Stochastic Model Based Opportunistic Spectrum Access in Wireless Networks

Manuj Sharma* and Anirudha Sahoo*
* Department of Computer Science and Engineering
Indian Institute of Technology Bombay, Mumbai, India
Email: manuj@it.iitb.ac.in, sahoo@cse.iitb.ac.in

Abstract—We present a stochastic model based opportunistic channel access and transmission scheme for cognitive radio enabled secondary users for single data channel. We refer to this scheme as RIBS (Residual Idle Time Distribution based Scheme). In this scheme, the SU randomly sensess the channel and if the channel is sensed idle, then it uses residual idle time distribution to estimate its transmission duration such that the probability of its interference with PU is below a predefined threshold. We derive analytical formulae for computing the channel occupancy (channel idle time) distribution due to Primary User traffic in two different scenarios. In the first scenario, we synthetically generate channel occupancy data due to PU transmissions using two standard distributions (2-phase Erlang and Uniform distribution). We validate the analytical formulations using simulation with synthetic channel occupancy. In the second scenario, we simulate a TDMA-based PU network that runs realistic applications (VoIP and Web browsing). A pair of sender and receiver SU uses RIBS to opportunistically transmit on the channel. We analyze the characteristics of white spaces obtained due to these realistic applications, list some of the challenges in using RIBS in realistic scenarios, and provide comprehensive methodology to use RIBS in primary network running real applications.

Index Terms—Dynamic Spectrum Access, Opportunistic Spectrum Access, Cognitive Radio, MAC

I. INTRODUCTION

Dynamic Spectrum Access (DSA), has emerged as a promising spectrum sharing paradigm to efficiently utilize the electromagnetic spectrum. In this approach, the unlicensed users (also referred as Secondary Users or SUs) can dynamically use licensed spectrum bands to increase spectrum utilization. One important category of DSA is interweave model (see [1], [2] and [3] for complete categorization of DSA model). In interweave model, an SU cannot use a licensed spectrum band as long as PU is using the band. This model is also called opportunistic spectrum access (OSA), as SUs can opportunistically use a band only when no PU is using it. OSA approach is usually used by secondary users in hierarchical spectrum sharing paradigm. In this paradigm, SUs share the spectrum with PUs. However, PUs have higher priority in accessing the spectrum band, and SUs must vacate the band as soon as any PU starts using it. The SUs must also abide by certain Interference Management (IM) policy, which specifies the maximum bound on the interference that a PU can tolerate due to SU transmissions. There are three main IM policies studied in the literature [4]: (i) Evacuation-time based policy that defines how fast an SU should vacate a channel after a PU is detected (ii) Power based policy that defines the power limits that each SU needs to take into account when using a PU channel, and (iii) Collision probability-based policy, which defines the upper bound on the probability of interference of SU to PU. The IM policy should be known a priori to both SUs and PUs.

OSA approach, though promising, introduces many new challenges for secondary users, such as spectrum sensing, opportunity detection, and opportunistic transmission by SUs while ensuring that the interference constraints specified in the IM policy are not violated. Channel occupancy (channel idle and busy time) distribution due to Primary User traffic has recently been used to devise opportunistic channel access schemes for secondary networks. Several authors have proposed opportunistic channel access schemes in which SUs keep track of the idle periods (white spaces1) on the channel and make use of a white space as long as probability of interference of their frame transmission with PU is below a predefined threshold value [5]. Such schemes use channel idle time distribution for opportunistic transmission. In these schemes, SU keeps track of the amount of idle time that has elapsed from the start of current idle cycle, and computes the probability of successful transmission of its next frame, conditioned on the elapsed idle time. The main limitation of such schemes, therefore, is that the SU needs to continuously sense the channel (irrespective of whether it has frames to transmit or not) in order to keep track of the start of each idle cycle. Continuous sensing of the channel significantly drains the energy of SU over long run, and is not suitable for energy-constrained SU devices. In this report, we propose a stochastic model based opportunistic spectrum access approach which can be used by SUs to access primary channel2 such that the impact on PU is within acceptable limits. There are two main motivations for our work: (1) To increase the utilization of a primary channel by having SUs use the channel within

1In this report, we use the terms channel idle time/white space and spectrum band interchangeably. A channel can be considered as a smallest unit of a spectrum band that can be used for communication by a wireless device. A spectrum band may have multiple channels.

2In this report, we refer to the terms spectrum band and channel interweave model synonymously.
acceptable limits of interference with PU and (2) to reduce the channel sensing overhead of SUs so that the channel access scheme can also be used by energy-constrained secondary networks such as cognitive sensor networks [6], or battery-operated low end wireless devices. We propose an opportunistic channel access scheme for secondary users called RIBS (Residual Idle Time Distribution based Channel Access Scheme). RIBS uses channel’s residual idle time distribution to compute transmission duration for an SU such that the probability of interference of SU’s transmission with PU is less than or equal to a pre-defined threshold. The residual idle time distribution is computed using the known idle time distribution for the channel. RIBS is a non-persistent carrier sensing scheme in which SU does not need to continuously sense the medium if it is busy nor does it require to keep sensing to track the start of each idle period. This makes RIBS an energy efficient scheme. Details of RIBS are presented in Section V.

The main contributions of our work, which is described in this report, are:

1) We propose an opportunistic channel access scheme that is based on residual idle time distribution of the channel.
2) The proposed scheme does not require the SU to continuously sense the channel and also ensures that the probability of interference of SU transmissions with PU is upper bounded by a predefined threshold. It has low sensing overhead and hence energy efficient.
3) We provide a methodology for opportunistic spectrum access when on-off traffic of PU can be of any general distribution.
4) We use simple backoff scheme to access channel idle periods and specify guidelines in choosing the backoff parameter in different operating scenarios.

II. RELATED WORK

FCC initiative on exploring the approaches for efficient spectrum utilization and interference management led to many researchers investigate and categorize the methods of spectrum sharing and management. Among many approaches, Dynamic Access Spectrum (DSA) was one of the most actively investigated approaches for efficient spectrum utilization. A comprehensive survey and research issues in almost all the aspects of DSA and cognitive radio networks is presented in [7] and [8]. Author in [9] presents detailed taxonomy and models for dynamic spectrum access networks. Zhao et al., presented a survey of dynamic spectrum access [1] in which they categorize the dynamic spectrum access models into three categories: dynamic exclusive use model, open sharing model (also referred to as spectrum commons model), and hierarchical access model. Similar categorization of spectrum sharing approaches is also made in [4]. All these studies have categorized the hierarchical access model as underlay and overlay model. In underlay model, an SU can transmit on a channel along with PU provided its transmission does not increase the interference to PU beyond a predefined threshold.

In overlay model, an SU can transmit on a channel only when no PU is using the channel, and it must evacuate the channel as soon as a PU appears on the channel. However, recent studies such as [2] and [3] categorizes hierarchical model into underlay, overlay and interweave models. In these studies, a distinction is made between the overlay and interweave models. In interweave model, the transmissions of SU and PU on a licensed channel are interweaved and the SU can use a channel only if no PU is using the channel. If an SU is using a channel, then it must relinquish the channel as soon as a PU starts transmitting on the channel. The interweave model is also called as opportunistic spectrum access, as the SUs can opportunistically use a channel only when no PU is using it. In overlay model, like underlay model, the SUs are allowed to transmit on a licensed channel even when PU is using the channel. However, using advanced coding techniques and cooperation with PUs, SUs assists in enhancing the performance of PUs. Our work presented in this report is based on interweave mode of operation and we adopt the categorization suggested in [2] and [3] in our work.

Interweave paradigm has been the most actively and widely studied paradigm for dynamic spectrum access. Figure 1 illustrates interweave communication. Red blocks on each channel depict that the channel is occupied by PU. The dotted green lines between the red blocks denote the spectrum white spaces when no PU is using the channel. Duration $t_{sw}$ denotes the channel switching time (including time to locate new opportunity and tuning the radio on the new channel).

![Fig. 1: An example of interweave communication](image)

3In [9], hierarchical access model is referred to as ‘shared use of licensed spectrum’ model.
shows such channel switching by an SU. In the illustration, SU vacates C4 and switches to channel C2. \( t_{sw} \) denotes the channel switching time. The SU opportunistically transmit on C2 until a PU appears on C2 (shown as point S), at which the SU vacates C2 and switches to C1.

Research in the area of DSA, specifically in overlay and interweave models, got major initial impetus from nXt Generation Communications (XG) program of DARPA. The goal of this program was to develop technologies and systems for dynamic spectrum access networks, and the initial test field results were reported in [10]. Similar initiatives were made by Ofcom (spectrum regulatory body in UK) [11]. In [12], authors proposed an architecture for Dynamic Intelligent Management of Spectrum for Ubiquitous Mobile-access network (DIMSUMnet). DIMSUMnet was one of the initial architectures proposed for DSA, which used the concept of Coordinated Access Bands (CAB) and statistical multiplexing of spectrum access in CAB. CAB is a contiguous chunk of spectrum reserved by regulating authorities and is managed by a spectrum broker for controlled dynamic spectrum access among SUs.

Research work on DSA using primary channel models include modeling of PU activity and primary channel occupancy based on detail measurements, and devising channel access and control schemes based on these models. In [13], authors analyze cellular primary usage that is based on call records collected at hundreds of base stations inside a cellular network. This is in contrast to many other measurement based studies where primary channel usage models are constructed using active sensing of the channel. The authors have modeled both call arrivals and call durations. The study reports that although the call arrivals can be characterized by Poisson process, yet call durations show high variability with the variance three times the mean, which is much higher than that of commonly used exponential distributions. The authors point out that knowledge of the PU usage model enables resource provisioning for SUs, and also help spectrum owners optimize there spectrum auction process to SUs (for example, for dynamic short term leases) such that PUs performance is not impaired. In [14], authors assume that opportunistic transmission of an SU spans multiple primary channels. Authors consider N independent frequency bins (FBs) in which PUs operate. The activity of a PU in each bin is modeled as an exponential ON/OFF process. The complete system state due to operation of PUs in all the bins is modeled as \( 2^N \)-state CTMC. Distribution of transmission opportunity (TO) for an SU is derived using lumped Markov chain model. TO is defined as the continuous time elapsed when at least one FB becomes idle (available for SU transmission) to the instant when all the FBs become unavailable to SU. The authors conclude that TO follows Hyperexponential distribution. Geirhofer et al. have empirically shown [15] that the channel occupancy in a WLAN system can be modeled as a 2-state (ON-OFF) Semi Markov chain, where ON state models the channel occupancy and OFF state models the white spaces (idle periods). The authors have fitted a generalized Pareto distribution to the idle time periods and approximated it with phase-type distribution (Hyper-Erlang distribution) as the later is analytically more tractable. In [16], authors have applied their model to enhance the coexistence of Bluetooth and WLAN in unlicensed ISM band. The authors use the white space distribution to devise adaptive hopping sequence for Bluetooth-based SUs to mitigate interference between SUs and PUs (WLAN devices). However, in [17], the authors have approximated the semi-Markov model by a Continuous Time Markov Chain (CTMC) in their Cognitive Medium Access (CMA) protocol in order to simplify analysis. The CMA approximation corresponds to fitting exponential idle and busy time distributions. The authors describe both fully observable CMA (where state of all the channels can be observed simultaneously at the beginning of each slot) and partially observable CMA (where state of only a subset of channels can be observed simultaneously at the beginning of each slot), and devise optimal access strategies for both the scenarios.

In [18] and [19], the authors have proposed channel sensing and transmission strategies under the Partially-Observable Markov Decision Process (POMDP) framework, but assumes that both the Primary as well as the Secondary networks are slotted. In [18], authors model the network state (consisting of N channels) as Markov chain with known transition probabilities and initial state probability vector. PU network is slotted and the secondary network is assumed to be synchronized with primary transmission slot structure. At the beginning of each slot, the SU selects a set of channels to sense. For the current state of the underlying Markov process, the SU observes the outcome of sensing and selects a set of channels to access. For a chosen action (where action refers to transmission on one of the selected channels), the SU gets a reward (which is equal to the number of bits delivered) at the end of the slot. The authors have devised a cognitive MAC to maximize the reward in T slots under the constraint that the probability of collision is bounded below a threshold. The authors in [19] also assume a slotted primary network and POMDP framework, but their objective is to maximize the gain, where gain is defined as the difference between the throughput achieved in each slot if the SU transmits in the slot, and the energy cost due to sensing. Authors in [20] model the interactions between PUs and SUs as a CTMC. For two SU case, the authors describe a 5-state CTMC, where state ‘0’ means no user operates in the spectrum, state ‘1’ means user \( \gamma \) operates in the spectrum with \( \gamma \in \{SU-A, SU-B, PU\} \), and state ‘2’ means both SU-A and SU-B operate in the spectrum. The authors devise schemes for state dependent spectrum access probabilities for SU-A and SU-B so as to maximize average SU throughput. The authors have also presented case for N SUs.

In [21] and [22], the authors consider an unslotted Primary Network with multiple channels and a slotted Secondary Network, which sense the channels at the beginning of each slot. In [21], the authors model the occupancy of the channels using CTMC. The Secondary Network is assumed to have a slotted transmission structure in which, at the beginning of each slot, the secondary node decides which channel to sense and potentially transmit over. The scheme essentially adopts a single secondary frame transmission per sensing operation, but the decision to transmit in a slot is based on an optimal joint sensing and access strategy of the secondary
networks. This strategy maximizes the expected total number of bits that can be delivered by the secondary node in T slots under a given collision constraint with the PU. In [22], the authors assume that channel idle and busy periods are exponentially distributed, and have proposed a learning-based approach in which each secondary user maintains an estimate of its own belief vector as well as that of other secondary nodes by learning collision events. Based on reward accrued (for successful transmissions) and the channel idle probabilities (computed using belief vectors), the secondary node computes the probability of sensing each channel so that collisions with other users are minimized. It then maps these channel sensing probabilities to a concrete decision in determining which channel to access, so that the total throughput of the secondary users in T slots is maximized. In [23], authors propose a scheme to compute ON and OFF durations of SU. The SU need not sense the channel at all. Whenever it has frames to send, it transmits for ON duration followed by a silent period of OFF duration. Unlike RIBS, SU transmission duration in this scheme does not vary with changing channel idle period profile, and is therefore, fixed.

Recently, Restless Multiarmed Bandit formulations are used for opportunistic channel access [24], [25], [26], [27]. These schemes are based on exploration versus exploitation trade off and do not require an SU to know the channel idle and busy period distributions a priori. However, logarithmic bounds for regret in these schemes are shown in literature only for Markovian and Bernoulli reward distributions. In contrast, RIBS is devised for any general continuous time distribution for idle periods and does not depend on distribution of busy periods. But, RIBS bound the interference probability to a specified threshold η and is not designed to provide logarithmic bounds for regret in these schemes. Therefore, RIBS is a Markovian Arrival Process (MAP) and the channel holding times are modeled by phase-type distributions. This work primarily focuses on the blocking probabilities of Primary and Secondary calls, and the waiting times of an SU call that is preempted due to arrival of a Primary call. In contrast, in our work we focus on how much an SU should transmit every time it senses the channel idle while keeping the probability of interference to PU below a predefined threshold. Instead of matrix-analytic method, we use residual idle time distribution in RIBS.

Many of the schemes mentioned above assume exponential distribution for idle and busy time periods for each channel, whereas others model the state of the complete system consisting of N PUs as 2N-state CTMC. In our work, the proposed channel access scheme RIBS works for any idle time distribution. If the idle time distribution does not have a closed form, then our scheme can be applied after approximating the continuous time distribution by appropriate Phase Type distribution.

Several schemes proposed in the literature use channel idle time distribution for opportunistic transmissions under interference constraint, which is defined as a part of Interference Management (IM) policy. In [30], authors propose schemes to provide probabilistic outage guarantee to PUs based on power sensing information obtained at the secondary user. Using the sensed Primary transmission power, SU estimates its distance from Primary transmitter, and the effective receiver range for Primary receivers and computes the outage probability of its transmission to PU receivers at a given transmission power. In essence, the proposed scheme ensures opportunistic transmission in spatial domain. In contrast, RIBS enables opportunistic transmission in temporal domain and computes the duration for which SU can transmit before PU appears on the channel so that the probability of interference to PU is bounded. In [5], the authors have used empirical PDF and CDF of a whitespace trace (idle periods) (obtained using whitespace measurements in simulation) to compute the sensing duration and the number of frames that an SU should transmit on sensing a channel idle, subject to a maximum PU interference probability bound. The authors compute, both analytically and using simulations, the Effective Secondary Throughput, and Primary User Interference. In [31] also, the SU determines the optimal transmission duration on finding a channel idle subject to maximum interference probability threshold. Both [5] and [31] use Collision probability-based IM policy, but they assume that the start of each idle period is known, and use channel idle time distribution in their MAC protocol design. In [32], the authors investigate the trade off between the sensing overhead and the overhead for link maintenance under the constraint of proper protection of PU communication using Evacuation-time based IM policy. In [33], the authors have proposed a distributed cognitive radio MAC (COMAC) protocol that enables SUs to dynamically utilize the spectrum while limiting the interference to PUs using a power based IM policy. In [34], the authors have proposed a cost and reward-based access policy to maximize the secondary network utility. The authors consider an unslotted Primary network, but their analytical formulation of the access scheme is based on the assumption that the secondary node can detect the beginning of each idle period. The RIBS scheme described by us in this report is designed for the Collision probability-based IM policy and does not need to sense the channel continuously to keep track of start of each idle period, as it is based on residual idle time distribution of the channel.

