission duration \((y_{max})\), and the expressions for AUPWS and PUIP when the channel idle and busy time durations are 2-Erlang distributed with rate parameters \(\lambda_i\) and \(\lambda_b\) respectively \((\lambda_i > 0, \lambda_b > 0)\).

6.1 Derivation of Residual Idle Time Density and Distribution Functions

The idle time distribution function and the mean value for 2-phase Erlang Distribution are given as:

\[
F_i(y) = 1 - e^{-\lambda_i y}[1 + \lambda_i y], \quad y \geq 0
\]
and

\[ E[I] = \frac{2}{\lambda_i} \]  

(7)

Using the above equations and equations (1) and (2), we obtain the density and distribution functions of residual idle time (see Appendix A), which are given below:

\[ f_{RI}(y) = \frac{\lambda_i e^{-\lambda_i y}}{2} \left[ 1 + \lambda_i y \right] \]  

(8)

and

\[ F_{RI}(y) = 1 - e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right] \]  

(9)

### 6.2 Secondary Node’s Transmission Duration Computation within Remaining White Space

Using equations (3) and (9), we obtain \( y_{\text{max}} \) by solving the following inequality for the maximum value of \( y \) that satisfies the PU interference constraint:

\[ 1 - e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right] \leq \eta \]

\[ \Rightarrow e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right] + (\eta - 1) \geq 0 \]  

(10)

### 6.3 Derivation of Expression for AUPWS

For the obtained value of \( y_{\text{max}} \), let us denote the numerator of (4) by \( N \):

\[ N = y_{\text{max}} (1 - F_{RI}(y_{\text{max}})) + \int_{q=0}^{y_{\text{max}}} q f_{RI}(q) dq \]  

(11)

So, (4) can be written as:

\[ AUPWS = \frac{N}{E[I]} \]  

(12)

Using expression for \((1 - F_{RI}(y_{\text{max}}))\) from (9), the numerator can be written as:

\[ N = y_{\text{max}} e^{-\lambda_i y_{\text{max}}} \left[ 1 + \frac{\lambda_i y_{\text{max}}}{2} \right] + \int_{q=0}^{y_{\text{max}}} q f_{RI}(q) dq \]  

(13)

Let us denote the integral in (13) as \( K \) so that (13) can be written as:

\[ N = y_{\text{max}} e^{-\lambda_i y_{\text{max}}} \left[ 1 + \frac{\lambda_i y_{\text{max}}}{2} \right] + K \]  

(14)
Using 8, we solve the integral $K$ as follows:

\[
K = \int_{q=0}^{y_{\text{max}}} q f_{RI}(q) dq
\]

\[
= \int_{q=0}^{y_{\text{max}}} q \lambda_i e^{-\lambda_i q} \frac{1}{2} [1 + \lambda_i q] dq
\]

\[
= \frac{\lambda_i}{2} \int_{q=0}^{y_{\text{max}}} q e^{-\lambda_i q} dq + \frac{\lambda_i^2}{2} \int_{q=0}^{y_{\text{max}}} q^2 e^{-\lambda_i q} dq
\] (15)

Let us denote the integrals in (15) as follows:

\[
M = \int_{q=0}^{y_{\text{max}}} q e^{-\lambda_i q} dq
\] (16)

and

\[
N = \int_{q=0}^{y_{\text{max}}} q^2 e^{-\lambda_i q} dq
\] (17)

so that the integral $K$ in (15) can be written as:

\[
K = \frac{1}{2} \lambda_i M + \frac{1}{2} \lambda_i^2 N
\] (18)

To solve $M$ and $N$ we use the following indefinite integral solutions (see [12, pp. 112]) and then apply the limits:

\[
\int x e^{ax} dx = e^{ax} \left( \frac{x}{a} - \frac{1}{a^2} \right)
\] (19)

and

\[
\int x^2 e^{ax} dx = e^{ax} \left( \frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right)
\] (20)

