Homework 3 Interval Graphs.
Submit by Midsem Date 11/9/2010.

1. Suppose you are given a perfect elimination ordering of a chordal graph and a transitive orientation of its complement. Show how you can construct an interval representation of the graph in $O(n^2)$ time.

For these problems, it maybe convenient to assume that the interval graph is represented by an ordering of the maximal cliques, such that cliques containing a given vertex appear consecutively.

2. Describe a polynomial-time algorithm to find a minimum cardinality dominating set in an interval graph. Extend the algorithm to find a minimum weight dominating set.

3. Describe a polynomial-time algorithm to find the largest subset of vertices of an interval graph that induces a subgraph with maximum clique size at most k, where k is an arbitrary specified number.

4. Consider an EPT graph, that is the edge intersection graph of paths in a tree. The vertices of the graph are paths in a tree and two vertices are adjacent iff the corresponding paths have an edge in common. Describe a polynomial-time algorithm to find a maximum clique in such a graph. Prove that your algorithm works correctly.

5. You should try this problem yourself and only after that search for a reference. Given an interval graph and a number k, find the minimum number of colors needed to properly color the vertices of the graph so that no color is used more than k times. This is motivated by the usual scheduling problem, with the additional restriction that at most k tasks can be done in parallel. Show that the problem is NP-Hard in general for arbitrary k. Show that it is solvable in polynomial-time for $k = 2$.