Some researchers have argued that none of the three channel sharing schemes, namely underlay, interweave, or overlay scheme, alone lead to full utilization of spectrum. In underlay schemes, SU transmits with a restricted power even if PU is not using the channel; in interweave schemes, SUs miss spectrum sharing opportunities when PU is using the channel; and, in overlay schemes, SUs generally use advanced coding techniques to encode their data with PU packets and do not exploit the spectrum white spaces when PU is not using the channel. Recently, schemes have been proposed which use interweave paradigm along with underlay or overlay paradigms. Such approaches have also been referred in literature as ‘mixed sharing of spectrum’. In [35], authors have proposed mixed
sharing of spectrum between PUs and SUs using a combination of underlay and interweave communications. When an SU senses a channel to be free (i.e., no PU is using the channel), then the SU transmits in interweave mode with higher power to increase its throughput. If the channel is busy due to PU transmissions, SU operates in underlay mode and transmits with restricted power so that the interference constraint at PU receiver is satisfied. In [36], authors have proposed a relay assisted spectrum sharing (RASS) scheme for SUs, in which a cooperative relay is introduced in mixed sharing secondary network. In RASS, when the channel is free, SU transmits with higher power using interweave mode of communication. When the channel is in use by PU, SU transmits its packets with restricted power to relay nodes, which decode and forward the packets with low power to receiver SU. Use of relay nodes in RASS leads to increased coverage of secondary nodes and reduces path loss between SUs, thereby enhancing SU throughput.

Mixed sharing schemes using overlay and interweave communications are also proposed in literature. In [37], authors propose Credit-based Overlay and Interweave (CONI) DSA protocol, which integrates the features of both overlay and interweave approaches. In this protocol, SUs assist PUs by relaying their messages in overlay mode and accumulate credits. In turn, they (SUs) are allowed (based on their credits) to transmit on the channel in interweave mode when no PU is transmitting, even if it leads to minor disruption in PU transmissions. Authors describe scenarios in which relaying of PU messages by SUs not only enables communication between a sender and a receiver PU which do not have a direct path between them, but may also lead to shorter path between those sender and receiver PUs who need to exchange messages through relatively larger number of intermediate primary relay nodes.

A number of researchers have proposed using geolocation-based approaches for accessing spectrum bands. In these approaches, a radio environment map (REM) is constructed that essentially characterizes the radio environment and a number of other information in the serviced region, such as available services, primary device locations, spectrum regulations, access policies, etc. These maps are stored in distributed databases and can be used by SUs operating in the region. Although this approach is not directly comparable to our work, it can be used to store primary channel models (idle and busy time distributions for the channels) and their parameters. A dedicated server can update these models whenever there is a change in the perceived radio environment. SUs can use these models to derive residual idle time distributions. Therefore REM-based approach can be used in conjunction with our work. Interested readers may refer to [38, pp. 325–366, Ch. 11], [39], and the references therein for more details on geolocation/REM-based approaches.

III. SYSTEM MODEL AND ASSUMPTIONS

We consider a hierarchical spectrum sharing paradigm in a wireless network. The network communicates using a single channel. The designated users of the single channel are set of users termed as Primary Users of the channel. There are other set of users known as Secondary Users which share the channel with PUs. However, the SUs operate in interweave mode and can access the channel only when no PU is using the channel. Thus, the SUs have to look for the so called white spaces (i.e., idle periods in the channel) and opportunistically transmit their packets. The PUs are not aware of SU’s transmission and can initiate their transmission whenever they require. It is SU’s responsibility to detect the PU transmission and evacuate the channel. The SUs must also ensure that their transmissions do not cause interference to any PU beyond a certain limit. This limit is denoted as $\eta$. Thus, $\eta$ is the upper bound on probability of interference of SU’s transmission with PUs. Here, probability of interference refers to the probability that SU’s opportunistic transmission in an idle cycle will interfere with PU. In other words, it is the probability that PU will start using the channel before SU finishes its opportunistic transmission in the idle cycle. Other works in literature have also assumed similar upper bound to limit SU’s transmissions (~[5], [4], [40]).

In our work, we make the following assumptions:

1) SU knows the $\eta$ value for the channel as well as the channel idle time distribution due to PU transmissions.
2) Channel occupancy distribution and parameters remain stationary (stable) for a relatively much larger duration than the opportunistic transmission duration of SU.
3) SU performs perfect sensing.
4) PU uses non CSMA protocol.

The second and third assumptions are made to apply model-based channel access scheme and to keep the analytical formulation tractable. We assume that PU uses a non CSMA protocol because of the following reason. A CSMA-based PU senses the channel and backs off if the channel is busy due to some secondary node’s transmission. This violates hierarchical model as the PU is forced to wait until SU finishes its transmission, and adversely affects PU’s performance such as throughput and delay. However, RIBS can be used with other types of primary networks such as slotted primary networks (Slotted Aloha or TDMA based networks) and unslotted primary networks (such as pure Aloha based networks), without requiring any synchronization with the primary network. This is so because in RIBS, scheduled sensing time of SU need not be aligned to PU slot boundary (if PU network is slotted). If PU appears on the channel in one of its slot while SU is transmitting on the channel, the SU evacuates the channel.

IV. OVERVIEW OF RIBS

In RIBS, whenever an SU has one or more packets for transmission, it generates a channel sensing schedule and finds the time instant at which it will sense the channel\(^4\). This sensing schedule is generated such that the scheduled sensing happens at random time\(^5\). The SU waits up to the scheduled

---

\(^4\)The SU does not immediately sense the channel as soon as it gets a frame to transmit. Instead, it generates a sensing schedule using a scheme that is described later in this section.

\(^5\)Random sensing of the channel is required for using RIBS to opportunistically transmit on the channel. Details are given in Section V.
time and then senses the channel for a predefined sensing duration. If the channel is sensed busy, the SU backs off and senses the channel again. The backoff duration value is generated using Exponential distribution. If the channel is sensed idle, then the SU transmits its frames opportunistically with PU (also referred as interference probability threshold) and transmits its frames for remaining $(y_{\text{max}} - s)$ duration if the channel is sensed idle.

In RIBS, we model the channel occupancy due to PU traffic as an Alternating Renewal Process (ARP). The process alternates between idle and busy states and renews after each renewal cycle of idle and busy periods. Hence, we derive and use the residual idle time distribution based on random incidence in a renewal cycle\(^6\) (see \[41, pp. 328–331\]). The residual idle time distribution is derived using the known channel idle time distribution of PU traffic. Channel idle and busy time distributions can be any general distribution rather than exponential distribution that is assumed in many works in the literature. Once the residual idle time distribution is computed, the SU computes the maximum duration for which it can use the channel when it randomly senses the channel to be idle. We denote the duration of this whole transmission operation by $y_{\text{max}}$. We use the term ‘transmission operation’ by an SU to denote the action of the SU to sense the channel for duration $s$ and then transmit for the remaining $(y_{\text{max}} - s)$ duration if the channel is idle\(^7\). One ‘transmission operation’ does not refer to the operation of each frame transmission;

\(^6\)The phrase ‘random incidence in a renewal cycle’ is used in \[41\] to denote the random time at which the renewal process is observed.

\(^7\)Whenever we refer to SU transmitting its frames within $y_{\text{max}}$ duration, we mean SU transmission within one transmission operation, including sensing of the channel for duration $s$ and transmission of frames for a maximum of the remaining $(y_{\text{max}} - s)$ duration.
instead it refers to the operation of sensing the channel for duration $s$ and transmitting (possibly multiple frames) for maximum of $(y_{max} - s)$ duration if the channel is idle. The method to compute $y_{max}$ is described in Section V. Depending on its frame size and channel data transfer rate, the SU transmits one or more data frames within the $(y_{max} - s)$ duration. The SU transmits these data frames within $(y_{max} - s)$ duration without any further sensing of the channel, thereby reducing the sensing overhead for each transmitted frame. It receives an acknowledgment from the receiver for each successfully received frame.

Figure 2 and Figure 3 show two possible transmission scenarios on a single data channel. In both the figures, $T_A$, $T_B$, and $T_c$ denote respectively the start of an idle period, end of the idle period (or, equivalently, start of the next busy period $B$ due to PU transmission), and end of the busy period within a renewal cycle. Assume that the scheduled time for the SU to sense the data channel is $t_1$. The SU senses the data channel at time $t_1$ for a predefined duration $s$ (from time $t_1$ to $t_2$), and since the channel is sensed idle, it transmits frames for a maximum duration of $(y_{max} - s)$ (which is equal to $t_3 - t_2$)\(^8\). The duration $(T_B - t_1)$, represented as $RI$, denotes the residual (remaining) idle time in the cycle, starting from the incidence time $t_1$ into the ARP\(^9\). The duration $(T_B - t_2)$, represented as $(RI - s)$, denotes the available residual idle time in the cycle after the SU has sensed the channel for duration $s$. In Figure 2, since $RI$ is more than $y_{max}$, i.e., $(RI - s)$ is more than $(y_{max} - s)$, the SU successfully transmits for the entire $(y_{max} - s)$ duration.

On the other hand, in Figure 3, since $RI$ is less than $y_{max}$, the SU transmits only for $(RI - s)$ duration and collides with the next busy period due to PU transmission, thereby interfering with the PU transmission. In both the above cases, if the SU has more frames to transmit, it generates a new sensing schedule for sensing the channel again.

Once the SU finishes the transmission (either for complete $(y_{max} - s)$ duration, or for $(RI - s)$ duration until it collides with PU), and it has more frames in its transmission queue to transmit, it generates a sensing schedule to sense the channel again. We explain the generation of channel sensing schedule using Figure 4 in which the SU senses the channel for duration $s$ (from $t_1$ to $t_2$). Since the channel is sensed idle, the SU transmits frames for $(y_{max} - s)$ duration (from instant $t_2$ to $t_3$). There are three possible situations that can arise during this transmission operation: (1) The channel is sensed busy during channel sensing (2) The channel becomes busy during SU transmission (due to re-appearance of PU on the channel) and as a result, the SU interferes with PU, and (3) SU successfully transmits for $(y_{max} - s)$ duration (up to $t_3$) within the current white space where PU does not appear on the channel in this duration\(^10\). In all of these situations, the SU needs to generate the next sensing schedule. In order to do this, it generates a random backoff value, which we denote by random variable $BO$. This value is generated using Exponential distribution with mean backoff duration of $E[BO]$. In Section IX, we will elaborate more on the selection of mean BO parameter value for exponentially distributed backoffs). The SU then compiles the next incidence time by adding the $BO$ value to the previous sensing time\(^11\). For example, in Figure 4, after transmitting successfully for $(y_{max} - s)$ duration (up to $t_3$), the SU generates $BO_1$ value and computes the next incidence time as $t_4 = t_3 + BO_1$. Since the next incidence time $t_4$ is greater than the current time $t_3$, the SU records $t_4$ as the next scheduled sensing time. It waits until time $t_4$ and then senses the channel. Generating the backoffs using exponential distribution ensures that the incidence times are random and memoryless.

---

\(^8\)If SU does not have enough frames to consume the duration of $y_{max}$, it stops after sending all the frames.

\(^9\)The term “incidence time into the ARP” refers to the time at which SU can potentially sense the channel whose occupancy is modeled using the ARP. At some incidence times, the SU can not sense the channel, as will be explained later in this section.

\(^10\)Figure 4 depicts the occurrence of situation (3).

\(^11\)The SU keeps record of the last time it sensed the channel. However, it does not keep record of the boundaries of idle and busy periods on the channel, which would have required it to sense the channel continuously.
Note that depending on the exponential backoff parameter value, it is possible that the generated BO\(_1\) value is less than \(y_{\text{max}}\), i.e., \((t_1 + BO_1) < t_3\). This case is shown as incidence time \(a\) in Figure 5. In such a case, the SU generates a new value of BO (say, BO\(_2\)) and computes the next incidence time as \((t_2 + BO_1 + BO_2)\). The SU repeats this backoff process until it surpasses the current time. This surpassed incidence time becomes the new sensing time for the SU. For example, since the incidence time \(b\), which corresponds to \((t_1 + BO_1 + BO_2)\), and incident time \(c\), which corresponds to \((t_1 + BO_1 + BO_2 + BO_3)\), are less than the current time \(t_3\), the SU generates another backoff value BO\(_4\) and further computes the next incidence to be at time \(t_4\) (which is equal to \(t_1 + BO_1 + BO_2 + BO_3 + BO_4\)). The incidence time \(t_4\) (shown as point \(d\)) is more than the current time \(t_3\), and therefore, the SU records \(t_4\) as the next scheduled sensing time. The SU waits until time \(t_4\) and then senses the channel for channel sensing duration \(s\). As the channel is sensed busy at \(t_4\), the SU generates another backoff value BO\(_5\) and computes the next incidence time as \(t_5 = t_4 + BO_5\) (shown as point \(e\)). The SU records \(t_5\) as the next scheduled sensing time. This process of generating the sensing schedules is repeated in the manner described above.

In previous descriptions (Figure 4 and Figure 5), we assumed that SU has more frames to transmit when its \(y_{\text{max}}\) duration is completed (at time \(t_3\)). Now we consider another scenario (see Figure 6). In this scenario, the SU senses the data channel (from \(t_1\) to \(t_2\)) and finds it to be idle, transmits its frames for maximum of \((y_{\text{max}} - s)\) duration (from \(t_2\) to \(t_3\)), but after the transmission is over, it does not have any more frame in its transmission queue to transmit. Suppose that the SU’s MAC layer gets new frame to transmit (from upper layer in the stack) at a later time \(t_4\). The SU then generates the sensing schedule for the channel by generating exponential backoff values from its previous sensing instant \(t_1\) until the backoff value exceeds the current time \(t_4\). The time instant \(t_5\) (shown as point \(e\)) becomes the scheduled sensing instant at which the SU would sense the channel and transmit the frames if the channel is sensed idle.

It is important to note the following points in the above description:

1) In Figure 5, the SU first senses the channel at time \(t_1\) and then senses it at time \(t_4\). Similarly, in Figure 6, the SU first senses the channel at time \(t_1\) and then senses it at time \(t_5\). The intermediate incidence time values (\(a\), \(b\) and \(c\) in Figure 5, and \(a\), \(b\), \(c\) and \(d\) in Figure 6) are generated as a part of random backoffs until we exceed the current time instant. Since these intermediate incidence time values have already elapsed on the time line, SU can not perform channel sensing at these times.

2) The backoff (BO) starts from the previous incidence time rather than the end of transmission time so that inter-incidence durations are exponentially distributed and incidence process into the ARP is Poisson. This enables us to use PASTA property to derive analytical formulation for average raw SU throughput (see Section VI-2).

3) SU performs non-persistent sensing.

We have emphasized in this section that the channel sensing schedule is generated such that the channel sensing is random. Random channel sensing is required to apply the RIBS scheme as described in the next section. After describing the channel access scheme, we will revisit the issue of random channel sensing in more detail in Section IX. Another important point to note from the above description is that in RIBS, the SU performs non-persistent carrier sensing, and hence does not sense the channel continuously. Instead, the SU generates a sensing schedule whenever it has one or more frames to transmit, and senses the channel only at the scheduled sensing time.

V. CHANNEL ACCESS BASED ON RIBS

Whenever an SU senses the channel to be idle for transmission, the duration for which it should transmit is determined such that the probability of its interference with PU transmission is less than or equal to a predefined threshold, which we denote as \(\eta\). In RIBS, the SU uses the residual idle time distribution to compute this duration. The residual idle time distribution is obtained using the known idle time distribution for the channel. Let random variable \(I\) represent channel idle time whose Cumulative Distribution Function (CDF) is given by \(F_I\). Suppose an SU senses the channel at a random time instant (such as at time \(t_1\) in Figure 2 to Figure 6) and finds it to be idle. We denote the channel residual idle time in the sensed idle cycle by random variable \(RI\). For random sensing in an idle cycle, the residual idle time density function \(f_{RI}\), and the residual idle time distribution function \(F_{RI}\) can then
be computed as follows \([41, \text{ pp. 331}]\):

\[
f_{RI}(y) = \frac{1 - F_I(y)}{E[I]} \quad (1)
\]

\[
F_{RI}(y) = \int_0^y f_{RI}(z)\,dz \quad (2)
\]

\(E[I]\) in (1) denotes the mean channel idle time. If SU uses the channel for duration \(y\), then \(F_{RI}(y)\) denotes the probability that the remaining idle time will end (due to appearance of PU on the channel) before SU transmission is over. In this case, SU transmission will interfere with PU transmission. In other words, \(F_{RI}(y)\) gives the probability of interference of SU’s transmission with PU if SU uses the channel for duration \(y\) on randomly sensing the channel to be idle. In RIBS, this interference probability is upper bounded by \(\eta\). That is,

\[
F_{RI}(y) \leq \eta \quad (3)
\]

Since, we also want to maximize the duration for which the SU transmits when it detects an idle period, we find the maximum value of \(y\) for which the inequality (3) holds. We denote this maximum value as \(y_{max}\). The channel access algorithm used by SU is broadly summarized in Algorithm 1. We now briefly describe the steps performed in Algorithm 1. Whenever an SU has frames to send, (say, at time \(t\)), it generates sensing schedule for the channel (Line 11). The sensing schedule is generated by starting the exponentially distributed backoffs from the previous sensing time \(s\). These backoffs are generated with mean parameter value \(BO\), as described in Section IV. The SU waits until the scheduled time \(t_s\) and then senses the channel. It also stores the sensing time \(t_s\) in variable \(t_p\), so that the current sensing time \(t_s\) can be used as previous sensing time for future backoffs. If the channel is sensed idle, then the SU computes the number of frames \((Z)\) that it will transmit during its current transmission operation (Line 15). The value of \(Z\) is minimum of the number of frames that the SU can transmit in the precomputed transmission duration \((y_{max} - s)\), or the number of frames that are currently queued up in its transmission queue. The SU then transmits each frame (Line 16 to 22). During transmission, if PU appears on the channel, then the SU interferes with PU transmission and PU stops its transmission (Line 18 to 21). If PU does not appear on the channel during SU transmission, then the SU transmits all its \(Z\) frames. If the channel is sensed busy (Line 24), then the SU generates fresh sensing schedule by repeating the while loop.

Following points should be noted in the above formulations and the algorithm:

1) The above formulations are valid for the case when SU senses the channel at a random instant and finds it to be idle. In other words, these formulations hold when SU detects an idle period on random sensing of the channel. The residual idle time density and distribution functions (equations (1) and (2)) are derived for random incidence in the idle cycle (based on [41, pp. 331]).

2) \(y_{max}\) value depends only on residual idle time distribution (which is derived using idle time distribution) and is independent of the channel busy time distribution. In general, \(y_{max}\) is directly proportional to \(\eta\) and \(E[RI]\) (which, in turn, is directly proportional to \(E[I]\); see Section VIII).