Using (19), we can solve (16) to obtain:

\[
M = \int_{q=0}^{y_{\text{max}}} q e^{-\lambda_i q} dq
\]

\[
= \left[ e^{-\lambda_i q} \left( -\frac{q}{\lambda_i} - \frac{1}{\lambda_i^2} \right) \right]_{q=0}^{q=y_{\text{max}}}
\]

\[
= \frac{1}{\lambda_i^2} - e^{-\lambda_i y_{\text{max}}} \left( \frac{1}{\lambda_i^2} + \frac{y_{\text{max}}}{\lambda_i} \right)
\] (21)
Similarly, using (20), we can solve (17) to obtain:

\[ N = \int_{q=0}^{y_{\text{max}}} q^2 e^{-\lambda_i q} dq = \left| -e^{-\lambda_i q} \left( \frac{2}{\lambda_i^3} + \frac{2q}{\lambda_i^2} + \frac{q^2}{\lambda_i} \right) \right|_{q=0}^{y_{\text{max}}} = \frac{2}{\lambda_i^3} - e^{-\lambda_i y_{\text{max}}} \left( \frac{2}{\lambda_i^3} + \frac{2y_{\text{max}}}{\lambda_i^2} + \frac{y_{\text{max}}^2}{\lambda_i} \right) \quad (22) \]

Using expressions for \( M \) and \( N \) from (21) and (22), we can write the expression for integral \( K \) from (18) as follows:

\[ K = \frac{\lambda_i}{2} \left[ \frac{1}{\lambda_i^2} - e^{-\lambda_i y_{\text{max}}} \left( \frac{1}{\lambda_i^2} + \frac{y_{\text{max}}}{\lambda_i} \right) \right] + \frac{\lambda_i^2}{2} \left[ \frac{2}{\lambda_i^3} - e^{-\lambda_i y_{\text{max}}} \left( \frac{2}{\lambda_i^3} + \frac{2y_{\text{max}}}{\lambda_i^2} + \frac{y_{\text{max}}^2}{\lambda_i} \right) \right] \]

\[ = \frac{1}{2\lambda_i} - \frac{1}{2\lambda_i} e^{-\lambda_i y_{\text{max}}} \left( \frac{1}{\lambda_i^2} + \frac{y_{\text{max}}}{\lambda_i} \right) + \frac{1}{2\lambda_i} - \frac{1}{2\lambda_i} e^{-\lambda_i y_{\text{max}}} \left( \frac{2}{\lambda_i^3} + \frac{2y_{\text{max}}}{\lambda_i^2} + \frac{y_{\text{max}}^2}{\lambda_i} \right) \]

\[ = \frac{3}{2\lambda_i} - \frac{1}{2\lambda_i} e^{-\lambda_i y_{\text{max}}} \left( \frac{1}{\lambda_i^2} + \frac{y_{\text{max}}}{\lambda_i} \right) + \lambda_i \left( \frac{2}{\lambda_i^3} + \frac{2y_{\text{max}}}{\lambda_i^2} + \frac{y_{\text{max}}^2}{\lambda_i} \right) \]

\[ = \frac{3}{2\lambda_i} - \frac{1}{2\lambda_i} e^{-\lambda_i y_{\text{max}}} \left( \frac{2}{\lambda_i^2} + \frac{3y_{\text{max}}}{\lambda_i} + \frac{3}{\lambda_i} \right) \]

\[ = \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left[ \frac{\lambda_i y_{\text{max}}}{2} + \frac{3y_{\text{max}}}{2} + \frac{3}{2\lambda_i} \right] \quad (23) \]

