PhD Admissions Written Test (Basic)

December 4, 2019 (Wednesday), 9:00 am-12:00 pm

Registration No. ____________________________

Important Instructions

– The paper consists of 28 “fill in the blank(s)” type questions with a total of 38 blanks to fill. All questions are compulsory.

– In each question, the first blank will be shown as ______ (i) ________, the second (if any) as ______ (ii) ________ and so on.

– Each blank carries 1 mark. Answer given must be absolutely correct to get 1 mark. Wrong or partially correct answers will get 0 (No negative marks).

– The answers for the blanks SHOULD ONLY be filled in the answer sheet provided. Please do NOT write your answers in the question paper. Answers written in question papers will NOT be evaluated.

– If a question has multiple blanks then the answers should be filled against the corresponding sub-part numbers in the answersheet. For example, suppose question number X has two blanks. Then write your answer for the second blank of the Xth question against ‘X(ii)’ in the answer sheet.

– For rough work you may use the blank sides of the question paper, or the sheet attached to it.

– Before leaving the exam hall you will need to turn-in your answer paper and your question paper back to the invigilators.

– You cannot carry any electronic gadgets like cellphones, calculators etc. In case you have them please deposit them with the invigilators and remember to collect them back before leaving the exam hall. The invigilators are not responsible for loss of any items.

– Please do not ask for any further clarifications in the questions. While the exam is in progress avoid moving out of the exam hall for any reason.

– Please do not use any unfair means like copying, consulting etc. during the exam. Students employing unfair means will be disqualified from the examination and the admissions process. The invigilator’s decision is final in such cases.
1. An affine transformation in 2D can be represented by a matrix as \( M = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{pmatrix} \). This can be applied to a point \( V \) to obtain a transformed point \( V' = M \cdot V \).

(a) Elements of the transformation \( M \) that represent translation are \( \text{________} \).

(b) If all vertices of a unit square are transformed by \( M \) then the area of the transformed square is given by \( \text{________} \).

2. During rasterization, a pixel is covered by two overlapping triangles, \( T_1 \) and \( T_2 \). The RGBA color value of the triangle closer to the eye, \( T_1 \), is \( (R_1, G_1, B_1, A_1) \) and that of the triangle behind it, \( T_2 \) is \( (R_2, G_2, B_2, A_2) \). Let triangle \( T_2 \) be rasterized first, followed by triangle \( T_1 \).

If the resulting color of the pixel is denoted by \( (R_p, G_p, B_p, A_p) \) then \( R_p = \text{________} \).

Assume the alpha (A) channel has the value 1 for fully opaque objects and the value 0 for fully transparent objects.

3. Given a language \( L \) over alphabet \( \Sigma \), let \( Perm(L) = \{ w' \in \Sigma^* \mid \exists w \in L \text{ s.t. } w' \text{ is a permutation of } w \} \). For \( L = (01)^* \), \( Perm(L) \cap 0^*1^* = \text{________} \).

4. Which ONE of the following implies all the other formulas? \( \text{________} \)

(a) \( x + y = 3 \Rightarrow x \geq 2 \)
(b) \( x + y \geq 3 \Rightarrow x \geq 2 \)
(c) \( x + y = 3 \Rightarrow x = 2 \)
(d) \( x + y \geq 3 \Rightarrow x = 2 \)

5. Consider a disk with 10 concentric cylinders numbered 1...10. Disk requests come in to the driver for cylinders 3, 9, 6, 7, 4, 10, 2, simultaneously. A seek takes 5\( \mu \)s per cylinder. Calculate the sequence of reads, and the total seek time, when the disk controller is using the elevator algorithm. Assume the disk arm is initially at cylinder 8, and is moving toward decreasing cylinder numbers.

(a) Sequence of reads: \( \text{________} \).
(b) Total seek time: \( \text{________} \).