3) For a given idle time distribution and parameter values, the residual idle time distribution and parameter values remain unchanged as long as the idle time distribution and parameter values do not change. Hence, the value of \(y_{max}\), which is computed by solving (3), is also fixed for a given channel idle time distribution and \(\eta\) value. Therefore, \(y_{max}\) can be computed by the SU once in the beginning (Line 9 in Algorithm 1) and need not be computed every time the channel is accessed by SU to opportunistically transmit on it.

4) On randomly sensing the channel to be idle, the SU transmits as many frames as it can in \((y_{max} - s)\) duration (see Figure 2). If the channel becomes busy due to PU transmission before completion of \(y_{max}\) duration (as shown in Figure 3), then the SU transmits as many frames as possible until the channel becomes busy. In both of the cases, the frames are transmitted without further sensing the channel (i.e., Line 17 within the ‘for’ loop of Algorithm 1 is performed without sensing the channel).

VI. ANALYTICAL FORMULATION OF PERFORMANCE METRICS

In this section, we derive the analytical formulations for various performance metrics of SU. Before describing each performance metric, we briefly explain some terms which are used in analytical formulations of the performance metrics.
1) The term transmission operation by an SU refers to the action of the SU to sense the channel for a fixed predefined duration $s$ and transmit for a maximum of $(y_{\text{max}}-s)$ duration if the channel is idle. Here, $y_{\text{max}}$ is the duration of one complete transmission operation (including sensing and transmission) and is calculated using (3). Depending on the SU data and acknowledgment frames sizes and the channel data rate, SU may transmit one or multiple frames in one transmission operation. One ‘transmission operation’ does not refer to the operation of each frame transmission; instead it represents the operation of transmitting (possibly multiple frames) for maximum of $(y_{\text{max}}-s)$ duration, after sensing the channel idle for duration $s$.

2) We use the term raw SU frames to refer to all the frames which are transmitted by an SU at the MAC layer during its each transmission operation. This includes successfully (i.e., completely) transmitted frames as well as any partially transmitted frame which could not be transmitted completely due to appearance of PU on the channel resulting in collision with PU.

3) We use the term successfully transmitted SU frames to refer to all the frames which are successfully transmitted by an SU at the MAC layer during each transmission operation.

All the metrics, which are computed using raw SU frames, take into account both successfully transmitted frames as well as partially transmitted frame, if any. The metrics, which are computed using successfully transmitted SU frames, take into account only the successfully transmitted frames and discount any partially transmitted frame. In our work, all the analytical formulations are derived using raw SU frames, and for saturated SU traffic. Analytical value of a performance metric is denoted with a notation having a widehat symbol over it. The corresponding simulation value is denoted with the same notation, but without the widehat.

1) Average Number of Raw Frames Transmitted per Transmission Operation by SU ($\hat{\varphi}$): We once again consider two possible transmission scenarios (depicted in Figure 2 and Figure 3) for SU when it transmits after sensing the channel idle. In first scenario (Figure 2), the remaining idle time duration $RI$ is more than the transmission operation duration $y_{\text{max}}$. Therefore, the SU transmits for the entire $(y_{\text{max}}-s)$ duration and there is no interference to PU. In second scenario (Figure 3), the remaining idle time duration $RI$ is less than the transmission duration $y_{\text{max}}$, and therefore, the SU transmits only for available residual idle time duration $(RI-s)$ after sensing. In this scenario, the SU transmission interferes with PU transmission because the idle period ends (due to PU’s appearance on the channel) before the SU transmission is over. Therefore, the average number of raw frames an SU transmits during transmission operation, can be computed as:

$$\hat{\varphi} = E[(\min(RI, y_{\text{max}})-s)|RI>s] \times \frac{\hat{\varphi}}{E[I]}$$

where

$$E[I] = \int_{q=s}^{y_{\text{max}}} (q-s) f_{RI}(q) dq$$

which can be solved to:

$$\hat{\varphi} = \left\{ (y_{\text{max}}-s) \int_{q=s}^{y_{\text{max}}} (q-s) f_{RI}(q) dq \right\} \times \frac{\hat{\varphi}}{E[I]}$$

(4)

Here, $s$ is the fixed channel sensing duration, $R$ is the data transmission rate of the wireless channel (in bits per second), and $S$ is sum of SU data frame size and acknowledgment frame size (in bits). Therefore, $R/S$ gives the number of SU frames transmitted per second. $y_{\text{max}}$ is computed by solving (3). Let us use the following representations:

$$\text{term}_1 = (y_{\text{max}}-s) \left(1 - F_{RI}(y_{\text{max}})\right) \left(1 - F_{RI}(s)\right)$$

(5)

and

$$\text{term}_2 = \frac{1}{(1 - F_{RI}(s))} \int_{q=s}^{y_{\text{max}}} (q-s) f_{RI}(q) dq$$

(6)

Note that $(1-F_{RI}(y_{\text{max}}))/(1-F_{RI}(s))$ is equal to the conditional probability $P(RI > y_{\text{max}}|RI > s)$ when SU transmits without interfering with PU. $\text{term}_1$ gives the average duration for which an SU transmits in the first scenario. Similarly, $\text{term}_2$ gives the average duration for which an SU transmits in the second scenario.

2) Average Raw SU Throughput ($\hat{\gamma}$): Raw SU throughput is defined as the total number of raw frames transmitted by an SU per unit of time at MAC layer. To derive analytical formulation for average raw SU throughput (in frames per second), we first formulate expression for the number of times an SU incidenices into idle cycles of the channel and senses the channel to be idle for transmission operation. Let us denote this by $N_{tx}$. In other words, $N_{tx}$ represents the number of times an SU senses the channel to be idle and initiates transmission operation. $N_{tx}$, when multiplied by $\hat{\varphi}$ (average number of raw SU frames transmitted per transmission operation), analytically gives the number of SU frames transmitted in the total duration for which the system is operational. Let $T$ denote the total duration in which the average SU throughput is to be computed, and $E[BO]$ denote the mean backoff duration for exponential backoffs (please refer to Section IV). So, the backoff process is a Poisson arrival process with rate parameter $1/E[BO]$. Then, the mean number of incidences (arrivals) into the alternating renewal process (ARP) in duration $T$ is $T \times 1/E[BO]$. Some of these incidences will be in idle periods and some of them will be in busy periods. We now compute the number of incidences in idle periods. Let us denote the mean idle and busy time values by $E[I]$ and $E[B]$ respectively. Then, the fraction of time the channel is in idle state is computed as $\frac{E[I]}{E[I]+E[B]}$ [42, pp. 428-429]. To compute the number of
SU incidences in idle periods, we make use of a property called Poisson Arrivals See Time Averages (PASTA) ([43] and [44, pp. 293]). PASTA property states that when arrivals to a stochastic process is Poisson, the fraction of arrivals which find the stochastic process in a given state is equal to the fraction of time the process is in that state [44, pp. 77-78]. The PASTA property holds for any stochastic process [44, pp. 250]. In our model, the PU traffic is an Alternating Renewal Process (ARP), which describes the state (idle or busy) and occupancy of the channel, and arrivals are the Poisson incidences by the SU into the ARP. Therefore, the fraction of SU incidences ('the arrivals') in idle periods of the channel is equal to the fraction of time the channel is in idle state, which, as mentioned above, is equal to \( \frac{E[I]}{E[BO]} \). As a result, the total number of SU incidences in idle periods (which we refer to as \( N_I \)) during operational duration \( T \) is given as the product of total number of incidences into the ARP and the fraction of the incidences which are in idle cycle:

\[
N_I = \frac{T}{E[BO]} \times \frac{E[I]}{(E[I] + E[B])} \quad (7)
\]

However, all the incidences into idle cycles do not result in sensing of the channel by SU. As explained in explained in context of Figure 5 in Section IV, an SU cannot perform channel sensing at those incidence times during exponential backoff which lie within the duration of the previous transmission operation. During backoff, an SU may either incidence into the just concluded transmission operation duration either once or more than once (as in Figure 5), or it may not incidence at all within this duration if the backoff value is sufficiently large to surpass the current time (as in Figure 4). In order to compute the average number of backoff incidences which occur within a transmission operation duration, we proceed as follows: Every time an SU senses the channel for a predefined duration \( s \) and finds it to be idle, it transmits, on the average, for a duration of \((\text{term}_1 + \text{term}_2)\), where \( \text{term}_1 \) and \( \text{term}_2 \) are given by equations (5) and (6) respectively. Let \( X \) be a random variable that denotes the duration of transmission operation, and let random variable \( Y \) denote the number of backoff incidences within transmission operation durations. So \( E[X] = (s + \text{term}_1 + \text{term}_2) \). The average number of backoff incidences which occur within a transmission operation duration is given as \( W_{inc} \) and can be computed as (see Appendix A-A):

\[
W_{inc} = \frac{(s + \text{term}_1 + \text{term}_2)}{E[BO]} \quad (8)
\]

We model each backoff incidence into the ARP as Bernoulli distributed with probability of success \( p \). A success is reported when an incidence lies within the transmission operation duration. It can be shown that \( p = \frac{W_{inc}}{1+W_{inc}} \) (see Appendix A-B). Hence, the probability that an SU actually senses the channel on a random backoff incidence (\( P_{sen} \)) is equal to the probability that the incidence is outside of transmission operation, which is \((1 - p)\). Thus,

\[
P_{sen} = \frac{1}{1 + W_{inc}} \quad (9)
\]

Using (7) and (9), the total number of idle cycle incidences within duration \( T \) on which the SU senses the channel is computed as:

\[
N_{I_{sen}} = N_I \times P_{sen}
\]

\[
= \left[ \frac{T}{E[BO]} \times \frac{E[I]}{(E[I] + E[B])} \right] \times P_{sen} \quad (10)
\]

So, \( N_{I_{sen}} \) gives the number of times the SU incidences into idle cycles and senses the channel. On each sensing, the channel is sensed idle if the residual idle time is greater than the sensing duration \( s \) (i.e., no PU appears on the channel during the sensing duration \( s \)). The probability that the residual idle time (RI) is greater than the sensing duration \( s \) is given by \( (1 - F_{RI}(s)) \). Therefore, the total number of idle cycle incidences within duration \( T \) on which the SU senses the channel and finds it to be idle (and therefore, initiates a transmission operation) is given as:

\[
N_{tx} = N_{I_{sen}} \times P(\text{RI} > s)
\]

\[
= \left[ \frac{T}{E[BO]} \times \frac{E[I]}{(E[I] + E[B])} \right] \times P_{sen} \times (1-F_{RI}(s)) \quad (11)
\]

During each such transmission operation, the average number of frames that the SU transmits is \( \hat{\gamma} \). Therefore, the total number of frames that the SU transmits in duration \( T \) is given as:

\[
N_{total} = N_{tx} \times \hat{\gamma}
\]

\[
= \left[ \frac{T}{E[BO]} \times \frac{E[I]}{(E[I] + E[B])} \right] \times P_{sen} \times (1-F_{RI}(s)) \times \hat{\gamma} \quad (12)
\]

For large \( T \), the average SU throughput (in frames/sec) is given as:

\[
\hat{\gamma} = \frac{N_{total}}{T}
\]

\[
= \left[ \frac{1}{E[BO]} \times \frac{E[I]}{(E[I] + E[B])} \right] \times P_{sen} \times (1-F_{RI}(s)) \times \hat{\gamma} \quad (13)
\]

Here, \( F_{RI} \), \( P_{sen} \), and \( \hat{\gamma} \) are given by equations (2), (9), and (4) respectively.

3) Average Sensing Overhead of SU (\( \hat{\delta} \)): We compute the average sensing overhead of SU as the percentage of time (with respect to one SU frame transmission time) that an SU spends in sensing the channel. This metric is reported as percentage. However, in this report, we will simply refer to it as ‘sensing overhead’. It is computed as:

\[
\hat{\delta} = \frac{T_{sensing}}{T_{tx}} \times 100
\]

Here, \( T_{sensing} \) and \( T_{tx} \) are the total time durations that the SU spends in sensing the channel and transmitting frames respectively during operational duration of \( T \). The total number of incidences at which SU senses the channel, \( N_{sen} \), is computed as:

\[
N_{sen} = N_{I_{sen}} + N_B \quad (15)
\]
where $N_{\text{sen}}$ is given by (10) and $N_B$ represents the number of incidences at which the SU senses the channel in busy cycle. $N_B$ is computed as:

$$N_B = \frac{T}{E[BO]} \times \frac{E[B]}{E[I] + E[B]} \quad (16)$$

where $\frac{E[B]}{E[I] + E[B]}$ is the fraction of SU incidences in busy cycles. Note that unlike idle cycle incidence, all the busy cycle incidences are sensed by the SU.

Substituting the expressions from (10) and (16) into (15), we get the formulation for the sensing overhead of SU as follows:

$$N_{\text{sen}} = \left[ \frac{T}{E[BO]} \times \frac{E[I]}{E[I] + E[B]} \right] \times P_{\text{sen}} + \left[ \frac{T}{E[BO]} \times \frac{E[B]}{E[I] + E[B]} \right] \times \left( 1 - E[I] \right)$$

$$\hat{\delta} = \frac{N_{\text{sen}} \times s}{N_{\text{total}} \times T_s} \times 100 \quad \text{(20)}$$

Substituting the values of $N_{\text{sen}}$ from (17) and $N_{\text{total}}$ from (12) into (20), we get the formulation for the sensing overhead of SU as follows:

$$\hat{\delta} = \left[ \frac{E[I]}{E[I] + E[B]} \right] \times P_{\text{sen}} \times \left( 1 - E[I] \right) \times \hat{\varphi} \times T_s \times 100 \quad \text{(21)}$$

### V. Example Distributions

In this section, we present analytical formulations for performance metrics of Secondary Users for three example channel idle time distributions, which are used in our simulations. Note that RIBS scheme depends only on channel idle time distribution and not on channel busy time distribution for opportunistic channel access. However, mean channel busy time value is required to derive analytical expressions for $\hat{\gamma}$ and $\hat{\delta}$ metrics (see (13) and (21)). We do not assume any specific channel busy time distribution in our derivations of expressions for $\hat{\gamma}$ and $\hat{\delta}$, and represent it generically by $E[I]$. Mean value of the collected busy time data sample can itself be used as $E[I]$ because sample mean is an unbiased estimator of the population mean. Note that $\hat{\varphi}$, $\hat{\gamma}$, and $\hat{\delta}$ are computed for each example distribution using the general analytical formulations given by (4), (13), and (21) respectively. We also specify the constraint inequality for each case, which is derived using (3). In each of these cases, $s$ and $T_s$ are constant values. Please refer to Table I for notations and see Appendix B for derivation of these formulations.

### A. Configuration-1: Channel Idle Time Values are 2-Erlang Distributed

Channel idle time distribution rate parameter: $\lambda_i$ ($\lambda_i > 0$) Constraint inequality for computing $y_{\text{max}}$:

$$y_{\text{max}} \geq \left[ 1 + \frac{\lambda_i y}{2} \right] + (\eta - 1) \geq 0 \quad \text{(22)}$$

$$\hat{\varphi} = \left[ (y_{\text{max}} - s) e^{-\lambda_i(y_{\text{max}} - s)} \right] + \left[ (y_{\text{max}} - s) e^{-\lambda_i(y_{\text{max}} - s)} \right] + \left[ (y_{\text{max}} - s) e^{-\lambda_i(y_{\text{max}} - s)} \right] + \left[ (y_{\text{max}} - s) e^{-\lambda_i(y_{\text{max}} - s)} \right]$$

$$\hat{\gamma} = \frac{1}{E[I]} \times \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times e^{-\lambda_i(1 + \frac{b^2}{2})} \times \frac{R}{S} \quad \text{(24)}$$

$$\hat{\delta} = \frac{1}{E[I]} \times \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times e^{-\lambda_i(1 + \frac{b^2}{2})} \times \frac{R}{S} \quad \text{(25)}$$

Here, $\hat{\varphi}$ is given by (23), and $E[I]$ and $P_{\text{sen}}$ are given by (59) and (79) respectively in Appendix B.

### B. Configuration-2: Channel Idle Time Values are Uniformly Distributed

Channel idle time distribution parameter values: (a,b) ($0 < a < b$)

Constraint inequality for computing $y_{\text{max}}$:

$$y^2 - 2by + \eta(b^2 - a^2) \geq 0 \quad \text{(26)}$$

$$\hat{\varphi} = \left[ (y_{\text{max}} - s) \left\{ \frac{b^2 - a^2 - 2bsy_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right\} \right] + \left[ (y_{\text{max}} - s) \left\{ \frac{b^2 - a^2 - 2bs + s^2}{b^2 - a^2 - 2bs + s^2} \right\} \right]$$

$$\hat{\gamma} = \frac{1}{E[I]} \times \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times \left[ 1 - \frac{(2bs - s^2)}{b^2 - a^2} \right] \times \hat{\varphi} \quad \text{(27)}$$

$$\hat{\delta} = \left[ \frac{E[I]}{E[I] + E[B]} \right] \times P_{\text{sen}} \times \left[ 1 - \frac{(2bs - s^2)}{b^2 - a^2} \right] \times \frac{R}{S} \quad \text{(28)}$$

### VII. Example Distributions

In this section, we present analytical formulations for performance metrics of Secondary Users for three example channel idle time distributions, which are used in our simulations. Note that RIBS scheme depends only on channel idle time distribution and not on channel busy time distribution for opportunistic channel access. However, mean channel busy time value is required to derive analytical expressions for $\hat{\gamma}$ and $\hat{\delta}$ metrics (see (13) and (21)). We do not assume any specific channel busy time distribution in our derivations of expressions for $\hat{\gamma}$ and $\hat{\delta}$, and represent it generically by $E[I]$. Mean value of the collected busy time data sample can itself be used as $E[I]$ because sample mean is an unbiased estimator of the population mean. Note that $\hat{\varphi}$, $\hat{\gamma}$, and $\hat{\delta}$ are computed for each example distribution using the general analytical formulations given by (4), (13), and (21) respectively. We also specify the constraint inequality for each case, which is derived using (3). In each of these cases, $s$ and $T_s$ are constant values. Please refer to Table I for notations and see Appendix B for derivation of these formulations.
C. Configuration-3: Channel Idle Time Values are \( r \)-phase Hyper Exponentially Distributed

Channel idle time distribution parameters: \((\beta_p, \lambda_p)\); \((p = 1 \ldots r; \lambda_p > 0, 0 < \beta_p \leq 1.0)\)

\(r\) is the total number of phases. \(\beta_p\) and \(\lambda_p\) are probability and rate parameter respectively of phase \(p\).