Substituting the expression for integral \( K \) from (23) in (14), we rewrite the expression for the numerator \( N \) as follows:

\[ N = y_{\text{max}} e^{-\lambda_i y_{\text{max}}} \left[ 1 + \frac{\lambda_i y_{\text{max}}}{2} \right] + \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left[ \frac{\lambda_i y_{\text{max}}}{2} + \frac{3y_{\text{max}}}{2} + \frac{3}{2\lambda_i} \right] \]

\[ = e^{-\lambda_i y_{\text{max}}} \left[ y_{\text{max}} + \frac{\lambda_i y_{\text{max}}}{2} \right] + \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left[ \frac{\lambda_i y_{\text{max}}}{2} + \frac{3y_{\text{max}}}{2} + \frac{3}{2\lambda_i} \right] \]

\[ = \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left[ \frac{\lambda_i y_{\text{max}}}{2} + \frac{3y_{\text{max}}}{2} + \frac{3}{2\lambda_i} - y_{\text{max}} - \frac{\lambda_i y_{\text{max}}}{2} \right] \]

\[ = \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left[ \frac{y_{\text{max}}}{2} + \frac{3}{2\lambda_i} \right] \quad (24) \]

Substituting the values of \( E[I] \) and \( N \) from (7) and (24) respectively into (12), we write the expression for AUPWS as follows:

\[ AUPWS = \frac{\lambda_i}{2} \left[ \frac{3}{2\lambda_i} - e^{-\lambda_i y_{\text{max}}} \left( \frac{y_{\text{max}}}{2} + \frac{3}{2\lambda_i} \right) \right] \quad (25) \]
6.4 Derivation of Expression for PUIP

For the obtained value of $y_{\text{max}}$, the expressions for PUIP can be obtained using (8) and (5):

$$PUIP = \int_{q=0}^{y_{\text{max}}} f_{RI}(q) dq$$

$$= F_{RI}(y_{\text{max}})$$

$$= 1 - \frac{e^{-\lambda_i y_{\text{max}}}}{2} (2 + \lambda_i y_{\text{max}})$$

(26)

7 Formulations for Uniformly Distributed Channel Idle and Busy Time Durations

In this section, we derive the equation for computing the transmission duration ($y_{\text{max}}$), and the expressions for AUPWS and PUIP when the channel idle and busy time durations are uniformly distributed between the positive values ($a, b$) and ($c, d$) respectively.

7.1 Derivation of Residual Idle Time Density and Distribution Functions

The idle time distribution function and the mean values for Uniform Distribution are given as:

$$F_I(y) = \frac{y - a}{b - a}, \quad a \leq y < b$$

(27)

and

$$E[I] = \frac{a + b}{2}$$

(28)

Using the above equations and equations (1) and (2), we obtain the density and distribution functions of residual idle time (or residual white space) (see Appendix B), which are given below:

$$f_{RI}(y) = \frac{2(b - y)}{b^2 - a^2}$$

(29)

and

$$F_{RI}(y) = \frac{(2by - y^2)}{b^2 - a^2}$$

(30)
7.2 Secondary Node’s Transmission Duration Computation within Remaining White Space

Using equations (3) and (30), we obtain $y_{\text{max}}$ by solving the following inequality for the maximum value of $y$ that satisfies the PU interference constraint:

\[
\frac{(2by - y^2)}{b^2 - a^2} \leq \eta
\]

\[
\Rightarrow y^2 - 2by + \eta(b^2 - a^2) \geq 0
\]

(31)

7.3 Derivation of Expression for AUPWS

Again, let us denote the numerator of (4) by $N$:

\[
N = y_{\text{max}}(1 - F_{RI}(y_{\text{max}})) + \int_{q=0}^{y_{\text{max}}} qf_{RI}(q)\,dq
\]

(32)

So, (4) can be written as:

\[
\text{AUPWS} = \frac{N}{\bar{E}[I]}
\]

(33)

