

Homework 4
Threshold graphs and Cographs.
Submit by October 18, 2010.

1. The threshold dimension of a graph G is the smallest number d such that there exist n d -dimensional vectors v_1, v_2, \dots, v_n and a d -dimensional vector T , such that vertex i is adjacent to vertex j iff $v_i + v_j \geq T$. Here $u \geq v$ for vectors u, v iff $u_i \geq v_i$ for each coordinate $1 \leq i \leq d$. Show that the threshold dimension of any graph with $n \geq 3$ vertices is at most $n - 2$. Can you prove a better upper bound? Try to construct a graph with n vertices having a large threshold dimension (increasing with n).
2. Suppose G is a graph with threshold dimension 2, and you are given a representation of G in terms of the vectors v_i and T . Show that the maximum clique and chromatic number of G can be computed in polynomial time.
3. Show that a graph G is a cograph iff for every subset S of at least two vertices, either the subgraph of G induced by S is disconnected or its complement is disconnected.
4. Describe a polynomial-time algorithm to decide whether the vertices of a given cograph can be partitioned into disjoint triangles.
5. Describe a polynomial-time algorithm to find a largest subset of vertices of a cograph that induces a subgraph with no independent set of size $k + 1$, where k is a given integer.