Constraint inequality for computing \(y_{\text{max}}\):

\[
\sum_{p=1}^{r} \alpha_p e^{-\lambda_p y_{\text{max}}} + (\eta - 1) \geq 0
\]  

(30)

where

\[
\alpha_q = \frac{(\beta_q / \lambda_q)}{\sum_{p=1}^{r} (\beta_p / \lambda_p)} \quad q = 1, \ldots, r
\]  

(31)

and

\[
\sum_{p=1}^{r} \alpha_p = 1
\]  

(32)

\[
\tilde{\varphi} = \left[ (y_{\text{max}} - s) \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y_{\text{max}}} \right] + \sum_{p=1}^{r} \alpha_p e^{-\lambda_p s} \left\{ \sum_{p=1}^{r} \alpha_p \lambda_p \left( e^{-\lambda_p s} - \frac{1}{\lambda_p} \sum_{p=1}^{r} \alpha_p \right) \right\} - \sum_{p=1}^{r} \left\{ \alpha_p \left( e^{-\lambda_p s} - e^{-\lambda_p y_{\text{max}}} \right) \right\} \times \frac{R_s}{S}
\]  

(33)

\[
\tilde{\gamma} = \frac{1}{E[I]} \times \frac{E[I]}{[E[I] + E[R_I]]} \times P_{\text{scen}} \times \left( \sum_{p=1}^{r} \alpha_p e^{-\lambda_p s} \right) \times \tilde{\varphi}
\]  

(34)

\[
\tilde{\delta} = \frac{E[I]}{[E[I] + E[R_I]]} \times P_{\text{scen}} \times \left( \sum_{p=1}^{r} \alpha_p e^{-\lambda_p s} \right) \times \tilde{\varphi} \times T_{sf}
\]  

(35)

\(\varphi\) is given by (33), and \(E[I]\) and \(P_{\text{scen}}\) are given by (96) and (109) respectively in Appendix B.

VIII. EFFECT OF SYSTEM PARAMETERS ON MAXIMUM TRANSMISSION DURATION \((y_{\text{max}})\) VALUE

In this Section, we elaborate on the system parameters on which \(y_{\text{max}}\) value depend, and how the variations in value of these parameters affect \(y_{\text{max}}\). Understanding this dependency will help in analysis of the results.

\(y_{\text{max}}\), as described in Section V, is the maximum duration of a transmission operation for an SU whenever it randomly senses the channel to be idle. This value is computed by solving (3) for the maximum value of \(y\) for which the inequality holds. The corresponding constraint inequalities for the example idle time distributions are (22), (26), and (30) respectively. It is clear from these equations that \(y_{\text{max}}\) value depends on the interference probability threshold value \(\eta\), and the parameter values of residual idle time distribution (which, in turn, is derived from channel idle time distribution). The larger the channel’s mean residual idle period \((E[R_I])\), the larger is the \(y_{\text{max}}\) value. Further, \(E[R_I]\) and \(E[I]\) are directly proportional to each other and are related using the following equation [45]:

\[
E[R_I] = E[I] \times \left[ \frac{C_{oV^2} + 1}{2} \right]
\]  

(36)

where \(C_{oV^2}\) is the squared Coefficient of Variation of the channel’s idle time distribution. Coefficient of Variation is defined as the ratio of standard deviation of the data set to its mean value. Equation (36) implies that if the channel’s mean idle time value \((E[I])\) is large, then the \(y_{\text{max}}\) value obtained by solving the constraint equation is also large. Similarly, if the interference probability threshold value \(\eta\) is large (i.e. PU interference probability constrains are relatively relaxed), then the computed \(y_{\text{max}}\) value is also large. In general, the \(y_{\text{max}}\) Value computed by solving constraint equation is directly proportional to \(E[I]\) and \(\eta\).

IX. DISCUSSION ON RANDOM CHANNEL SENSING

In Section V, we specified the general formulation to compute the residual idle time distribution using (2) and \(y_{\text{max}}\) using (3). We also emphasized that equations (1) and (2) are valid only if the channel sensing by SU is random and memoryless. In this section, we elaborate more on the reasons for selecting random channel sensing as the basis for our proposed channel access scheme RIBS, and the mechanism for ensuring that SU senses the channel randomly.

The main advantage of using the theory of random incidence described in Section V is as follows: If an SU senses the channel at random instant and finds it to be idle, and if the channel idle time distribution (i.e. \(F_I\)) is known, then the residual idle time distribution \((F_{RI})\) can be computed (using (2)) without knowing the start of the idle cycle in which the SU has randomly sensed the channel. This distribution is used by SU to compute the maximum duration \((y_{\text{max}})\) for which it can use the channel on randomly sensing the channel to be idle, such that the probability of its interference with PU transmission is below a predefined threshold \((\eta)\). This relieves the SU from continuously sensing the channel to keep track of start of each idle period on the channel. This is in contrast to those schemes reported in the literature, which use idle time distribution for opportunistic transmission [5], and need to sense the channel continuously.

In order to enforce memoryless random sensing of the channel, the SU backs off using exponentially distributed

![Fig. 7: Backoffs Using Large Mean Backoff Parameter Value](image-url)
values as described in detail in Section IV. The choice of backoff parameter of the exponential distribution is important for the performance of the system. If the backoff parameter is large, then the SU may miss some idle periods on the channel, thereby decreasing the average SU throughput. But, large backoffs also decrease sensing overhead and decrease SU’s chances of interfering with the PU as the SU is likely to skip small idle periods. On the other hand, if backoff parameter is small, the SU tends to use more transmission opportunities, thereby increasing the average SU throughput. However, it also leads to more random incidences into the ARP that translates into more channel sensing. For example, consider the actions performed by the SU between time instants $t_4$ and $t_5$ in Figure 7. After every transmission operation of $y_T$ performed by the SU between time instants $t_1$ and $t_2$, and $t_3$ and $t_4$, and $t_5$ and $t_6$, and $t_7$ and $t_8$, and $t_9$ and $t_{10}$, and $t_{11}$ and $t_{12}$, and $t_{13}$ and $t_{14}$, and $t_{15}$ and $t_{16}$ in idle cycle $I_1$, and ($t_{17}$ to $t_{18}$) in idle cycle $I_2$. It transmits for $(y_{max} - s)$ duration seven times - from $t_2$ to $t_3$, and $t_5$ to $t_6$, and $t_9$ to $t_{10}$, and senses the channel at four random time instants - $t_1$, $t_4$, $t_7$ and $t_8$. On the other hand, if the backoff parameter is small, the SU tends to use more transmission opportunities, thereby increasing the average SU throughput. However, it also leads to more channel sensing. For example, in Figure 8, the SU misses relatively smaller portions of white spaces such as ($t_2$ to $t_3$), ($t_4$ to $t_5$), ($t_6$ to $t_7$), and ($t_8$ to $T_d$) in idle cycle $I_1$, and ($T_c$ to $t_{10}$), ($t_{11}$ to $t_{12}$), ($t_{13}$ to $t_{14}$), and ($t_{15}$ to $T_d$) in idle cycle $I_2$. As we can see from the figure, the SU transmits for $(y_{max} - s)$ duration seven times but senses the channel eight times (at random incidences $t_1$, $t_3$, $t_5$, $t_7$, $t_9$, $t_{10}$, $t_{12}$, and $t_{14}$). The incidences which lie within each $y_{max}$ duration can not be sensed by the SU, as explained earlier in Section IV in context of Figure 5 and Figure 6. Note that since the inter incidence durations are exponentially distributed, the incidence process is Poisson. The theory is applicable to primary traffic with any distribution in contrast to some of the works in literature which assumes exponential ON-OFF traffic for PU.

We need to mention that for some idle time distributions, such as Uniform distribution, $y_{max}$ can be computed exactly (in closed form). For such distributions, the backoff parameter value can be decided analytically. For a given backoff parameter value, the SU can analytically compute the achievable SU throughput and the sensing overhead (using general expressions (13) and (21) respectively) and can select that backoff value for which the analytically computed sensing overhead is acceptable. For such distributions, the interference probability of the SU with PU will not be violated for the selected backoff parameter value.

However, for some other idle time distributions (such as Erlang or HED), it is not possible to compute the exact value of $y_{max}$ (in closed form) under interference constraint. For such cases, the constraint inequalities are solved using iterative numerical methods (such as Newton Raphson method) to compute the $y_{max}$ value, and hence, the $y_{max}$ value is only approximate. For such distributions, the backoff parameter value should be carefully selected, as the error in $y_{max}$ value may result in violation of the interference probability constraint ($\eta$) if the selected backoff value is very small. In such cases, the SU should start with relatively large backoff parameter value (for example, mean idle cycle time of the channel) and gradually adapt the value during run time so that the SU throughput increases without violating the $\eta$ constraint and without increasing the sensing overhead to unacceptable level.

X. USING RIBS IN PU NETWORKS WITH REAL APPLICATION TRAFFIC

In real world scenarios, the channel occupancy data is collected from actual PU traffic. In such scenarios, the distributions fitted to the measured data are only approximations of the ideal fit. In this section, we describe the challenges that are encountered in using RIBS in realistic scenario. We then present some guidelines and a comprehensive methodology to be followed by the SU so that it can use RIBS approach in realistic scenarios [46].

A. Characteristics of Real White Spaces and Distribution Fitting

The channel idle periods due to actual Primary applications can be low or high in terms of variability. The amount of variability depends on the nature of the applications that are using the channel and the load that they generate on the channel. This variability is captured by squared Coefficient of Variation ($CoV^2$) value of the white space data. The higher the $CoV^2$ value, the higher is the variability.

One of the widely accepted approaches to model the characteristics of highly variable data (with $CoV^2 > 1.0$) is to use Hyperexponential Distribution (HED) [47]. An $r$-phase HED is a mixture of $r$ exponentials, each with associated rate and probability parameters. The higher the variability, the larger the number of phases (exponentials) required to model the data. However, increasing the number of phases also makes the model more sensitive to the parameter values of these phases. Distributions with less number of phases are analytically more tractable but introduces more approximation in the fitted distribution. If the $CoV^2$ of the idle period data is equal to 1.0, then Exponential distribution can be fitted to the data. If $CoV^2 < 1$, then sum of exponentials (such as Erlang or Hypoexponential distributions) are preferred distribution for idle period data set [48]. However, we emphasize that the distributions fitted to the real idle time data are only approximate. Therefore, the residual idle time distributions derived from these fitted idle time distributions are also approximate.
B. Challenges in Using RIBS in Realistic Primary Networks and Some Guidelines

Approximation in the distribution fitted to the real idle time data poses new challenges in using RIBS when the idle time values are highly variable. Approximate idle time distribution does not accurately represent the variability of the idle times on the channel due to PU traffic. In this section, we briefly describe the challenges encountered when RIBS uses approximate idle time distribution.

The main challenge in using RIBS is with those idle time profiles which have large $\text{CoV}^2 (> 1.0)$ and very small mean idle time $E[I]$, and where the fitted idle time distribution is approximate. Such a white space profile indicates that the idle periods on the channel consist of a mix of very small white spaces (which makes the $E[I]$ value low) and relatively medium or large white spaces (which make $\text{CoV}^2 > 1.0$). Since the fitted distribution is only approximate with respect to the actual channel idle periods, the $y_{\text{max}}$ value computed in RIBS using such a distribution is also approximate. When an SU senses the channel to be idle and transmits for these approximate $(y_{\text{max}} - s)$ durations, then it may violate the interference probability constraint, specially when it transmits in very small white spaces. Therefore, using approximate idle time distribution in RIBS may increase the probability of interference of SU’s transmission with PU, specially when channel white spaces are highly variable with very low mean idle time. The analytical formulation described in Section VI can be applied in this scenario; however the approximations lead to discrepancies in the analytical and simulation results.

There are two possible approaches to reduce the increase in interference probability when RIBS uses approximate idle time distribution for highly variable data. Both of these approaches are given below and can be used together:

1) Make the exponential backoff parameter large (for example, integer multiple of $y_{\text{max}}$, or mean cycle period $E[C]$). Statistically, this will make the SU skip very small idle periods. However, as described in Section IX (Figure 7), this will lead to low SU throughput, but low sensing overhead.

2) Throw out some outliers (for example, discard the top 1 to 5 percentile data and consider only 99 to 95-percentile of idle time values) and fit the appropriate distribution to this truncated data. Discarding top few percentile of largest idle time values from channel idle time data set makes the fitted model more conservative. That is, the idle time distribution fitted to such a truncated data set gives lower $y_{\text{max}}$ value, and therefore, the SU transmits conservatively on sensing the channel idle. However, care should be taken not to throw away too many values as it will make the fitted model to deviate even more from the ideal fit.

C. Methodology to Use RIBS in Realistic Primary Networks

Figure 9 shows the flow chart, which describes the steps to be followed by an SU to model the channel occupancy due to PU traffic, and to use the RIBS approach for its opportunistic transmission. Note that the method proposed in the flow chart is not new. We merely have borrowed it from the literature of distribution fitting (please refer to [49] and [47] for fitting phase-type distributions) and presented here so that the readers get a comprehensive view of how to use RIBS scheme in practice. In RIBS, computation of $F_{RI}$ (see (1) and (2)) requires closed form expression of the Cumulative Distribution Function (CDF) of the fitted idle time distribution. If the fitted distribution does not have a closed form distribution function in general, then such distributions must be approximated using Phase type (PH) distributions and the approximated PH distribution should be used to compute $F_{RI}$. Another alternative is to directly fit PH distribution to the idle time data set. Note that RIBS does not depend on the channel busy time distribution. Therefore, it is not required to fit a distribution for the collected busy time data sample. Mean value of the collected busy time data can itself be used as $E[B]$ because sample mean is an unbiased estimator of the population mean.

12 A continuous time distribution can be approximated to Phase Type distribution (for example, Exponential, or r-phase sum of exponentials, or r-phase mixture of exponentials) using tools such as EMpht [50].
XI. SIMULATION EXPERIMENTS AND RESULTS

In this section, we describe the simulation experiments for synthetic PU traffic and real PU application traffic. We have conducted Monte Carlo simulations (applied in a discrete event model) using OPNET simulator [51]. We compare the analytical and simulation values of the performance metrics and analyze the results. The SU computes these values for different interference probability constraints (η) and for saturated PU traffic. Analytical values of these performance metrics are obtained for each η using the analytical expressions given in Section VII. The performance metric values computed using simulations are obtained with 98% Confidence Interval. In some figures, the confidence interval is too narrow to be noticeable.

A. Case 1: Synthetic PU Traffic

1) Simulation Model: In order to validate the RIBS scheme and the analytical formulations for SU performance metrics, we simulate a single data channel network, which is opportunistically used by an SU to transmit its frames. The idle and busy periods on the channel due to PU traffic are synthetically generated using example distributions. Therefore, the exact time instants when the channel becomes idle or busy (due to completion of PU transmission, or reappearance of PU on the channel respectively) are known in simulation from the synthetically generated channel occupancy data. We simulate two different configurations to obtain the channel idle and busy time values synthetically. These configurations are as follows:

- **Configuration-1:** In this configuration, we generate the primary channel idle and busy time values alternately using 2-Erlang distribution with rate parameters λ₁ and λ₂ respectively (λ₁ > 0, λ₂ > 0). The $y_{max}$ value for a given η value is computed by solving (22) using Newton Raphson method.

- **Configuration-2:** In this configuration, we generate the primary channel idle and busy time values alternately using Uniform distribution with parameters (a, b) and (c, d) respectively. For this configuration, we solve the inequality (26) by equating its LHS to 0, and select the root which lies within the valid range of (a, b), as the value of $y_{max}$.

Table II shows the parameter values for these configurations, which are used in our simulation experiments. The analytical formulations of performance metrics for these configurations are given in Section VII.

For each of the above configurations, we use OPNET simulator [51] for our experiments. The channel occupancy due to PU traffic is modeled as an Alternating Renewal Process. The SU data and acknowledgment frame sizes are 2048 bits and 128 bits respectively. The wireless channel data transfer rate is 11 Mbps. For each value of interference probability constraint η, we obtain the value of the following performance metrics using simulations (please refer to Table I for notations):

1) Probability of interference of SU transmissions with PU:

\[
p_{intf} = \frac{N_{col}}{N_{tx}}
\]  

2) Average number of raw SU frames transmitted per transmission operation:

\[
\varphi_r = \frac{N_{rsf}}{N_{tx}}
\]  

3) Average number of successfully transmitted SU frames per transmission operation:

\[
\varphi_s = \frac{N_{ssf}}{N_{tx}}
\]  

4) Average raw SU throughput (frames/sec):

\[
\gamma_r = \frac{N_{rsf}}{T}
\]  

5) Average SU goodput (frames/sec):

\[
\gamma_s = \frac{N_{ssf}}{T}
\]  

6) Average SU sensing overhead (in %) for raw frames:

\[
\delta_r = \frac{T_{sensing}}{T_{tx}} \times 100
\]  

7) Average SU sensing overhead (in %) for successfully transmitted frames:

\[
\delta_s = \frac{T_{sensing}}{T_{tx}} \times 100
\]  

Note that the above notations represent the values obtained using simulation. The subscripts rsf and ssf stands for raw SU frames and successful SU frames respectively. The first simulation metric value, $p_{intf}$, is compared against the interference probability constraint η. We now briefly explain some important points regarding comparison of other performance metrics with their analytical counterparts.