Using expression for $(1 - F_{RI}(y_{\text{max}}))$ from (30) and using (29), the numerator can be written as:

\[
N = y_{\text{max}} \frac{(b^2 - a^2) - 2by_{\text{max}} + y_{\text{max}}^2}{(b^2 - a^2)} + \int_{q=0}^{y_{\text{max}}} \frac{2q(b - q)}{b^2 - a^2} \, dq
\]

\[
= \frac{(b^2 - a^2)y_{\text{max}} - 2by_{\text{max}}^2 + y_{\text{max}}^3}{(b^2 - a^2)} + \frac{2}{(b^2 - a^2)} \int_{q=0}^{y_{\text{max}}} q(b - q)\,dq
\]

\[
= \frac{1}{(b^2 - a^2)} \left[ (b^2 - a^2)y_{\text{max}} - 2by_{\text{max}}^2 + y_{\text{max}}^3 + 2 \int_{q=0}^{y_{\text{max}}} bq\,dq - 2 \int_{q=0}^{y_{\text{max}}} q^2\,dq \right]
\]

\[
= \frac{1}{(b^2 - a^2)} \left[ (b^2 - a^2)y_{\text{max}} - 2by_{\text{max}}^2 + y_{\text{max}}^3 + by_{\text{max}}^2 - \frac{2y_{\text{max}}^3}{3} \right]
\]

(34)

Substituting the values of $\bar{E}[I]$ and $N$ from (28) and (34) respectively into (33), we write the expression for AUPWS as follows:

\[
\text{AUPWS} = \frac{2}{(a + b)(b^2 - a^2)} \left[ (b^2 - a^2)y_{\text{max}} - by_{\text{max}}^2 + \frac{y_{\text{max}}^3}{3} \right]
\]

(35)


### Table 1: Main Simulation Parameters

<table>
<thead>
<tr>
<th>Configurations and Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Config-1: 2-Erlang</strong></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td></td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>$200 \text{ Sec}^{-1}$</td>
</tr>
<tr>
<td>$\lambda_b$</td>
<td>$500 \text{ Sec}^{-1}$</td>
</tr>
<tr>
<td>SN Frame Size</td>
<td>2048 bits</td>
</tr>
<tr>
<td><strong>Config-2: Uniform</strong></td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>0.0</td>
</tr>
<tr>
<td>$b$</td>
<td>2.0 Sec</td>
</tr>
<tr>
<td>$c$</td>
<td>0.0</td>
</tr>
<tr>
<td>$d$</td>
<td>0.8 Sec</td>
</tr>
<tr>
<td>SN Frame Size</td>
<td>2048 bits</td>
</tr>
</tbody>
</table>

7.4 Derivation of Expression for PUIP

For the obtained value of $y_{max}$, the expressions for PUIP can be obtained using (29) and (5):

$$PUIP = \int_{q=0}^{y_{max}} f_{RI}(q) dq$$
$$= F_{RI}(y_{max})$$
$$= \frac{2by_{max} - y_{max}^2}{(b^2 - a^2)}$$

8 Simulation Results and Performance Evaluation

We obtain simulation results with two different configurations for channel idle and busy time distributions. In Configuration-1, we consider the primary channel idle and busy time values to be 2-Erlang distributed with rate parameters $\lambda_i$ and $\lambda_b$, respectively ($\lambda_i > 0, \lambda_b > 0$). The analytical formulations for this configuration is derived in Section 6. In Configuration-2, we consider the primary channel idle and
busy time values to be uniformly distributed with parameters \((a, b)\) and \((c, d)\) respectively. The analytical formulations for this configuration is derived in Section 7. The values of the parameters are given in Table 1.