6. In a recovery log, \(<Ti \text{ start}> \) denotes the start of a transaction \( Ti\), \(<Ti \text{ commit}> \) and \(<Ti \text{ abort}> \) denote commit and abort of \( Ti\), and log record \(<Ti, X, v1, v2> \) indicates that \( Ti\) updated \( X\) whose old value was \( v1\) to new value \( v2\).

Consider the following state of the recovery log during the execution of transactions:

\(<\text{start T1}>\)
\(<\text{T1, A, 2, 5}>\)
\(<\text{start T2}>\)
\(<\text{T2, A, 5, 8}>\)

The above log corresponds to a schedule that is: \( \text{________} \) (choose one option from each of these pairs: recoverable/non-recoverable, cascade-free/non-cascade-free; for example, your answer could be recoverable, cascade-free).

Now if the log has the following extra record at the end: \(<\text{T2 commit}>\) the schedule is \( \text{________} \) (choose one option from each of these pairs: recoverable/non-recoverable, serializable/non-serializable).
7. Consider the following two WHERE clause predicates in SQL:
   
   \[ A ! = 4 \quad \text{and} \quad \text{not}(A = 4). \]

   Given that null values are allowed in SQL, fill in the blanks to indicate if the two are
   (a) equivalent if three-valued logic is used in SQL, with comparisons with null returning unknown, and
   (b) equivalent if two-valued logic is used in SQL, with comparisons with null returning false.

   Your answer should be one of (Y,Y), (Y,N), (N,Y), (N,N) where Y denotes equivalent and N denotes not
   equivalent. _________ (i) _________

8. What is the output of the following code snippet? _________ (i) _________

   ```c
   int main(int argc, char *argv[]) {
      int n = 0;
      int ret = fork();

      if(ret == 0) {
         n++;
         printf("n=%d\n", n);
      }
      else {
         wait();
         n += 2;
         printf("n=%d\n", n);
      }

      return 0;
   }
   ```

9. You are given an input polynomial: \( ax + bx^2 + cx^3 \) with real, non-zero coefficients. The minimum number
   of multiplications needed to evaluate this polynomial on an arbitrary, real input \( x \) (assuming you have access
   to only addition and multiplication operations over reals) is: _________ (i) _________

10. Suppose the running time of an algorithm on inputs of size \( n \) follows the recurrence relation
    \( T(n) \leq 1 + T(\lfloor \sqrt{n} \rfloor) \) with the base case \( T(1) = 1 \). Then, the tightest bound for \( T(n) \) is \( O ( _________ (i) _________ ) \).

11. Consider an array of \( n \) distinct elements sorted in ascending order. The first element in the array is swapped
    with another element in the array. The worst case time for the best algorithm to find the minimum element in
    the resulting array is \( \Theta ( _________ (i) _________ ) \).

12. Let \( x \in [0,2] \) be a continuous random variable and \( y \in \{-1,+1\} \) a discrete binary variable. Next, consider
    the probability mass function \( P(y) \) and the two conditional probability density functions \( f_{+1}(x|y = +1) \) and
    \( f_{-1}(x|y = -1) \) defined as follows:

    \[
    P(y = +1) = 0.7 \quad f_{+1}(x|y = +1) = 0.5 \quad f_{-1}(x|y = -1) = \begin{cases} 
    2 & \text{if } 0 \leq x \leq 0.5 \\
    0 & \text{otherwise}.
    \end{cases}
    \]

    (a) Compute the value of the conditional probability mass function
    \( P(y = -1|x = 0.4) \). _________ (i) _________