In our work, all the analytical values (α, γ, and δ) are computed using raw SU frames. That is, the values of these metrics are computed by considering all the frames (successful and collided) transmitted by SU during its each transmission operation. It was hard to derive the exact analytical expressions for these metrics by considering only the successfully transmitted SU frames and discarding the partial frames. In simulation however, we compute the metric values by using raw SU frames as well as successfully transmitted SU frames. These performance metrics, when computed in simulation using raw SU frames, are denoted as $\bar{\varphi}$, $\bar{\gamma}$, and $\bar{\delta}$ (see (38), (40), and (42) respectively), and when computed using successfully transmitted SU frames, are denoted as $\bar{\varphi}_s$, $\bar{\gamma}_s$, and $\bar{\delta}_s$ (see (39), (41), and (43) respectively). Whether a metric value is more useful when computed using raw frames or successfully transmitted frames, depends on the performance parameter that the metric represents. The performance metrics (such as average goodput), which are computed using successfully transmitted frames, are sometimes considered more useful, as they represent the useful bits transmitted over the channel. But, for some other performance metrics (such as sensing overhead), raw frames are more useful, as they denote the energy spent by the device in transmitting each bit, irrespective of whether the bit turns out to be useful or not at the receiver.

\[\text{Note:} \quad \text{The corresponding analytical expressions were denoted by placing a widetilde over the corresponding symbol (see Section VI).}\]
TABLE II: Channel Occupancy Distribution Parameters for Synthetic PU Traffic

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Distribution Parameters</th>
<th>Mean Idle Time (Sec)</th>
<th>Mean Busy Time (Sec)</th>
<th>( \eta_{\text{max}} ) (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
<td>2-Erlang ( \lambda_i = 1.0 \text{ sec}^{-1}, \lambda_u = 50.0 \text{ sec}^{-1} )</td>
<td>2.0</td>
<td>0.04</td>
<td>0.100159</td>
</tr>
<tr>
<td>Configuration 2</td>
<td>Uniform ((a,b) = (0.01, 0.1) \text{ sec}, (c,d) = (0.001, 0.009) \text{ sec})</td>
<td>0.055</td>
<td>0.005</td>
<td>0.005079</td>
</tr>
</tbody>
</table>

In our simulation experiments, we compare the analytical values of the performance metrics (\( \bar{\phi}, \bar{\gamma}, \) and \( \delta \)) with both types of simulation values - those which are obtained using raw SU frames (\( \bar{\phi}_r, \bar{\gamma}_r, \) and \( \delta_r \)), and those which are obtained using successfully transmitted SU frames (\( \bar{\phi}_s, \bar{\gamma}_s, \) and \( \delta_s \)).

To validate the analytical formulations for different backoff parameter values, we run our simulation experiments with three different backoff parameter values: \( E[C], \eta_{\text{max}} \) and \( \eta_{\text{max}}/10 \). Here, \( E[C] \) denotes the mean cycle time of the channel, which is equal to sum of mean idle time and mean busy time of the channel (i.e., \( E[C] = E[I] + E[B] \)). \( E[C] \) is independent of \( \eta \) and \( \eta_{\text{max}} \) value, and is constant for a given channel idle and busy time distribution and parameter values. In our experiments, we assume the channel occupancy profile (i.e., channel idle and busy time distribution and their parameter values) to remain unchanged, and therefore \( E[C] \) is constant in our simulation experiments. The other two backoff parameter values, namely \( \eta_{\text{max}} \) and \( \eta_{\text{max}}/10 \), depend on the \( \eta \). This relation will be often used in analysis of the results, therefore we explicitly state it: As \( \eta \) increases for a given channel idle time distribution, \( \eta_{\text{max}} \) value also increases. As a result, the exponential backoff parameter values which are directly proportional to \( \eta_{\text{max}} \) (such as \( y_{\text{max}} \) and \( \eta_{\text{max}}/10 \)), also increase with increasing \( \eta \). We further note that \( E[C] \) is always greater than \( \eta_{\text{max}} \). Therefore, \( E[C] \) is largest of all the three backoff parameter values used in our simulation experiments and \( \eta_{\text{max}}/10 \) is the smallest one. These values provide a good range of the mean backoff values.

In our experiments, we have considered \( \eta \) values in the range of 0.05 to 0.3 (in steps of 0.05). High \( \eta \) values are not of much practical interest in hierarchical networks. Higher values of \( \eta \) indicate that PU can accept significant interference from SU, which is usually not true in practice.

In our simulation experiments, we use saturated SU traffic profile, in which SU always has enough frames in its transmission queue to consume the available white space. This allows us to study RIBS scheme for maximum SU load. Each simulation scenario is configured with a given channel occupancy configuration, \( \eta \) value, and backoff parameter value. A total of 36 scenarios are simulated (2 configurations \( \times \) 6 \( \eta \) values \( \times \) 3 backoff parameter values). Each simulation scenario is run with 30 different random seeds and value of performance metrics in each scenario are computed with 98% Confidence Interval using OPNET simulator.

2) Results for Performance Metrics of SU: Figure 10 and Figure 11 show the probability of interference of SU transmission with PU for Configuration-1 and Configuration-2 respectively. We see from these figures that for each \( \eta \), the probability of SU’s interference with PU remains less than \( \eta \) for all the backoff parameter values, i.e., the interference constraint is not violated. For a given \( \eta \) value, the interference probability for smaller backoff parameter values is marginally less than that for larger backoff parameter value\(^{14}\). This is so because SU performs more transmission operations (\( N_{tx} \)) when the backoff parameter value is small (such as \( \eta_{\text{max}}/10 \)) as compared to when the backoff parameter value is large (such as \( \eta_{\text{max}} \) or \( E[C] \)). Although number of collisions (\( N_{col} \)) also goes up, this increase is smaller than \( N_{tx} \). Hence, as per

\(^{14}\)Note that \( E[C] \) is the largest backoff parameter value and \( \eta_{\text{max}}/10 \) is the smallest one of all the three backoff parameter values used in experiments.

---

Fig. 10: Probability of Interference of SU with PU (\( \eta_{\text{intf}} \)) for Configuration 1

Fig. 11: Probability of Interference of SU with PU (\( \eta_{\text{intf}} \)) for Configuration 2

Fig. 12: Average Number of Secondary Frames Transmitted per Transmission Operation for Configuration 1
Fig. 13: Average Number of Secondary Frames Transmitted per Transmission Operation for Configuration 2

Fig. 14: Average SU Throughput for Configuration 1

Fig. 15: Average SU Throughput for Configuration 2

Fig. 16: Average SU Sensing Overhead for Configuration 1

Fig. 17: Average SU Sensing Overhead for Configuration 2

(37), $p_{\text{inf}}$ is smaller compared to the scenario with higher BO (e.g., $y_{\text{max}}$).

Figure 12 and Figure 13 show (for Configuration-1 and Configuration-2 respectively) the analytical and simulation values for the average number of SU frames transmitted per transmission operation for different $\eta$ values. This metric depends on $\eta$ (as $y_{\text{max}}$ value depends on $\eta$) and the channel’s residual idle time distribution parameter values, but is independent of Exponential backoff parameter (see equation (4)). Therefore, for each $\eta$, there is single analytical value $\hat{\phi}$ for all the BO values. In simulation, we compute the number of raw SU frames transmitted per transmission operation ($\phi_r$) and the number of successfully transmitted SU frames per transmission operation ($\phi_s$). These are indicated in the figures by (rsf) and (ssf). The simulation values are computed for three different backoff parameters. These graphs show that for each $\eta$, the simulation values are nearly equal to each other for all the three backoff values. This indicates that like the analytical value, the value obtained for this metric using simulation is also independent of backoff value. Further, the simulation values matches reasonably well with the analytical value.

In Figure 14 and Figure 15, we compare the analytical value of average raw SU throughput ($\hat{\gamma}$) with the simulation values $\gamma_r$ and $\gamma_s$ for different $\eta$ values for Configuration-1 and Configuration-2 respectively. The throughput metric depends on the backoff parameter value (see (13)). As we see from the graphs, for each $\eta$, the analytical and simulation values match reasonably well for all the backoff parameter values. We also note that for a given $\eta$ value, smaller backoffs (such as one with mean value of $y_{\text{max}}/10$) yield larger SU throughput as the SU tends to use the idle periods more frequently. For larger backoff value, the SU skips more idle periods (and therefore misses more transmission opportunity), thereby resulting in lesser average throughput. More analysis of this metric is given later.

Figure 16 and Figure 17 show the average sensing overhead for SU. This metric also depends on the parameter value of exponentially distributed backoffs (see (21)). We observe good match between analytical and simulation values. We also note that for a given $\eta$ value, smaller backoffs results in higher sensing overheads for SU as the SU has more random incidences into the ARP which translates to more sensing

\footnote{Note that $P_{\text{SEN}}$ in (21) is dependent on $E[BO]$ as it is computed by substituting the value of $W_{\text{inc}}$ from (8) into (9), and $W_{\text{inc}}$ is dependent on $E[BO]$.}
The analytical value of average SU throughput and sensing overhead depends on the average number of frames transmitted per transmission operation by SU ($\hat{\phi}$) as well as on the backoff parameter value $E[BO]$ (see equations (13) and (21)). To study this dependency, we now further analyze the results for average SU frames transmitted per transmission operation (Figure 12 and Figure 13), average SU throughput (Figure 14 and Figure 15), and average SU sensing overhead (Figure 16 and Figure 17) together. We analyze these results for Configuration-1 only (Figure 12, Figure 14, and Figure 16); the analysis applies to Configuration-2 as well. In Figure 12, we observe that for each BO parameter value, the average SU frames transmitted per transmission operation increases as $\eta$ value increases. This is due to the fact that $y_{\text{max}}$ value increases as $\eta$ increases. Hence, on the average, SU transmits for longer duration in each transmission operation. Therefore, it is expected that for each backoff parameter value, the SU throughput (both analytical and simulation values) will also increase with the increasing $\eta$. However, in Figure 14, we note that as $\eta$ value increases, the average SU throughput value increases only when the SU uses backoff parameter value of $E[C]$ and remains nearly the same when SU uses backoff parameter value proportional to $y_{\text{max}}$ (i.e., $BO = y_{\text{max}}$ and $BO = y_{\text{max}}/10$). The reason for this is as follows: When SU uses a BO value which is directly proportional to $y_{\text{max}}$ (such as $y_{\text{max}}$ or $y_{\text{max}}/10$), then the BO value increases with increasing $\eta$ (since the increase in $\eta$ also increases $y_{\text{max}}$). Therefore, for larger $\eta$, the SU backs off for larger durations and hence, number of times the SU senses and transmits on the channel decreases. So although, on the average, the SU transmits more number of frames on each transmission operation as $\eta$ increases, yet the number of times it performs such transmission operation on sensing the channel idle decreases. On the other hand, for smaller $\eta$ values, the $y_{\text{max}}$ values are small, and therefore, the backoff parameter values are also small. Hence, the average number of frames transmitted by SU on each transmission operation is less, but the SU performs these transmission operations more frequently (because of small backoffs). As a result, the total number of successfully transmitted SU frames do not vary much when $\eta$ value increases, and the average SU throughput remains more or less the same for different values of $\eta$. This result indicates that by making the backoff parameter value proportional to $y_{\text{max}}$, we can achieve similar average SU throughput even for strict interference constraint scenarios (small $\eta$ value) as we achieve for relaxed interference scenarios (large $\eta$ values). When SU uses $BO = E[C]$, the backoff parameter is independent of $\eta$ and does not increases with increasing $\eta$. However, with increasing $\eta$, the number of SU frames transmitted during each of these transmission operation increases. As a result, the SU throughput value increases with increasing $\eta$ for $BO = E[C]$.

As explained above, the average SU throughput remains more or less the same with increasing $\eta$ values when backoff parameter value is proportional to $y_{\text{max}}$, but the number of times the SU senses the channel decreases. Therefore, the total time spent in sensing the channel decreases but the total time spent in transmitting the frames remains nearly the same (as the average throughput remains nearly the same). As a result, for each backoff parameter value, the sensing overhead (shown in Figure 16 and Figure 17) decreases as $\eta$ increases.

By definition, analytical values of performance metrics ($\hat{\phi}$, $\gamma_s$, and $\delta_s$) will differ from $\varphi_s$, $\gamma_s$, and $\delta_s$ as the analytical values are computed using raw frames and simulation values are computed using successfully transmitted frames. However, as we see from the results, this difference is statistically negligible for both the configurations. The $y_{\text{max}}$ value computed in these configurations is large enough to transmit multiple SU frames (transmission time for one SU frame on the simulated data channel is 186.2$\mu$s, whereas $y_{\text{max}}$ value for Configuration-1 and Configuration-2 is 100.159 ms and 5.079 ms respectively (see Table II)). Therefore, the fraction of partially transmitted frame, if any, is amortized by multiple successfully transmitted frames. As a result, the difference between these analytical and simulation values is statistically negligible. In general, relative difference between these sets of performance metrics is significant only when the channel mean idle period is very small and at the same time, the $\eta$ constraint is very strict (i.e., $\eta$ value is small). In such a configuration, the computed $y_{\text{max}}$ value is very small$^{16}$ and very few frames are transmitted per transmission operation. Therefore, fractional part of partially transmitted frame, if any, is non-negligible with respect to completely transmitted frames. Hence, the relative difference between analytical values (computed using successfully as well as partially transmitted frames) and simulation values (computed using only the successfully transmitted frames) is noticeable. For configurations in which $y_{\text{max}}$ value is larger with respect to a single SU frame transmission time, the SU transmits more number of frames during $y_{\text{max}}$, and the fraction of partially transmitted frame, if any, is amortized by multiple successfully transmitted frames. Therefore, the relative difference between the analytical and simulation values is negligible.

3) Comparison of RIBS with Other Schemes: We compare RIBS scheme with three alternative channel access schemes. The first scheme, which we refer to as adle time distribution based scheme (ITBS), is broadly similar to the Contiguous SU Transmission Strategy (CSTS) proposed in [5]. In an access scheme, SU keeps track of elapsed idle time in current idle cycle and computes the probability of interference of its next frame with PU, given the elapsed idle time. It then transmits the frame only if this probability is less than or equal to the predefined upper bound $\eta$, and updates the elapsed idle time value. If the probability is more than $\eta$, the SU continuously stops transmission.

<table>
<thead>
<tr>
<th>Access Scheme</th>
<th>Avg SU Goodput (Frames/sec)</th>
<th>Avg Sensing Overhead (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RIBS</td>
<td>1825.88 +/- 0.849</td>
<td>19.09 +/- 0.032</td>
</tr>
<tr>
<td>RCA</td>
<td>1904.75 +/- 0.44425</td>
<td>29.85 +/- 0.0067</td>
</tr>
</tbody>
</table>

$^{16}$ $y_{\text{max}}$ value is directly proportional to the channel mean idle time and $\eta$ value; please see Section VIII for more on $y_{\text{max}}$ dependency on these two parameters.
senses the channel until it detects the next idle cycle. In ITBS, the SU needs to sense the channel continuously to keep track of start of each idle cycle. The second scheme is based on the theory presented in [23]. We refer to this scheme as MIL (MILCOM Scheme). We simulate MIL and apply the presented theory for exponentially distributed PU activity for a single Base Station (BS) Scenario. Both ITBS and MIL schemes are compared for a single channel scenario.

The third scheme is a restless multi-arm bandit scheme called Regenerative Cycle Algorithm (RCA) [24], [25]. Since RCA is meaningful only when there are multiple arms (channels), we simulate both RCA and RIBS in a three channel scenario. We compare these schemes with RIBS with respect to average SU goodput and sensing overhead. Values of both the metrics are obtained using simulations. We compare the RIBS and ITBS schemes for Configuration-1. The results for RIBS are analyzed for each backoff parameter value. No backoff is performed in ITBS scheme as SU continuously senses the channel. For MIL and RCA simulations, the configuration parameters are specified with the results.

Figure 18 shows the average SU goodput for Configuration-1. We note that ITBS yields higher average network goodput as compared to RIBS for all values of \( \eta \) when RIBS uses \( BO = E[C] \). In both the cases (i.e., when SU uses ITBS, and when it uses RIBS with \( BO = E[C] \)), the average SU goodput increases as \( \eta \) increases. However, due to large backoff value, the SU using RIBS senses the channel much less often, and therefore performs fewer transmission operations. This results in lower SU goodput as compared to ITBS scheme.

When SU uses RIBS with \( BO = y_{max} \), the average SU goodput remains almost the same as \( \eta \) increases. With backoff parameter value equal to one-tenth of \( y_{max} \), backoff values are sufficiently small even for larger values of \( \eta \). Therefore, the SU performs enough number of transmission operations such that its average goodput with RIBS exceeds its average goodput with ITBS for all \( \eta \) values shown in the graph. Note that if we further increase the \( \eta \) value, then the ITBS curve may overtake the ‘\( BO = y_{max}/10 \)’ curve at some larger value of \( \eta \) as it did for ‘\( B = y_{max} \)’ curve. However, large \( \eta \) values are not of practical interest in hierarchical network of PUs and SUs, and therefore, not considered.

Figure 19 shows the sensing overhead of the SU when it uses the ITBS and RIBS (with different backoff parameter values). It is intuitive that ITBS, which performs continuous channel sensing to keep track of the start of each idle period, will have significantly higher sensing overhead, as compared to the RIBS scheme, which senses the channel only when it has frames to transmit. As the \( \eta \) value increases, the number of frames transmitted by the SU per transmission operation also increases, and therefore, the sensing overhead per SU frame transmission time decreases. As previously mentioned in the context of Figure 16 and Figure 17, for each BO value in RIBS, the sensing overhead decreases as the \( \eta \) value increases. However, the values are very less as compared to the ITBS, and therefore, the RIBS curves overlap with each other and appear as one curve (due to large scale of Y-axis).

In Table III, we compare the RIBS scheme with RCA algorithm [24], [25] with 95 % Confidence Interval. We run both the schemes for three channel scenario. Channel data rates are taken to be 11 Mbps and each SU frame size is 4096 bits. For RCA, we set the transition probabilities from IDLE/BUSY state to BUSY/IDLE state for the three channels as \((0.02, 0.05), (0.07, 0.03), (0.1, 0.9)\) respectively. These values lead to bursty channels and are used in [24]. The channel idle periods obtained using RCA simulations are used to fit idle time distribution for use in RIBS simulation. In RIBS simulation, we fit exponential distribution to the idle time values, set \( \eta \) threshold to 0.1 and backoff value to \( y_{max}/10 \). As shown in the figure, RIBS gives marginally lesser average SU
throughput than RCA at significantly lesser sensing overhead. In RCA, SU senses the channel at every slot and uses every idle slot for its frame transmission. However, in RIBS, the SU may skip some idle durations during backoff thereby resulting in lesser throughput. But in RIBS scheme, SU transmits one or more frames on each idle sensing, thereby resulting in lesser sensing overhead per frame transmission.