For each of the above configurations, we use OPNET simulator [13] to sim-
ulate the PU activity on a 11 Mbps wireless channel. The activity is modeled as an Alternating Renewal Process, which consist of 10000 busy and 10000 idle periods on the channel that appear alternatingly. For Configuration-1, we use the idle and busy time distribution parameters as $\lambda_i = 200 \text{ sec}^{-1}$ and $\lambda_b = 500 \text{ sec}^{-1}$ respectively, with the mean values of 0.01 and 0.004 seconds respectively. For Configuration-2, we use the idle time distribution parameters $(a, b) = (0, 2.0 \text{ sec})$, and busy time distribution parameters $(c, d) = (0, 0.8 \text{ sec})$ with the mean values of 1.0 and 0.4 seconds respectively. For each value of the acceptable PU interference constraint ($\eta$), using simulation we obtain the average channel utilization by SN per white space, and the probability of interference to PU due to SN transmissions. For configuration-1, the value of transmission duration $y_{\text{max}}$ for a given value of $\eta$, is obtained by solving (10) using Newton Raphson method (see Appendix C). To verify the $y_{\text{max}}$ values obtained theoretically using Newton Raphson method, we plot in Figures 4, 5, and 6, for three different values of $\eta$, the variation of LHS expression values of inequality (10) with different values of $y$. We denote the LHS expression of inequality (10) as $f_n(y)$. The $f_n(y)$ values (ordinates) are plotted on the vertical axis whereas the values of $y$ (abscissa) are plotted on the horizontal axis. Though the graphs are plotted for negative values of $y$ to observe the pattern of variations, only the positive values of $y$ are valid for our work. From these graphs, we want to find the maximum value of $y$ for which $f_n(y) \geq 0$ (see (10)).
Table 2: SN Bursty Traffic Profile Parameters for Two Primary Channel Occupancy Configurations (all units in seconds)

<table>
<thead>
<tr>
<th></th>
<th>Profile-1</th>
<th>Profile-2</th>
<th>Profile-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ_{ON}</td>
<td>0.003</td>
<td>0.005</td>
<td>0.006</td>
</tr>
<tr>
<td>µ_{OFF}</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

As we see from Figures 4 and 5, the maximum value of $y$ for which the function $f_n(y)$ is $\geq 0$ is 0.011986 and 0.002047 seconds respectively. These are the abscissa values at which the $f_n(y)$ curve (in the respective figures) slopes down and intersects with the horizontal axis (toward the right side of the graph). We denote these values as $y_{max}$ for the respective $\eta$ values. For any $y > y_{max}$ (in the respective figures), the $f_n(y) < 0$. These values matches with the values obtained theoretically using Newton Raphson method. The value of $\eta = 0$ indicates that the PU can not tolerate any interference. Therefore, the SN completely shuts-off and does not transmit any frames. Solving expression (10) using Newton Raphson method gives $y_{max} = 0$. This is also reflected in Figure 6, which shows that the maximum value of $y$ for which the function $f_n(y)$ is $\geq 0$ is 0. For Configuration-2, we solve the quadratic equation (31), and select that root as the value of $y_{max}$ which lies within the valid range of $(a, b)$.

We consider four different SN traffic profiles: one saturated SN traffic profile, and three bursty SN traffic profiles, which are modeled using 2-state Markov chain with ON and OFF states. We represent the mean sojourn time for ON and OFF states as $\mu_{ON}$ and $\mu_{OFF}$ respectively. Parameter values for the three bursty SN traffic profiles for Configuration-1 and Configuration-2 are shown in Table 2. The performance statistics are also calculated (for the respective $\eta$ values) using analytical expressions given in (25) and (26) (for Configuration 1), and (35) and (36) (for Configuration 2). The average channel utilization by SN per white space is computed using the maximum transmission opportunity the SN gets in each white space.