    (b) What is the conditional expectation of \( x \) given \( y = -1 \), i.e., \( E(x|y = -1) \)? _________ (ii) _________
13. The following function takes an integer array $a$ and its length $n > 0$, and another array $c$ of length $n + 1$ as input, and computes and stores in array $c$, in decreasing order of degree, the coefficients of the following polynomial:

$$f(x) = (x + a[0]) \times (x + a[1]) \times \cdots \times (x + a[n - 1]).$$

For example, if $a = \{1, -1, 2\}$, $n = 3$ then the function should compute $c$ as $\{1, 2 - 1, -2\}$. Fill in the blanks to complete the function.

```c
void coeff(int *a, int *c, int n)
{
    // a is of size n while c is of size n+1
    c[0] = 1;
    c[1] = a[0];
    for(int k=1; k<n; k++)
    {
        c[k+1] = a[k] * c[k];
        for(int i=k; i>0; i--)
        {
            c[i] = c[i] + a[k] * (ii);
        }
    }
}
```

Hint: every time the loop body is entered, following condition is true:

$$(x + a[0]) \times (x + a[1]) \times \cdots \times (x + a[k - 1]) = c[0] \times x^k + c[1] \times x^{(k-1)} + \cdots + c[k] \times x^0$$

14. For a string $b$ of balanced nested parentheses, let $d_{\text{max}}(b)$ denote its maximum nesting depth. The following function that takes a string $b$ of balanced parentheses and the string-length as the input, and returns the total number of parentheses pairs () that occur at depth $d_{\text{max}}(b)$. Some examples:

<table>
<thead>
<tr>
<th>String $b$</th>
<th>$d_{\text{max}}(b)$</th>
<th>return value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(()())()()</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>()(()())</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>()()()()()</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Fill in the blanks to complete the function.

```c
int maxdepthcount(char *b, int length) {
    int maxdepth = 0, count = 0, currentdepth=0;
    for (int i=0; i<length; i++)
    {
        if(b[i]=="(')
            currentdepth++;
        if(b[i]==")")
            currentdepth--;
        if (maxdepth == currentdepth)
            count = (i);
        else if (maxdepth < currentdepth) {
            maxdepth = currentdepth;
            count = (ii);
        } else {} // do nothing
    }
    return count;
}
```
15. Given a histogram of quiz marks of students of a class, we want to write a program that calculates any given percentile. For example, if the histogram is as follows:

<table>
<thead>
<tr>
<th>marks</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

The \( p \)th percentile of this distribution is the smallest marks \( M \) such that at least \( p \)% of students have marks less than or equal to \( M \). For this example, to calculate, say, the 67th percentile, we observe first that 67% of 61 is 40.87, and that 7+19 = 26 students have less than or equal to 2 marks, and 7+19+22 = 48 students have less than or equal to 3 marks. Thus, 3 marks is the 67th percentile.

Fill in the blanks below to compute \( p \)th percentile of the distribution. \( \text{num} \), \( \text{marks} \), \( \text{freq} \), \( p \) represent number of entries in the histogram, list of marks in increasing order, list of corresponding frequencies, and the percentile \( p \) required, respectively.

```c
int marks[]={1,2,4,7,8};
int freq[]={13,9,13,37,7};
int pp = percentile(5, marks, freq, 41);
// pp will be equal to 4.
```

```c
int percentile(int num, int *marks, int *freq, float p) {
    float mean=0;
    int totalNum=0;
    for (int i=0; i < num; i++) {
        totalNum += freq[i];
    }
    int n = ceil(totalNum*p/100.0);
    int c = 0;
    for (int i = 0; i < num; i++) {
        c = (i) if ( (ii) ) {
            (iii)
        }
    }
}
```

16. Which ONE of the following statements is true about every \( n \times n \) matrix with only real eigenvalues? (i)

(A) If the trace of the matrix is positive, all its eigenvalues are positive
(B) If the determinant of the matrix is positive, all its eigenvalues are positive
(C) If the trace of the matrix is negative and its determinant positive, at least two of its eigenvalues are negative
(D) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive
17. What is the output of the following program? 

```c
int f (int *a, int M, int *b, int N) {
    if ((M==0)||(N==0)) return 1;
    if (*a==*b) return f (a+1, M-1, b+1, N-1);
    else return 0;
}

int main () {
    int X[7]={1,0,1,0,1,1,1};
    int Y[2]={1,1};
    int Z[3]={0,1,0};
    int P[1]={1};
    printf("%d%d%d", f(X,7,Y,2), f(X,7,Z,2), f(X,7,P,1) );
}
```

18. A non-zero polynomial \( f(x) \), of degree 5, has roots at \( x = 5 \), \( x = 12 \), \( x = 23 \), \( x = 50 \) and \( x = 66 \). Which ONE of the following must be true? 