Figure 20 shows the average SU throughput for RIBS and MIL schemes. In these simulations, channel data rates are taken to be 4 Mbps and each SU frame size is 4096 bits. Since the PU constraint in RIBS and MIL scheme are different (\(\eta\) in RIBS, and \(\rho\) and \(\theta\) in MIL; please refer to [23] for definition of parameters \(\theta\) and \(\rho\) ), we first simulate RIBS with \(\eta = 0.1\) and compute equivalent \(\rho\) value that comes out to be 0.16. We use \(\rho = 0.16\) in MIL simulation, \(\theta\) value is set to 0.08. In MIL scheme, the SU transmission duration (ON period) depends on \(\lambda\) and \(\rho\) values and is independent of mean idle (OFF) period of PU. The figure shows the average SU goodput for different PU busy duty cycle (Busy duty cycle, DC, of a PU is the proportion of time when the PU is active) for given \(\lambda\) and \(\rho\) values. \(y_{\text{max}}\) duration in RIBS is more than SU’s ON duration in MIL scheme, and \(y_{\text{max}}\) value increases as busy duty cycle decreases. With both scheme, the SU throughput decreases with the increasing busy duty cycle. However, the average SU throughput for RIBS scheme is greater than the MIL scheme for all the duty cycle values, although the difference is much smaller at heavy duty cycles. Figure 21 shows the sensing overhead in RIBS scheme for different duty cycles. We note from the figure that for low PU duty cycles, the sensing overhead is negligible. However, for higher PU duty cycles, the sensing overhead for SU becomes significantly large (from approximately 4% for duty cycle of 0.7 to 60% for duty cycle of 0.9). This is due to the fact that as we increase PU duty cycle in RIBS scheme, the channel gets busier due to increased PU activity. As a result, higher number of channel sensing operations by SU result in busy sensing, and consequently, the sensing overhead for SU increases. Since MIL scheme does not sense at all, there is no sensing overhead. From these results, we can conclude that MIL scheme is preferable in high busy duty cycle scenarios whereas RIBS scheme is better in low busy duty cycle scenarios.

B. Case 2: Real PU Application Traffic

1) Simulation Model: We simulate a Primary network, which consist of a single 4 Mbps data channel. The channel is shared by two pairs of PUs using a simple TDMA protocol. One pair of PU nodes run Voice-over-IP application (using UDP), whereas the second pair of PU nodes run TCP/IP-based Web browsing application using HTTP 1.1 protocol. Therefore, the channel carries a mix of Voice and HTTP traffic. The parameter values for primary applications are given in Table IV. The parameter values for Web browsing application are taken from [52] and the parameter values for VoIP application are as per default configuration of OPNET simulator. The network model also consists of a sender and a receiver SU node, which opportunistically transmits on the channel using RIBS approach.
TABLE V: Summary Statistics of White Spaces (99 Percentile) for Primary Network

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Median</th>
<th>Variance</th>
<th>CoV$^2$</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.1643 ms</td>
<td>49.88 ms</td>
<td>5.22 ms</td>
<td>1.88 ms</td>
<td>0.0795 ms</td>
<td>2.916</td>
<td>3.59</td>
</tr>
</tbody>
</table>

TABLE VI: Parameter values for 2-HED Idle Time Distribution

<table>
<thead>
<tr>
<th>β</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>Phase 2</td>
</tr>
<tr>
<td>0.1803592</td>
<td>0.819608</td>
</tr>
<tr>
<td>36.833 sec$^{-1}$</td>
<td>399.876 sec$^{-1}$</td>
</tr>
</tbody>
</table>

![Distribution function](image)

Fig. 22: Comparison of ECDF of Idle Time Data and the Fitted 2-Phase HED

We simulate saturated SU traffic profile in which the SU always have enough frames to consume the available idle time on the channel. SU frame size is 1024 bits and acknowledgment frame size is 128 bits. The simulations are first run without enabling the SU transmissions to obtain the channel occupancy data (idle and busy periods) due to PU transmissions. The obtained data is then statistically analyzed and an appropriate idle time distribution is fitted to the channel idle time values using the methodology specified in Figure 9. This idle time distribution is used by SU to compute the residual idle time distribution. Once the residual idle time distribution is known, the simulations are run again wherein SUs opportunistically transmit on the channel using RIBS.

2) White Space Characterization: The idle time data obtained using simulation of the Primary network shows high variability with Squared Coefficient of Variation (CoV$^2$) equal to 3.129406. In accordance with the methodology given in Figure 9, we have taken 99 percentile of idle time data in our experiments so as to remove the outliers and reduce the variability of the data. The truncated data set had 81941 idle time values and the CoV$^2$ of the truncated data set is 2.915973. Table V shows the statistical summary of the truncated data set of idle time values. The median of the data set is considerably smaller than the mean value and coefficient of skewness is greater than 1.0, which indicates that even the truncated data is right-skewed, although with marginally lesser variability. We fit a 2-phase HED to the measured idle time data set using a tool called EMpht [50], which is widely used to fit phase type distributions. The rate (λ) and probability (β) values for each phase is given in Table VI. The log likelihood of the 2-HED fit is 362278.1. In order to assess the Goodness of Fit (GoF) of the fitted 2-phase HED, we compare the first and second moments of the fitted distribution with that of the idle time data values [53].

Table VII shows the comparison of these moments for idle time data and the fitted 2-phase HED. Although the first moment matches reasonably well, we note that the fitted 2-phase HED is only an approximate fit as the second moment values differ by 14.13 %. Authors in [55] have also reported similar deviations for highly variable HED fits. Figure 22, which shows the Cumulative Distribution Function (CDF) of the fitted distribution and the empirical CDF of the idle data set, also indicates that 2-HED is an approximate fit.

RIBS does not require busy time distribution to compute $y_{max}$ for an SU. Therefore, it is not required to fit distribution to the busy time value. However, mean busy time value ($E[B]$, required to compute the analytical values of $\gamma$ and $\delta$). The mean busy time value is also required if SU uses mean cycle time $E[C]$ as the backoff parameter for exponentially distributed backoffs. For such cases, mean value of the measured busy time values can be used as $E[B]$ because the sample mean is unbiased estimator of the population mean. We briefly summarize the main statistical properties of the measured busy time data. A total of 85154 busy time data values were obtained out of which 93.7 % of the values were constant (0.00012 seconds), 4.4 % values were less than 0.00012 seconds and 2.9 % values were more than 0.00012 seconds. The mean value of the collected busy time data sample is 0.0001346 seconds.

3) Results: Since the fitted idle time distribution for realistic PU applications is 2-phase HED, we empirically selected the backoff parameter value (please refer to last paragraph of Section IX). We checked four backoff values: $y_{max}$, $y_{max}/10$, $2 \times y_{max}$, and $E[C]$, BO = $y_{max}$ and BO = $y_{max}/10$ resulted in violation of $\eta$. Therefore, we have used the other two backoff parameter values in our experiments: $E[C]$ and $(2 \times y_{max})$. The $y_{max}$ value for the fitted 2-HED

17As mentioned in [54, pp. 357], for large data sets such as in our case, the quantitative Goodness of Fit tests like Chi-square test and KS test, will almost always reject the hypothesis that the data values are i.i.d random values from the fitted distribution. Therefore, we use the moment matching method to assess the GoF of the fitted 2-HED distribution, as suggested in [53].
is computed for different $\eta$ values by solving the constraint inequality (30) for $r = 2$ using Newton Raphson Method. Figure 23 shows the simulation value of probability of SU’s interference with PU transmissions. We see that for each $\eta$, the probability of SU’s interference with PU is less than or equal to $\eta$.

In Figure 24 to Figure 27, we present results for SU performance metrics. In these graphs, we compare the analytical values (based on raw SU frames) of the performance metrics ($\hat{\phi}$, $\gamma$ and $\delta$) with the simulation values computed using successfully transmitted SU frames ($\hat{\phi}_s$, $\gamma_s$ and $\delta_s$). In our simulation of the network model, which is described in the beginning of Section XI-B1, we have simulated MAC layer operation of the network. Computing the number of raw frames transmitted by the SU in simulation would require the knowledge of exact time instant at which the SU collides with the PU at physical layer. This would require instrumenting the PHY layer code of OPNET modeler. Therefore, we have not recorded the number of raw SU frames in simulation, and do not report the corresponding simulation values ($\varphi_r$, $\gamma_r$ and $\delta_r$).

Figure 24 shows the values of $\hat{\phi}$ and $\varphi_s$ for different $\eta$ values. This metric is independent of the backoff parameter value, and therefore, there is single analytical value for both the backoff parameter values. We also obtain the simulation values of these metrics for both the backoff parameters values. We observe several points in this figure. First, for $\eta = 0.1$, the simulation values are much lower than the analytical value. Note that the analytical value is computed using average raw frames transmitted per transmission operation (including successfully and partially transmitted frames), whereas the simulation value is computed using successfully transmitted frames per transmission operation (without any partially transmitted frame). As mentioned in the end of Section XI-A2, if the channel mean idle period is very small (such as in this case; see Table V), then for low $\eta$ value, the average number of frames transmitted per transmission operation are also very small. Therefore, the fractional part of any partially transmitted frame per transmission operation is non-negligible with respect to completely transmitted frames, and the average number of raw SU frames transmitted per transmission operation is more than the average number of successfully transmitted frames. As can be seen from Figure 24, the percentage difference between these values (raw and successfully transmitted frames per transmission operation) is significantly more for small $\eta$ values as compared to large $\eta$ values. This difference decreases as $\eta$ value increases because with increasing $\eta$, $y_{\max}$ value increases and the fraction of partially transmitted frame, if any, is amortized by multiple successfully transmitted frames.

In order to confirm that the relative difference between $\hat{\phi}$ and $\varphi_s$ in Figure 24 for low $\eta$ value is due to partially transmitted frame as mentioned above, we once again simulate a single channel network for which the channel idle periods are synthetically generated using the same distribution and parameter values as obtained for real PU traffic (shown in Table VI). For channel busy time periods, we use constant value equal to 0.00012 seconds as 93.7% of the channel busy time values in real PU application are equal to 0.00012 seconds (see Section XI-B2). We plot the values of $\hat{\phi}$, $\varphi_r$ and $\varphi_s$, as shown in Figure 25. We see from the figure that even for exact distribution, the value of the metric $\varphi_s$ computed in simulation using only successfully transmitted frames differs from the analytical value $\hat{\phi}$, which is computed using raw SU frames) for low $\eta$, and this difference decreases when $\eta$ increases. On the other hand, the value of $\hat{\phi}$ and $\varphi_r$, both of which are computed (analytically and in simulation respectively) using raw SU frames, matches very well. This indicates that discarding of the partial frames in computation of $\varphi_s$ brings in the difference between $\hat{\phi}$ and $\varphi_s$. This difference decreases as $\eta$ increases and the fraction of partially transmitted frame is amortized by larger number of successfully transmitted frames.

Figure 26 and Figure 27 plot the performance of SU in terms of throughput and sensing overhead respectively. Analytical value of both of these metrics depend on $\hat{\phi}$ (see (13) and (21)). As noted in the context of Figure 24, for $\eta = 0.1$, $\hat{\phi}$ and $\varphi_s$ are

\[\text{Fig. 23: Probability of Interference of SU with PU for 2-HED Idle Time Distribution (VoIP + Web browsing PU Applications)}\]

\[\text{Fig. 24: Average Frames Transmitted by SU per Transmission Operation for 2-HED Idle Time Distribution (VoIP + Web browsing PU Applications)}\]

\[\text{Fig. 25: Average Frames Transmitted by SU per Transmission Operation for Synthetic Channel Occupancy based on 2-HED}\]
different for both the backoff values. This difference manifests itself in computation of average raw SU throughput and average SU sensing overhead for $\eta = 0.1$. Therefore, in Figure 26 and Figure 27, the analytical and simulation values also differ for $\eta = 0.1$. This difference, which occurs primarily at low $\eta$ for channel configurations with very low mean idle periods, reduces with increasing $\eta$. Also note that like synthetic channel occupancy case (Figure 14 and Figure 15), the average SU throughput increases with increasing $\eta$. Since ITBS uses the idle time distribution, it is able to transmit more number of frames as compared to RIBS, which uses residual idle time distribution. On the other hand, for 2-HED, $\text{CoV}^2 > 1.0$ and therefore, $E[I] > E[R(I)]$. As a result, RIBS outperforms ITBS when channel idle times are 2-HED distributed. We also note that for very strict interference constraint ($\eta = 0.1$), the ITBS could not transmit any frame. Similar to synthetic traffic case, sensing overhead of ITBS scheme is order of magnitude more than RIBS.

Interference with PU affects the user applications running in the PU network. Since the application performance is important from end user’s perspective, we study the performance of applications when RIBS is used. Figure 30 to Figure 32 show the performance of Primary Network applications when RIBS is used. We study this performance for the backoff parameters $BO = 2 \times y_{\text{max}}$ and $BO = E[C]$. In these figures, the value corresponding to $\eta = 0$ represents the performance metric value when only PU nodes are operational and SU nodes are silent. The other abscissa values ($\eta = 0.10$ to 0.35) correspond to the simulation scenarios in which both PU and SU nodes are operational.

First we consider the case when SU uses the exponen-
tial backoff parameter value of $2 \times y_{max}$. We see from ‘$BO = 2 \times y_{max}$’ curve in Figure 30 and Figure 31 that as $\eta$ value increases from 0.1 to 0.35, the average page download response time and the average voice packet end-to-end delay first increases (i.e., the application performance degrades), and then marginally decreases. The variation in metric values is more in case of HTTP page download response time and very less in end-to-end delay of voice packets. This observation is counter intuitive. As the interference probability constraint is relaxed (i.e., $\eta$ increases), it is expected that the PU will suffer more interference from SU and primary application performance will decrease (or equivalently, the page download response time, and the voice packet end-to-end delay will increase). We trace the reason for this observation to two facts: One, the backoff parameter value ($2 \times y_{max}$) used by SU in this case is proportional to $y_{max}$ and second, the way the TCP and UDP protocols react to collisions at the MAC layer\textsuperscript{19}. As $\eta$ increases, $y_{max}$ value increases, and therefore, the backoff value also increases. For larger values of $\eta$, the backoff parameter value is also relatively large. Larger backoffs by SU results in lesser number of transmission operations by it, leading to lesser number of collisions. So, when $BO = 2 \times y_{max}$, then for larger values of $\eta$, the number of collisions of SU with PU decreases with increasing $\eta$ (see the curve for $BO = 2 \times y_{max}$ in Figure 32 for $\eta > 0.25$). Decrease in number of collisions for higher $\eta$ decreases the number of retransmissions that TCP layer of PU node needs to perform for web browsing applications. This improves the average page download response time (or equivalently, the performance of web browsing application). On the other hand, the decrease in number of collisions does not significantly affect the average end-to-end delay of voice packets in VoIP application (see the curve for $BO = 2 \times y_{max}$ in Figure 31) as it uses UDP.

We now consider another case where SU uses $E[C]$ as the exponential backoff parameter value. $E[C]$ is a constant value (for a given channel configuration), independent of $\eta$ or $y_{max}$. For small $\eta$ value, the $y_{max}$ value is small and SU transmits conservatively. However, it backs off by a duration that is exponentially distributed with parameter $E[C]$, which is larger than the $y_{max}$. Therefore, for small $\eta$, the SU transmits not only conservatively but also less often. This results in smaller number of collisions of SU with PU for low $\eta$ values (see the curve for $BO = E[C]$ in Figure 32). As the $\eta$ value increases, the $y_{max}$ value increases and the SU transmits more number of frames per transmit operation, but it still backs off with same backoff parameter value of $E[C]$. This results in larger number of collisions of SU with PU as $\eta$ increases. Since the number of collisions increases as $\eta$ value increases (as shown in Figure 32) for $BO = E[C]$, the number of retransmissions made by TCP for web browsing application also increases with $\eta$. This results in increased average HTTP page download response time (as shown by the curve for $BO = E[C]$ in Figure 30) and performance of the web browsing application degrades as $\eta$ value increases. The end-to-end packet delay for VoIP application does not vary much (see the curve for $BO = E[C]$ in Figure 31) as the UDP neither waits for acknowledgment of transmitted PU frames nor retransmits any lost frame.

In Figure 33 and Figure 34, we study the performance of PU applications for RIBS (with $BO = E[C]$) and ITBS approach. In these figures, the value corresponding to $\eta = 0$ represents the performance metric value when only PU nodes are operational and SU nodes are silent. The other abscissa values ($\eta = 0.15$ to 0.35) correspond to the simulation scenarios in which both PU and SU nodes are operational. We make two observations from these figures:

1) HTTP page download response time in TCP/IP-based Web browsing application increases considerably (i.e., the application performance degrades) when SU uses RIBS as compared to when the SU uses ITBS. The impact of both the schemes on end-to-end packet delay in VoIP application is similar. However, the reduced impact of ITBS on Web browsing application comes at the cost of higher sensing overhead. This was pointed out in the discussion on Figure 28 and Figure 29.
2) In Figure 34, we note that in both RIBS and ITBS schemes, as the SU starts using the primary channel opportunistically (from \( \eta = 0.15 \) onwards), the end-to-end VoIP packet delay decreases (for both RIBS and ITBS) from approximately 0.127 sec for \( \eta = 0.0 \) to 0.1235 sec for \( \eta = 0.1 \). This is due to the following reason: Whenever an SU packet collides with the HTTP packet, the SU node backs off and the HTTP client goes into congestion control mode. TCP layer at the HTTP client doubles its RTO value and decreases its congestion window size, due to which the HTTP client offers less load on the channel. As a result, the VoIP application gets more bandwidth until either the SU node tries to re-access the channel after its backoff or TCP congestion window size of HTTP client increases.

XII. CONCLUSIONS AND FUTURE WORK

In this report, we presented an opportunistic channel access and transmission scheme for cognitive radio-enabled secondary users for single data channel. This scheme is used by SUs during interweave mode of operation when no PU is using the channel. We refer to this scheme as RIBS (Residual Idle Time Distribution based Scheme). In this scheme, SU uses residual idle time distribution to estimate its transmission duration such that the probability of its interference with PU is less than a predefined threshold value. We derived analytical expression for computing the average raw SU frames per transmission operation, average raw SU throughput, and the average sensing overhead for SU. Using simulation experiments, we showed that using RIBS, the SU can use the channel opportunistically without violating the interference probability constraint. We provided comprehensive methodology to use RIBS in primary network running real applications.