8.1 Results

Figure 7 and Figure 8 compare (for Configuration 1) the analytically estimated and simulation-based performance parameters for different PU interference constraints ($\eta$) and for different SN traffic profiles. Figure 9 and Figure 10 show these val-
values for Configuration 2. We make several observations from these figures. First, the larger the value of \( \eta \) (i.e., larger acceptable PU interference), the higher the channel utilization by SN per white space. This is so because the analytical formulations ((10) and (31)) predict larger transmission times (i.e., larger \( y_{\text{max}} \) values) and therefore, SN transmits aggressively in each white space. However, such aggressive transmission by the SN leads to higher PU interference. For smaller values of \( \eta \), the estimated values of \( y_{\text{max}} \), and consequently the SN transmissions, are conservative (so that it can meet the low interference requirement of the Primary Network), which leads to low white space utilization by the SN but low interference to the PU. Second, the analytically computed value for \( P_{\text{UIP}} \) serves as an upper bound for probability of interference to PU’s transmission (due to secondary transmissions). The proposed theory ensures that the PU interference bound is
never violated due to SN transmissions. The analytically computed \textit{AUPWS} values matches reasonably well with the simulation-obtained values for saturated SN traffic. For SN traffic profiles with low duty cycle (or low burstiness), such as Profile 1, the SN does not have enough frames to consume the estimated maximum transmission duration $y_{max}$ in the remaining white space. As the the burstiness of SN traffic increases (from Profile 1 to Profile 3), the values of both of these performance parameters increase, and approaches the saturated SN traffic case.

The residual idle time distribution-based OSA approach proposed in this report is not directly comparable to most of the approaches proposed in the literature, which are based on idle time distribution. Both the approaches have their advan-
tages and disadvantages, as explained in Section 1. The two approaches exhibit the well known trade off between the sensing overhead, and the SN throughput and PU interference. The detailed study of this trade off is left for future work. In this report, we compare the proposed scheme with a commonly used Listen-Before-Talk (LBT) scheme, which senses the channel for every frame transmission till it either collides with the PU transmission or senses the channel busy (due to PU transmission). In both these cases, the SN backs off exponentially with mean channel busy time parameter, and senses the channel again\(^4\). We compare two statistics: \textit{average throughput of the Secondary Node} (total number of SU frames transmitted / total simulation time) and \textit{the number of SN frames transmitted per sensing operation} (total number of SN frames transmitted / total number of sensing operations performed). The second parameter can be used to compute the sensing overhead (in seconds) per transmitted SN frame. Since LBT single frame transmission scheme does not comply with any PU-specified interference constraint, therefore in simulations, we observe the PU interference generated by the LBT scheme and then run the proposed scheme for the same value of the PU interference constraint. This enable us to compare the two schemes with respect to the above statistics for similar PU interference constraints.

Figures 11 and Figure 12 shows respectively the \textit{average SN throughput} and \textit{SN frames transmitted per sensing operation} statistics for LBT and the residual idle time distribution-based proposed access scheme when the channel idle and busy times are 2-Erlang distributed. Figures 13 and Figure 14 show these statistics for Uniform Primary channel occupancy distribution. Since the LBT scheme transmits single frame per sensing operation, it incurs significant sensing overheads, as compared to the proposed scheme that transmits multiple frames per sensing operation. Therefore, both the statistics have lower values for LBT scheme as compared to the proposed scheme. Note that since the \textit{SN frames transmitted per sensing operation} statistic value is computed by dividing the total number of SN frames transmitted by the total number of sensing operations (including those for which the channel was sensed busy), therefore, its value for LBT scheme is slightly less than one.

\(^4\)We assume that in LBT scheme, the SN knows the channel occupancy distribution and parameters, but does not use this information for estimating transmission duration; it just transmits a single frame per sensing operation if the channel is sensed idle, and uses the mean channel busy time only for backoff purpose.
9 Conclusions and Future Work

Opportunistic channel access approaches based on channel idle and busy time distributions have recently been investigated. Most of these approaches have used channel idle time distribution to estimate how long (or how many secondary frames) to transmit in a white space, once the white space is identified. They usually make assumption that the start of the white space is known (which require continuous sensing of the channel). Such schemes are appropriate for the secondary devices, which are not energy constrained (for e.g., which are connected to a source of power supply). In this report, we describe a channel access scheme which uses the residual (remaining) idle time distribution to estimate how long to transmit in the remaining white space, once the white space is detected. The SN using this scheme senses the channel only when it has one or more frames to transmit, and therefore, incurs much less sensing overheads. Such a scheme is useful for energy-constrained and mobile secondary devices, which can not be recharged frequently. Our simulation results validate that the opportunistic SN transmissions based on the proposed theory do not violate the acceptable interference bound set by the PU. We also show the benefits of the proposed scheme over a commonly used Listen-Before-Talk scheme in which the SN transmits one frame per sensing operation (if the channel is sensed idle).