(A) \( f(10)f(35) < 0 \)
(B) \( f(10)f(35) > 0 \)
(C) \( f(5) + f(66) > 0 \)
(D) \( f(5) + f(66) < 0 \)

19. Let \( X \) and \( Y \) be discrete random variables uniformly distributed in \{7,8,9,10,...,36\} and \{4,5,6,7,8\} respectively. The probability \( P(X + Y = 23) \) is 

(A) 1/20
(B) 1/30
(C) 1/100
(D) 1/150

20. Consider a random variable \( X \) with mean 10 and variance 20. Then \( E(X^2) = \) 

21. \( C \) is a noisy communication channel which “flips” every bit sent through it independently with probability \( \frac{1}{10} \), i.e. if a bit \( b \) is sent through \( C \) it is received as \( b \) with probability \( \frac{9}{10} \) and as \( 1 - b \) with probability \( \frac{1}{10} \). When \( n \) 0s are sent through \( C \), what is the expected number of 1s received at its output? 

22. Let \( U \) be a set of \( n \) (\( \geq 10 \)) distinct objects. Suppose 10 objects out of \( n \) are colored red and all others are colored blue. In how many ways can we pick some subset of \( U \) such that exactly 3 red objects are chosen in that subset? 

23. Consider the following recurrence.

\[ T(n) = T(3n/4) + n \]

\( T(n) \) is asymptotically growing as fast as 

(a) \( \Theta(n^2) \), (b) \( \Theta(n) \), (c) \( \Theta(n \log n) \), (d) \( \Theta(n \log \log n) \).

24. Recall that \( K_n \) is complete undirected graph on \( n \) vertices, i.e.

\[ K_n = (V = \{1, \ldots, n\}, E = \{(i,j) \mid 1 \leq i < j \leq n\}) \]

For \( n \geq 3 \), the number of cycles of length \( n \) in \( K_n \) is 

25. A fair coin is tossed $N$ times. Let Head outcome be denoted by H and Tail by T. The expected number of times two consecutive heads (HH) will appear in any sequence of $N$ tosses is \((i)\). Note that in the output sequence HHH, the number of HH is 2.

26. An error-correction code that can correct 5 bit errors can definitely detect \((i)\) bit errors. (Give the maximum possible integer value.)

27. Consider the following forwarding-table:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Next-hop</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0.0.0</td>
<td>A</td>
</tr>
<tr>
<td>12.0.0.0/8</td>
<td>B</td>
</tr>
<tr>
<td>12.1.0.0/16</td>
<td>C</td>
</tr>
<tr>
<td>12.2.3.0/24</td>
<td>D</td>
</tr>
</tbody>
</table>

(a) The next hop for a packet with destination IP address 12.2.4.5 is \((i)\).
(b) The next hop for a packet with destination IP address 12.2.3.1 is \((ii)\).

28. Consider the grammar $G$ shown below in which the only terminal symbols are $( $ and $ )$.

$$P \rightarrow ( P ) P \mid ( P ) \mid \epsilon$$

The grammar is ambiguous. Let $L(G)$ denote the language generated by the grammar. Now fill in the blanks below:

(a) The string of minimum length that illustrates the ambiguity of $G$ is \((i)\).
(b) The minimum length non-empty string in $L(G)$ that cannot be used to illustrate the ambiguity of $G$ is \((ii)\).