We draw the following conclusions from our work reported in this report:

1) Residual idle time distribution based scheme (RIBS) in interweave mode enables an SU to opportunistically transmit on a primary channel, and still ensures that the probability of interference of SU with PU remain below the predefined threshold \( \eta \). It also significantly decreases the channel sensing overhead for SU as compared to the idle time distribution based approach.

2) For some idle time distributions, such as Uniform, \( y_{\text{max}} \) can be computed exactly (in closed form). For such distributions, the backoff parameter value (for exponentially distributed backoffs) can be decided analytically. For a given backoff parameter value, the achievable SU throughput and the sensing overhead can be computed analytically (using general expressions (13) and (20) respectively). The SU can select that backoff value for which the analytically computed sensing overhead is acceptable. For such distributions, the interference probability of the SU with PU will be not be violated for the selected backoff parameter value. However, for some other idle time distributions (such as Erlang or HED), the \( y_{\text{max}} \) value is only approximate as it is computed by solving the constraint inequalities using iterative numerical methods (such as Newton Raphson method). For such distributions, the backoff parameter value should be carefully selected, as the error in \( y_{\text{max}} \) value may result in violation of the interference probability constraint (\( \eta \)) if the selected backoff value is very small. In such cases, the SU should start with relatively large backoff parameter value (for example, mean idle cycle time of the channel) and gradually adapt the value during run time so that the SU throughput increases without violating the \( \eta \) constraint and without increasing the sensing overhead to unacceptable level.

3) Parameter of exponentially distributed backoffs not only affects the SU throughput and sensing overhead but also primary application performance. Our experimental study shows that impact on performance of TCP application (running on PU network) is low for small \( \eta \) values and gradually increases with increasing \( \eta \). UDP application is not very sensitive to whether BO parameter is independent of \( y_{\text{max}} \). When backoff parameter depends on \( y_{\text{max}} \), the impact on performance of TCP application is higher for small \( \eta \) values and gradually decreases with increasing \( \eta \). UDP application is not very sensitive to whether BO parameter is dependent on \( y_{\text{max}} \) or not. Hence, for a PU network (where both type of application would be running), SU should choose BO parameter independent of \( y_{\text{max}} \) when \( \eta \) is small and \( y_{\text{max}} \) dependent BO parameter when \( \eta \) is large. The exact value should be determined based on the guideline given in the previous point.

We describe three main areas of future work, which are related to the research work reported in this report:

1) In RIBS, we have assumed perfect channel sensing by SU. However, real world deployment may not be ideal. There can be missed detection (i.e., channel is sensed idle when it is actually busy due to PU transmissions) and false alarm (i.e., channel is sensed busy when it is actually idle) during channel sensing. Missed detections are particularly harmful and their occurrence may degrade the PU’s performance. RIBS can be modified to incorporate missed detection and false alarm.

2) Opportunistic transmissions by SUs in CSMA-based primary network under Hierarchical Access Model poses many different kinds of challenges. For example, a CSMA-based PU senses the channel and backs off (such as in WLAN) if the channel is sensed busy due to some SU’s transmission. This clearly is a violation of the hierarchical model, because SU makes PU to hold back its transmission. Moreover, this also delays PU’s transmission in the future (because of backoff), thereby affecting its performance (e.g., delay and throughput) further. Note that in such situation, PU can go into multiple rounds of backoff while SU enjoys sending multiple frames at the cost of PU transmission. Unless the primary network protocol is modified, there is no means for the PU to indicate to the transmitting SU that it (PU) needs to use the channel. Some research work, specifically the ones conducted by Geirhofer et al. ([16], [17]), does address the issue of opportunistically using channel idle periods in a primary WLAN by other
devices, such as Bluetooth-based devices, operating in the ISM band. However, the authors have not assumed a strict hierarchical access model in which SUs must evacuate the channel as soon as a PU appears on the channel. In [16], the authors have used the idle time distribution to devise adaptive hopping sequence for Bluetooth-based SUs so as to mitigate interference between SUs and PUs (WLAN devices). In [17], authors model the channel idle and busy time durations using a CTMC and assume that the SU network is slotted. Based on the channel model, an SU decides at the beginning of each slot whether the channel is idle and whether to transmit in the slot or not. Since the SU’s decision is probabilistic, the SU may collide with the primary if its decision to transmit in a slot turns out to be wrong. The long term fraction of slot collisions per unit time is referred as cumulative interference constraint (CIC). The authors propose a reward-based transmission policy that maximizes SU rewards subject to the CIC value less than a predefined threshold. However, the authors do not address the issue of deferring of channel access by PU (WLAN nodes) when SU is already using the channel, and the performance degradation of PU due to such deferments and exponential backoffs. Deferral of access to a busy channel is key characteristic of any CSMA-based network.

Ideally, the opportunistic transmission protocol for SUs, which operate in CSMA-based primary network, should be designed such that the total PU backoff duration should also be upper bounded. Recently, some researchers have started exploring opportunistic channel access by SUs in CSMA-based primary networks [56] under hierarchical access model, but it still remains largely an open area of research.

3) For a last couple of years, it has been increasingly realized in research community that in order to make DSA networks more deployable in practice, it is required to provide incentive to PU to embrace and support DSA framework. Government agencies in USA, such as National Telecommunications and Information Administration (NTIA) and United State Government Accountability Office, have started to look into the possible incentives for efficient spectrum use (see [57] and [58]). In absence of any incentive, it is not realistic to assume that primary networks will willingly allow dynamic access of spectrum by secondary users, which may result in interference to PUs and performance degradation of primary networks. Overlay communication paradigm promises to enable SUs to use primary channels along with PUs and at the same time, assist PUs in achieving better coverage and higher data rates using advanced network coding and cooperative relaying techniques. On the other hand, interweave communication provides mechanism for SU to use the channel when no PU is using the channel. An important area of future work is to devise efficient mixed schemes that combines overlay and interweave communications so that secondary and primary networks can mutually benefit from each other and the spectrum can be fully utilized.

APPENDIX A
DERIVATION OF EXPRESSION FOR $W_{inc}$ AND $P_{sen}$

A. Derivation of expression for $W_{inc}$

$W_{inc}$ denotes the average number of backoff incidences which occur within a transmission operation duration. Let the continuous random variable $X$ denote the duration of a transmission operation, and discrete random variable $Y$ denote the number of backoff incidences within transmission operation durations. Note that:

1) For a given transmission duration $X = x$, $Y$ is Poisson (with parameter $\lambda = 1/E[BO]$) within that interval.
2) Expected value of $Y$ is given as (see [41, pp. 328–331]):

$$E[Y] = E[E[Y|X]]$$  
$$= \int_{-\infty}^{\infty} E[Y|X=x]f_X(x)dx$$  \hspace{1cm} (44)

3) $E[X] = s + \text{term}_1 + \text{term}_2$, where $s$ is channel sensing duration, and $\text{term}_1$ and $\text{term}_2$ are given by (5) and (6) respectively.

4) The average number of backoff incidences ($W_{inc}$) which occur within a transmission duration is given as: $W_{inc} = E[Y]$.

Let $f(t)$ denotes the expected value of backoff incidences in a given transmission operation duration $x$. Then,

$$f(t) = E[Y|X=x]$$
$$= \sum bP\{Y = b|X = x\}$$
$$= \sum bP\{Y_x = b\}$$
$$= \sum b \frac{e^{-\lambda t}(\lambda t)^b}{b!}$$
$$= \lambda x$$  \hspace{1cm} (45)

Here $P\{Y_x = b\}$ denotes the probability that the number of Poisson arrivals (incidences into the ARP) in duration $x$ is $b$, and its value is equal to $\frac{e^{-\lambda t}(\lambda t)^b}{b!}$. Therefore,

$$E[Y|X=x] = \lambda x$$  \hspace{1cm} (46)

Now,

$$W_{inc} = E[Y]$$
$$= E[E[Y|X]]$$
$$= \int_{-\infty}^{\infty} E[Y|X=x]f_X(x)dx$$
$$= \int_{-\infty}^{\infty} \lambda x f_X(x)dx$$
$$= \lambda \int_{-\infty}^{\infty} x f_X(x)dx$$
$$= \lambda E[X]$$
$$= \frac{(s + \text{term}_1 + \text{term}_2)}{E[BO]}$$  \hspace{1cm} (47)
B. Derivation of $P_{\text{sen}}$

$P_{\text{sen}}$ denotes the probability that an SU actually senses the channel on a random incidence into the ARP. Let us denote the event of incidence of SU into a transmission operation duration during backoff as a ‘successful’ event and let $p$ denotes the probability of success. In other words, the backoff incidences into the ARP are Bernoulli distributed with success probability ‘p’. Note that during each of these ‘successful’ events, the SU does not sense the channel as these incidence times (within the transmission operation duration) are already expired on the time line. The probability that an SU actually senses the channel on a random backoff incidence ($P_{\text{sen}}$) is equal to the probability that the incidence is outside of transmission operation, which is $(1 - p)$. Thus,

$$P_{\text{sen}} = 1 - p$$  \hspace{1cm} (48)

Let $X$ be a random variable that denotes the number of consecutive successes (i.e., the number of consecutive backoffs which occur within a transmission operation duration). Then, the probability that there are $m$ consecutive successes is given as:

$$P[X = m] = p^m (1 - p)$$

Therefore, the expected number of consecutive backoff incidences which occur within a transmission operation duration, denoted as $W_{\text{inc}}$, is given by:

$$W_{\text{inc}} = E[X] = \sum_{m=0}^{\infty} mp^m (1 - p) = (1 - p) \sum_{m=0}^{\infty} mp^m = (1 - p) \frac{p}{(1 - p)^2} = \frac{p}{1 - p}$$  \hspace{1cm} (49)

Rewriting (49), we get:

$$p = \frac{W_{\text{inc}}}{1 + W_{\text{inc}}}$$ \hspace{1cm} (50)

Using (48):

$$P_{\text{sen}} = 1 - p = 1 - \frac{W_{\text{inc}}}{1 + W_{\text{inc}}} = \frac{1}{1 + W_{\text{inc}}}$$  \hspace{1cm} (51)

APPENDIX B

ANALYTICAL FORMULATIONS

A. Configuration 1: Channel idle time values are 2-Erlang distributed

We first derive density and distribution function for an $r$-phase Erlang distribution and then obtain these functions for 2-Erlang distribution by using $r = 2$. We then derive the analytical formulations for the performance metrics.

Idle time distribution rate parameter: $\lambda_i$ \hspace{1cm} ($\lambda_i > 0$)

1) Derivation of Residual Idle Time Distribution ($F_{RI}$):

The distribution function and mean value expression for an $r$-Erlang distributed channel idle time period is given as:

$$F_{RI}(q) = 1 - \sum_{p=0}^{r-1} \frac{\lambda_i^p q^p e^{-\lambda_i q}}{p!}$$ \hspace{1cm} (52)

and

$$E[I] = \frac{r}{\lambda_i}$$ \hspace{1cm} (53)

Substituting above expressions in (1), we get,

$$f_{RI}(y) = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p y^p e^{-\lambda_i y}}{p!}$$ \hspace{1cm} (54)

The distribution function can be obtained using the density function as follows:

$$F_{RI}(y) = \int_{q=0}^{y} f_{RI}(q) dq = \int_{q=0}^{y} \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p q^p e^{-\lambda_i q}}{p!} dq = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p}{p!} \int_{q=0}^{y} q^p e^{-\lambda_i q} dq$$ \hspace{1cm} (55)

The integral in the above equation has the following solution (see [59, pp. 357]):

$$\int_{q=0}^{y} q^p e^{-\lambda_i q} dq = \frac{p!}{\lambda_i^{p+1}} - \sum_{k=0}^{p} \frac{p!}{k! \lambda_i^{k+1}}$$ \hspace{1cm} (56)

Substituting the integral value in (55), the distribution function is written as:

$$F_{RI}(y) = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p}{p!} \frac{p!}{\lambda_i^{p+1}} \left[ 1 - \sum_{k=0}^{p} \frac{p!}{k! \lambda_i^{k+1}} \right] = \frac{\lambda_i}{r} \sum_{p=0}^{r-1} \frac{\lambda_i^p e^{-\lambda_i y} y^k}{p! \lambda_i^{k+1}}$$ \hspace{1cm} (57)
Expanding the summation term on the RHS of the above equation, we get

\[
F_{RI}(y) = 1 - \frac{1}{r} \left[ e^{-\lambda_i y} + \left\{ 1 + \lambda_i y \right\} e^{-\lambda_i y} + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!} e^{-\lambda_i y} \right]
\]

\[
= 1 - \frac{e^{-\lambda_i y}}{r} \left[ 1 + \left\{ 1 + \lambda_i y \right\} + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!} \right]
\]

\[
= 1 - \frac{e^{-\lambda_i y}}{r} \left[ (r + (r - 1)\lambda_i y + (r - 2)\frac{(\lambda_i y)^2}{2} + \cdots + \frac{(\lambda_i y)^{r-1}}{(r-1)!} \right]
\]

\[
= 1 - \frac{1}{r-1} \sum_{p=0}^{r-1} \frac{(r-p)}{p!} e^{-\lambda_i y} (\lambda_i y)^p
\]

(58)

For 2-Erlang Channel idle time distribution, we substitute \( r = 2 \) in (53), (54), (58) and to get the following:

\[
E[I] = \frac{2}{\lambda_i}
\]

(59)

\[
f_{RI}(y) = \frac{\lambda_i e^{-\lambda_i y}}{2} [1 + \lambda_i y]
\]

(60)

\[
F_{RI}(y) = 1 - e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right]
\]

(61)

2) \(y_{max}\) Computation: Using equations (3) and (61), we obtain \(y_{max}\) by solving the following inequality for the maximum value of \(y\) that satisfies the PU interference constraint:

\[
1 - e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right] \leq \eta
\]

\[
\Rightarrow e^{-\lambda_i y} \left[ 1 + \frac{\lambda_i y}{2} \right] + (\eta - 1) \geq 0
\]

(62)

3) Average Number of Raw Frames Transmitted per Transmission Operation by an SU (\(\bar{\gamma}\)): For the obtained value of \(y_{max}\), sum of term1 and term2 (equations (5) and (6) respectively) is given by \(Z\):

\[
Z = (y_{max} - s) \left( \frac{1 - F_{RI}(y_{max})}{y_{max}} \right) + \frac{1}{1 - F_{RI}(s)} \int_{q=s}^{y_{max}} (q - s) f_{RI}(q) dq
\]

(63)

Using (61),

\[
Z = (y_{max} - s) e^{-\lambda_i (y_{max} - s)} \left\{ \frac{2 + \lambda_i y_{max}}{2 + \lambda_i s} \right\} + \frac{2}{e^{-\lambda_i s} (2 + \lambda_i s)} \int_{q=s}^{y_{max}} (q - s) f_{RI}(q) dq
\]

(65)

Let us denote the integral in (65) as \(K\) so that (65) can be written as:

\[
Z = (y_{max} - s) e^{-\lambda_i (y_{max} - s)} \left\{ \frac{2 + \lambda_i y_{max}}{2 + \lambda_i s} \right\} + \frac{2}{e^{-\lambda_i s} (2 + \lambda_i s)} K \]

(66)

Using (60), we solve the integral \(K\) as follows:

\[
K = \int_{q=s}^{y_{max}} (q - s) f_{RI}(q) dq
\]

\[
= \int_{q=s}^{y_{max}} (q - s) \left\{ \frac{\lambda_i e^{-\lambda_i q}}{2} + \frac{\lambda_i^2 q e^{-\lambda_i q}}{2} \right\} dq
\]

\[
= \left( \frac{\lambda_i - s \lambda_i^2}{2} \right) \int_{q=s}^{y_{max}} q e^{-\lambda_i q} dq + \frac{\lambda_i^2}{2} \int_{q=s}^{y_{max}} q^2 e^{-\lambda_i q} dq - \frac{s \lambda_i}{2} \int_{q=s}^{y_{max}} e^{-\lambda_i q} dq
\]

\[
= \left( \frac{(\lambda_i - s \lambda_i^2)}{2} \right) \int_{q=s}^{y_{max}} q e^{-\lambda_i q} dq + \frac{\lambda_i^2}{2} \int_{q=s}^{y_{max}} q^2 e^{-\lambda_i q} dq - \frac{s \lambda_i}{2} e^{-\lambda_i s} e^{-\lambda_i y_{max}}
\]

(67)

Let us denote the integrals in (67) as follows:

\[
P = \int_{q=s}^{y_{max}} q e^{-\lambda_i q} dq
\]

(68)

and

\[
Q = \int_{q=s}^{y_{max}} q^2 e^{-\lambda_i q} dq
\]

(69)

so that (67) can be written as:

\[
K = (\lambda_i - s \lambda_i^2) \int_{q=s}^{y_{max}} e^{-\lambda_i q} dq - \frac{s}{2} e^{-\lambda_i s} e^{-\lambda_i y_{max}}
\]

(70)

To solve \(P\) and \(Q\) we use the following indefinite integral solutions [59, pp. 112] and then apply the limits:

\[
\int x e^{ax} dx = e^{ax} \left( \frac{x}{a} - \frac{1}{a^2} \right)
\]

(71)

and

\[
\int x^2 e^{ax} dx = e^{ax} \left( \frac{x^2}{a^2} - \frac{2x}{a^2} + \frac{2}{a^3} \right)
\]

(72)

Using \(x = q\) and \(a = -\lambda_i\) in (71) and applying the limits, we can solve (68) to obtain:

\[
P = \int_{q=s}^{y_{max}} q e^{-\lambda_i q} dq
\]

\[
= e^{-\lambda_i q} \left( \frac{q}{\lambda_i} - \frac{1}{\lambda_i^2} \right) \bigg|_{q=y_{max}}^{q=s}
\]

\[
= \left\{ e^{-\lambda_i s} \left( \frac{s}{\lambda_i} + \frac{1}{\lambda_i^2} \right) - e^{-\lambda_i y_{max}} \left( \frac{y_{max}}{\lambda_i} + \frac{1}{\lambda_i^2} \right) \right\}
\]

(73)
Similarly, using (72), we can solve (69) to obtain:

\[ Q = \int_{q=s}^{y_{\text{max}}} q^2 e^{-\lambda_1 q} dq = \left[ -e^{-\lambda_1 q} \left( \frac{2}{\lambda_1^2} + \frac{2q}{\lambda_1^3} + \frac{q^2}{\lambda_1^4} \right) \right]_{q=s}^{y_{\text{max}}} = \left\{ e^{-\lambda_1 s} \left( \frac{s^2}{\lambda_1^2} + \frac{2s}{\lambda_1^3} + \frac{2}{\lambda_1^4} \right) - e^{-\lambda_1 y_{\text{max}}} \left( \frac{y_{\text{max}}^2}{\lambda_1^2} + \frac{2y_{\text{max}}}{\lambda_1^3} + \frac{2}{\lambda_1^4} \right) \right\} \]  

(74)

Using expressions for \( P \) and \( Q \) from (73) and (74), we can write the expression for integral \( K \) in (70) as follows:

\[ K = \frac{(\lambda_1 - s^2 \lambda_1^2)}{2} \left\{ e^{-\lambda_1 s} \left( \frac{s}{\lambda_1} + \frac{1}{\lambda_1^2} \right) - e^{-\lambda_1 y_{\text{max}}} \left( \frac{y_{\text{max}}}{\lambda_1} + \frac{1}{\lambda_1^2} \right) \right\} + \frac{1}{2} \lambda_1^2 \left\{ e^{-\lambda_1 s} \left( \frac{s^2}{\lambda_1} + \frac{2s}{\lambda_1^2} + \frac{2}{\lambda_1^3} \right) - e^{-\lambda_1 y_{\text{max}}} \left( \frac{y_{\text{max}}^2}{\lambda_1} + \frac{2y_{\text{max}}}{\lambda_1^2} + \frac{2}{\lambda_1^3} \right) \right\} - \frac{s}{2} \left\{ e^{-\lambda_1 s} - e^{-\lambda_1 y_{\text{max}}} \right\} \]  

where \( \text{term}_1 + \text{term}_2 \), denoted as \( Z \), is given by (76).