As a part of our ongoing and future work, we plan to compare our approach proposed in this report with an alternative approach reported in literature (such as [10]), which uses channel idle time distribution and performs continuous sensing to opportunistically use the white spaces. These two approaches exhibit the well known trade off between the sensing overhead, and the SN throughput and PU interference. We plan to compare these approaches for saturated as well as bursty SN traffic profiles.

A Derivation of Residual Idle Time Density and Distribution Function when Primary Channel Occupancy (Idle and Busy Time Periods) is 2-Erlang Distributed

We first derive density and distribution function for an r-phase Erlang distribution. Subsequently, we obtain these functions for 2-Erlang distribution by using $r = 2$.

The distribution function and mean value expression for an r-Erlang distributed
channel idle time period is given as:

\[ F_I(q) = 1 - \sum_{p=0}^{r-1} \frac{\lambda_i^p q^p e^{-\lambda_i q}}{p!} \]  \hspace{1cm} (37)

and

\[ E[I] = \frac{r}{\lambda_i} \]  \hspace{1cm} (38)

Substituting above expressions in (1), we get,

\[ f_{RI}(y) = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p y^p e^{-\lambda_i y}}{p!} \]  \hspace{1cm} (39)

The distribution function can be obtained using density function as follows:

\[ F_{RI}(y) = \int_0^y f_{RI}(q) dq \]

\[ = \int_0^y \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p q^p e^{-\lambda_i q}}{p!} dq \]

\[ = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p}{p!} \int_0^y q^p e^{-\lambda_i q} dq \]  \hspace{1cm} (40)

The integral in the above equation has the following solution (see [12, pp. 357]):

\[ \int_0^y q^p e^{-\lambda_i q} dq = \frac{p!}{\lambda_i^{p+1}} - e^{-\lambda_i y} \sum_{k=0}^{p} \frac{p! y^k}{k! \lambda_i^{p-k+1}} \]  \hspace{1cm} (41)

Substituting the integral value in (40), the distribution function is written as:

\[ F_{RI}(y) = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p}{p!} \left[ \frac{p!}{\lambda_i^{p+1}} e^{-\lambda_i y} \sum_{k=0}^{p} \frac{p! y^k}{k! \lambda_i^{p-k+1}} \right] \]

\[ = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \left[ \frac{1}{\lambda_i} - \frac{\lambda_i^p e^{-\lambda_i y}}{p!} \sum_{k=0}^{p} \frac{p! y^k}{k! \lambda_i^{p-k+1}} \right] \]

\[ = \frac{\lambda_i}{r} \left[ \frac{r}{\lambda_i} - \sum_{p=0}^{r-1} \sum_{k=0}^{p} \frac{\lambda_i^p e^{-\lambda_i y} y^k}{p! k! \lambda_i^{p-k+1}} \right] \]

\[ = 1 - \frac{\lambda_i^r}{r} \sum_{p=0}^{r-1} \sum_{k=0}^{p} \frac{\lambda_i^k e^{-\lambda_i y} y^k}{\lambda_i k!} \]

\[ = 1 - \frac{1}{r} \sum_{p=0}^{r-1} \sum_{k=0}^{p} \frac{(\lambda_i y)^k e^{-\lambda_i y}}{k!} \]  \hspace{1cm} (42)
Expanding the summation term on the RHS of the above equation, we get

\[
F_{RI}(y) = 1 - \frac{1}{r} \left[ e^{-\lambda_i y} + \{1 + \lambda_i y\} e^{-\lambda_i y} + \{1 + \lambda_i y + \frac{(\lambda_i y)^2}{2!}\} e^{-\lambda_i y} + \cdots + \{1 + \lambda_i y + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!}\} e^{-\lambda_i y} \right]
\]

