Homework 2
Submit by Monday 23rd August 2010

1. Modify the Lex-BFS or MCS algorithm to find a chordless cycle of length at least four if the input graph is not chordal. Try to do it in $O(n+m)$ time.

2. A graph G is said to be a split graph if its vertices can be partitioned into two parts A and B such that A induces a complete graph and B is an independent set. Describe an $O(n+m)$ time algorithm to recognize whether a given input graph is a split graph.

3. Consider the problem of finding the largest induced subgraph not containing a complete graph of size k, where k is arbitrary. Determine whether the problem is NP-Hard for split graphs or can be solved in polynomial-time. Note that split graphs are a subclass of chordal graphs.

4. Some problems that are efficiently solvable for general graphs can be solved even faster for special classes of graphs. Show that for a chordal graph, the min-cut can be computed in $O(n + m)$ time. The min-cut is the smallest set of edges whose removal gives a disconnected graph.

5. The Hamiltonian cycle problem is the problem of finding a cycle that passes through all vertices of a graph. Show that the Hamiltonian cycle problem is NP-Hard for chordal graphs. To do this, show that the problem for general graphs can be reduced to that for chordal graphs.