From (9), we get the value of \( P_{\text{sen}} \):

\[ P_{\text{sen}} = \frac{1}{1 + W_{\text{inc}}} \]  

(79)

where \( W_{\text{inc}} \) is given by (78).

Using the expression for \( F_{\text{RT}} \) from (61) in (13), we obtain the the average raw SU throughput as follows:

\[ \hat{\gamma} = \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times e^{-\lambda_1 s} \left( 1 + \frac{\lambda_1 s}{2} \right) \times \hat{\varphi} \]  

(80)

where \( E[I] \), \( \hat{\varphi} \), and \( P_{\text{sen}} \) are given by (59), (77) and (79) respectively. Appropriate value for \( E[B] \) should be be substituted based on the channel busy time distribution used or from the busy time data values, \( E[B] \) is a system design parameter.

5) Average SU Sensing Overhead \( \hat{\delta} \): Using the expression for \( F_{\text{RT}} \) from (61) in (21), we obtain the the average raw SU throughput as follows:

\[ \hat{\delta} = \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times e^{-\lambda_1 s} \left( 1 + \frac{\lambda_1 s}{2} \right) \times \hat{\varphi} \times T_{\text{sf}} \]  

(81)

Here, \( E[I] \), \( \hat{\varphi} \), and \( P_{\text{sen}} \) are given by (59), (77) and (79) respectively. Appropriate value for \( E[B] \) should be be substituted based on the channel busy time distribution used or from the busy time data values. \( T_{\text{sf}} \) is one SU frame transmission time.

B. Configuration 2: Channel idle time values are Uniformly distributed

Idle time distribution parameters: \( (a, b) \) \( \quad (0 < a < b) \)

1) Derivation of Residual Idle Time Distribution \( (F_{\text{RT}}) \): The idle time distribution and mean value for Uniform Distribution between parameters \((a, b)\) are given respectively as follows:

\[ F_{\text{RT}}(y) = \frac{y - a}{b - a}, \quad y \geq 0 \]  

(82)

\[ E[I] = \frac{a + b}{2} \]  

(83)
Substituting expressions for $F_I$ and $E[I]$ form (82) and (83) into (1), we get,

$$f_{RI}(y) = \frac{2}{(a + b)} \left[ 1 - \frac{(y - a)}{(b - a)} \right]$$

$$= \frac{2(b - y)}{(b^2 - a^2)}$$  \hspace{1cm} (84)

The distribution function can be obtained using density function as follows:

$$F_{RI}(y) = \int_{q=0}^{y} f_{RI}(q) dq$$

$$= \int_{q=0}^{y} \frac{2(b - q)}{(b^2 - a^2)} dq$$

$$= \frac{2}{(b^2 - a^2)} \left[ b \int_{q=0}^{y} dq - \int_{q=0}^{y} q dq \right]$$

$$= \frac{2}{(b^2 - a^2)} \left[ by - \frac{y^2}{2} \right]$$

$$= 2by - \frac{y^2}{2} \frac{(b^2 - a^2)}{2}$$  \hspace{1cm} (85)

2) $y_{\text{max}}$ Computation: Using equations (3) and (85), we obtain $y_{\text{max}}$ by solving the following inequality for the maximum value of $y$ that satisfies the PU interference constraint:

$$\frac{(2by - y^2)}{b^2 - a^2} \leq \eta$$

$$y^2 - 2by + \eta(b^2 - a^2) \geq 0$$  \hspace{1cm} (86)

3) Average Number of Raw Frames Transmitted per Transmission Operation by an SU ($\tilde{\varphi}$): For the obtained value of $y_{\text{max}}$, sum of term1 and term2 (equations (5) and (6) respectively) is given by $Z$:

$$Z = (y_{\text{max}} - s) \left( \frac{1 - F_{RI}(y_{\text{max}})}{1 - F_{RI}(s)} \right) +$$

$$\frac{1}{1 - F_{RI}(s)} \int_{q=s}^{y_{\text{max}}} (q - s)f_{RI}(q) dq$$  \hspace{1cm} (87)

So, (4) can be written as:

$$\tilde{\varphi} = Z \times \frac{R}{S}$$  \hspace{1cm} (88)

Using (84) and (85), (87) can be written as

$$Z = (y_{\text{max}} - s) \left( \frac{2b^2 - a^2 - 2by_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right) +$$

$$\frac{2}{(b^2 - a^2 - 2bs + s^2)} \int_{q=s}^{y_{\text{max}}} (q - s)(b - q) dq$$

$$= (y_{\text{max}} - s) \left( \frac{b^2 - a^2 - 2by_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right) +$$

$$\frac{2}{(b^2 - a^2 - 2bs + s^2)} \left[ bsy_{\text{max}} + \frac{(b + s)y_{\text{max}}^2}{2} - \frac{y_{\text{max}}^3}{3} \right]$$

$$= (y_{\text{max}} - s) \left( \frac{b^2 - a^2 - 2by_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right) +$$

$$\frac{2}{(b^2 - a^2 - 2bs + s^2)} \left[ bsy_{\text{max}} + \frac{(b + s)y_{\text{max}}^2}{2} - \frac{y_{\text{max}}^3}{3} \right]$$  \hspace{1cm} (89)

Substituting the value of $Z$ from (89) into (88), we get,

$$\tilde{\varphi} = \left[ (y_{\text{max}} - s) \left( \frac{b^2 - a^2 - 2by_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right) + \right.$$

$$\frac{2}{(b^2 - a^2 - 2bs + s^2)} \left[ bsy_{\text{max}} + \frac{(b + s)y_{\text{max}}^2}{2} - \frac{y_{\text{max}}^3}{3} \right] \times \frac{R}{S}$$  \hspace{1cm} (90)

4) Average Raw SU Throughput ($\gamma$): Using (8), $W_{\text{inc}}$ is given as:

$$W_{\text{inc}} = \left[ s + (y_{\text{max}} - s) \left( \frac{b^2 - a^2 - 2by_{\text{max}} + y_{\text{max}}^2}{b^2 - a^2 - 2bs + s^2} \right) + \right.$$

$$\frac{2}{(b^2 - a^2 - 2bs + s^2)} \left[ bsy_{\text{max}} + \frac{(b + s)y_{\text{max}}^2}{2} - \frac{y_{\text{max}}^3}{3} \right] \times \frac{1}{E[BO]}$$  \hspace{1cm} (91)

where $(\text{term}_1 + \text{term}_2)$ is given by (89).

From (9), we get the value of $P_{\text{sen}}$:

$$P_{\text{sen}} = \frac{1}{1 + W_{\text{inc}}}$$  \hspace{1cm} (92)

where $W_{\text{inc}}$ is given by (91).

Using the expression for $F_{RI}$ from (85) in (13), we obtain the the average raw SU throughput as follows:

$$\gamma = \frac{E[I]}{E[I] + E[BO]} \times \frac{E[I]}{E[I] + E[BO]} \times P_{\text{sen}} \times \left[ 1 - \frac{(2bs - a^2)}{b^2 - a^2} \right] \times \tilde{\varphi}$$  \hspace{1cm} (93)

Here, $E[I]$, $\tilde{\varphi}$, and $P_{\text{sen}}$ are given by (83), (90) and (92) respectively. Appropriate value for $E[BO]$ can be substituted based on the channel busy time distribution used or from the busy time data values. $E[BO]$ is a system design parameter.

5) Average SU Sensing Overhead ($\delta$): Using the expression for $F_{RI}$ from (85) in (21), we obtain the the average raw SU throughput as follows:

$$\delta = \frac{E[I]}{E[I] + E[B]} \times P_{\text{sen}} \times \left[ 1 - \frac{(2bs - a^2)}{b^2 - a^2} \right] \times \tilde{\varphi} \times \frac{1}{T_{sf}} \times 100$$  \hspace{1cm} (94)

$E[I]$, $\tilde{\varphi}$, and $P_{\text{sen}}$ are given by (83), (90) and (92) respectively. Appropriate value for $E[B]$ should be be substituted based on the channel busy time distribution used or from the busy time data values. $T_{sf}$ is one SU frame transmission time.
C. Configuration 3: Channel idle time values are r-phase Hyper Exponentially distributed (HED)

Channel idle time distribution parameters: \((\beta_p, \lambda_p); \quad (p = 1 \ldots r; \lambda_p > 0, 0 < \beta_p \leq 1.0)\)
r is the total number of phases. \(\beta_p\) and \(\lambda_p\) are probability and rate parameter respectively of phase \(p\).

1) Derivation of Residual Idle Time Distribution (\(F_{RI}\)):
The distribution function and mean value expression for an r-phase HED distributed channel idle time period is given as:
\[
F_l(q) = 1 - \sum_{p=1}^{r} \beta_p e^{-\lambda_p q}
\]
and
\[
E[I] = \sum_{p=1}^{r} \frac{\beta_p}{\lambda_p}
\]
Substituting above expressions in (1), we get,
\[
f_{RI}(y) = \sum_{p=1}^{r} \frac{\beta_p y e^{-\lambda_p y}}{\sum_{p=1}^{r} \frac{\beta_p}{\lambda_p}}
\]
The distribution function can be obtained using density function as follows:
\[
F_{RI}(y) = \int_0^y f_{RI}(q) dq
= \sum_{p=1}^{r} \frac{\beta_p}{\lambda_p} \left(1 - e^{-\lambda_p y}\right)
= \sum_{p=1}^{r} \frac{\beta_p}{\lambda_p} \left(1 - \alpha_p e^{-\lambda_p y}\right)
= 1 - \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y}
\]
where
\[
\alpha_q = \frac{\beta_q}{\sum_{p=1}^{r} \beta_p / \lambda_p} \quad q = 1, \ldots, r
\]
and
\[
\sum_{p=1}^{r} \alpha_p = 1
\]
Therefore, if idle time distribution is r-phase HED with parameters \((\beta_p, \lambda_p); \quad p = 1 \ldots r\), then residual idle time distribution is also r-phase HED with parameters \((\alpha_p, \lambda_p); \quad p = 1 \ldots r\), where \(\alpha_p\) is given by (100). So, the residual idle time density can be written in terms of \(\alpha\) as follows:
\[
f_{RI}(y) = \sum_{p=1}^{r} \alpha_p \lambda_p e^{-\lambda_p y}
\]

2) \(y_{max}\) Computation: Using equations (3) and (99), we obtain \(y_{max}\) by solving the following inequality for the maximum value of \(y\) that satisfies the PU interference constraint:
\[
1 - \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y} \leq \eta
\]
\[
\Rightarrow \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y} + (\eta - 1) \geq 0
\]

3) Average Number of Raw Frames Transmitted per Transmission Operation by an SU (\(\tilde{Z}\)):
For the obtained value of \(y_{max}\), sum of term1 and term2 (equations (5) and (6) respectively) is given by \(Z\):
\[
Z = \left(y_{max} - s\right) \left(1 - F_{RI}(y_{max})\right)
= \frac{1}{\left(1 - F_{RI}(s)\right)} \int_{q=s}^{y_{max}} (q - s) f_{RI}(q) dq
\]
So, (4) can be written as:
\[
\tilde{Z} = Z \times \frac{R}{S}
\]
\(R\) is the channel data rate (in bits/sec) and \(S\) is the total size of SU data and acknowledgment frames. Using (99) and (102), (104) can be written as:
\[
Z = \left(y_{max} - s\right) \sum_{p=1}^{r} \frac{\alpha_p e^{-\lambda_p y_{max}}}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} +
\frac{1}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} \int_{q=s}^{y_{max}} (q - s) \sum_{p=1}^{r} \alpha_p \lambda_p e^{-\lambda_p q} dq
\]
\[
= \left(y_{max} - s\right) \sum_{p=1}^{r} \frac{\alpha_p e^{-\lambda_p y_{max}}}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} +
\frac{1}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} \left[ \sum_{p=1}^{r} \{ \alpha_p \lambda_p \int_{q=s}^{y_{max}} q e^{-\lambda_p q} dq \} -
\sum_{p=1}^{r} \{ s \alpha_p \lambda_p \int_{q=s}^{y_{max}} e^{-\lambda_p q} dq \} \right]
\]
Computing the integral term in the above equation and solving it further, we get:
\[
Z = \left(y_{max} - s\right) \sum_{p=1}^{r} \frac{\alpha_p e^{-\lambda_p y_{max}}}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} +
\frac{1}{\sum_{p=1}^{r} \alpha_p e^{-\lambda_p s}} \left[ \sum_{p=1}^{r} \{ \alpha_p \lambda_p \left( e^{-\lambda_p \left( \frac{s}{\lambda_p} + \frac{1}{\lambda_p^2} \right)} -
\lambda_p y_{max} \left( e^{-\lambda_p \left( \frac{s}{\lambda_p} + \frac{1}{\lambda_p^2} \right)} -
\sum_{p=1}^{r} \{ s \alpha_p \left( e^{-\lambda_p s} - e^{-\lambda_p y_{max}} \right) \} \right) \right] -
\sum_{p=1}^{r} \{ s \alpha_p \left( e^{-\lambda_p s} - e^{-\lambda_p y_{max}} \right) \} \right] •
Substituting the value of $Z$ from (106) into (105), we get
\[
\hat{\varphi} = \left[ (y_{max} - s) \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y_{max}} + \sum_{p=1}^{r} \frac{1}{\alpha_p e^{-\lambda_p s}} \left( \sum_{p=1}^{r} \alpha_p \lambda_p e^{-\lambda_p s} \left( \frac{s}{\lambda_p} + \frac{1}{\lambda_p^2} \right) \right) - \sum_{p=1}^{r} \left( s \alpha_p e^{-\lambda_p s} - e^{-\lambda_p y_{max}} \right) \right] \times \frac{R}{S} \quad (107)
\]

4) Average Raw SU Throughput ($\hat{\gamma}$): Using (8), $W_{inc}$ is given as:
\[
W_{inc} = \left[ s + (y_{max} - s) \sum_{p=1}^{r} \alpha_p e^{-\lambda_p y_{max}} + \sum_{p=1}^{r} \frac{1}{\alpha_p e^{-\lambda_p s}} \left( \sum_{p=1}^{r} \alpha_p \lambda_p e^{-\lambda_p s} \left( \frac{s}{\lambda_p} + \frac{1}{\lambda_p^2} \right) \right) - \sum_{p=1}^{r} \left( s \alpha_p e^{-\lambda_p s} - e^{-\lambda_p y_{max}} \right) \right] \times \frac{1}{E[BO]} \quad (108)
\]
where $(term1 + term2)$ is given by (106).

From (9), we get the value of $P^{sen}$:
\[
P^{sen} = \frac{1}{1 + W_{inc}} \quad (109)
\]
where $W_{inc}$ is given by (108).

Using the expression for $F_{RT}$ from (99) in (13), we obtain the the average raw SU throughput as follows:
\[
\hat{\gamma} = \frac{1}{E[BO]} \times \frac{E[I]}{E[I] + E[B]} \times P^{sen} \times \left( \sum_{p=1}^{r} \alpha_p e^{-\lambda_p s} \right) \times \hat{\varphi} \quad (110)
\]
Here, $E[I]$, $\hat{\varphi}$, and $P^{sen}$ are given by (96), (107) and (109) respectively. Appropriate value for $E[B]$ can be substituted based on the channel busy time distribution used. $E[BO]$ is a system design parameter.

5) Average SU Sensing Overhead ($\hat{\delta}$): Using the expression for $F_{RT}$ from (99) in (21), we obtain the the average raw SU throughput as follows:
\[
\hat{\delta} = \frac{E[I]}{E[I] + E[B]} \times P^{sen} \times \left( \sum_{p=1}^{r} \alpha_p e^{-\lambda_p s} \right) \times \hat{\varphi} \times T_{sf} \quad (111)
\]
\[
E[I]$, $\hat{\varphi}$, and $P^{sen}$ are given by (96), (107) and (109) respectively. Appropriate value for $E[B]$ should be be substituted based on the channel busy time distribution used or from the busy time data values. $T_{sf}$ is one SU frame transmission time.

REFERENCES