\[
= 1 - \frac{e^{-\lambda_i y}}{r} \left[ 1 + \{1 + \lambda_i y\} + \{1 + \lambda_i y + \frac{(\lambda_i y)^2}{2!}\} + \cdots + \{1 + \lambda_i y + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!}\} \right]
\]

\[
= 1 - \frac{e^{-\lambda_i y}}{r} \left[ r + (r-1)\lambda_i y + (r-2)\frac{(\lambda_i y)^2}{2} + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!} \right]
\]

\[
= 1 - \frac{e^{-\lambda_i y}}{r} \sum_{p=0}^{r-1} (r-p) \frac{(\lambda_i y)^p}{p!}
\]

\[
= 1 - \sum_{p=0}^{r-1} (1 - \frac{p}{r}) \frac{(\lambda_i y)^p}{p!} e^{-\lambda_i y}
\]

(43)

For 2-Erlang Channel idle time distribution, we substitute \( r = 2 \) in (39) and (43) to get the following:

\[
f_{RI}(y) = \frac{\lambda_i e^{-\lambda_i y}}{2} \left[ 1 + \lambda_i y \right]
\]

(44)

and

\[
F_{RI}(y) = 1 - e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right]
\]

(45)

**B Derivation of Residual Idle Time Density and Distribution Function When Primary Channel Occupancy (Idle and Busy Time Periods) is Uniformly Distributed**

Substituting expressions for \( F_I \) and \( E[I] \) form (27) and (28) into (1), we get.

\[
f_{RI}(y) = \frac{2}{(a + b)} \left[ 1 - \frac{(y - a)}{(b - a)} \right]
\]

\[
= \frac{2(b - y)}{(b^2 - a^2)}
\]

(46)
The distribution function can be obtained using density function as follows:

\[ F_{RI}(y) = \int_{q=0}^{y} f_{RI}(q)dq \]

\[ = \int_{q=0}^{y} \frac{2(b-q)}{(b^2-a^2)}dq \]

\[ = \frac{2}{(b^2-a^2)} \left[ b \int_{q=0}^{y} dq - \int_{q=0}^{y} qdq \right] \]

\[ = \frac{2}{(b^2-a^2)} \left[ by - \frac{y^2}{2} \right] \]

\[ = \frac{2by - y^2}{b^2 - a^2} \quad (47) \]

### C Newton Raphson Method

Let us assume that \( f_n(y) \) if the function whose root is to be found, and \( f_n'(y) \) is its derivative with respect to \( y \). Let us consider two values: \( \epsilon \), which denotes the desired accuracy in the value of the root, and \( \delta \), which denotes the value to categorize the slope as very small (see [14]). Let \( y_0 \) denote the initial estimate (guess value) of the root, and \( max\_iters \) denote the maximum number of iteration for which the convergence should be tried. In our simulations, \( \epsilon = 0.000001 \), \( \delta = 0.00001 \) and \( y_0 = 0.0001 \). (The initial guess value of the root can be selected as a value between 0.0 and the mean idle time.) \( max\_iters \) is set to a sufficiently large value. The Newton Raphson algorithm is given on the next page.

### References


Algorithm 2 Newton Raphson Algorithm

1: while $iter \leq max_{iters}$ do
2: Compute $f_n(y_0)$. 
3: Compute $f'_n(y_0)$. 
4: if $abs(f'_n(y_0)) \leq \delta$ then 
5: Slope too small. 
6: Break 
7: else 
8: $y_1 \leftarrow y_0 - \frac{f_n(y_0)}{f'_n(y_0)}$ 
9: if $abs(f_n(y_1)) \leq \epsilon$ then 
10: Convergence achieved. 
11: Return $y_1$ 
12: else 
13: $y_0 = y_1$ 
14: $iter \leftarrow iter + 1$ 
15: end if 
16: end if 
17: end while


