The lexicographic breadth-first-search ordering of a graph is an ordering of the vertices of the graph obtained by the following procedure.

1. Label any vertex v as 1 and mark it visited.
2. While there exists an unvisited vertex, select an unvisited vertex whose list of visited neighbors is lexicographically the smallest, assign it the next number in sequence and mark it visited.

The lexicographic comparison of two lists is defined as follows. Arrange the two lists in increasing order, and find the smallest position at which the two lists differ. Then the list that contains the smaller number at that position is lexicographically smaller. If one list is a proper prefix of another then the *longer* list is considered to be lexicographically smaller.

Theorem 1 *A graph G is chordal iff the reverse of the lexicographic breadth-first-search ordering is a perfect elimination ordering.*

If reverse lexicographic breadth-first search is a perfect elimination ordering then the graph is obviously chordal. Suppose the graph is chordal and reverse lexicographic breadth-first-search is not a perfect elimination ordering. Then there exist vertices $n_0 > n_1 > n_2$ such that n_0 is adjacent to n_1 and n_2 but n_1 is not adjacent to n_2. Here we refer to vertices by the number assigned to them in the lexicographic breadth-first-search.

We say a sequence of vertices in descending order $n_0 > n_1 > n_2 > \cdots > n_{l-1}$ is *bad* if $l \geq 3$ and the subgraph induced by $\{n_i | 0 \leq i < l\}$ contains only the edges $n_0 n_1$ and $n_i n_{i+2}$ for $0 \leq i < l-2$. We use the same lexicographic ordering on these sequences as before, except that the sequences are now in descending order. If reverse lexicographic breadth-first search is not a perfect elimination ordering then there exists a bad sequence of vertices. Since the number of possible sequences is finite, there exists a bad sequence $n_0 > n_1 > n_2 > \cdots > n_{l-1}$ that is minimum in the lexicographic ordering. We show that this gives a contradiction.

Consider the time at which vertex n_{l-2} was visited in lexicographic breadth-first search. At this time, the vertex n_{l-3} was not visited and it had a neighbor n_{l-1} that was visited but not adjacent to n_{l-2}. Since n_{l-2} was chosen as the next vertex to visit, there must exist a vertex $n_l < n_{l-1}$ adjacent to n_{l-2} but not adjacent to n_{l-3}. We claim that n_l cannot be adjacent to any vertex n_i for $0 \leq i < l-2$.

Suppose n_l is adjacent to the vertex n_{l-3-2i} for some $i \geq 1$. Let i be the smallest such number. Then $n_{l-3-2i}, n_{l-1-2i}, n_l$ is a lexicographically smaller bad sequence, a contradiction. If n_l is adjacent to the vertex n_{l-2-2i} for some $i \geq 1$, then let i be the largest such integer. Then $n_0 > n_1 > n_2 > \cdots > n_{l-2-2i} > n_{l-1-2i} > n_l$ is a lexicographically smaller bad sequence.

If n_l is adjacent to n_{l-1}, we get a chordless cycle of length at least four, contradicting the assumption that the graph is chordal. Therefore n_l is adjacent only to n_{l-2} and thus $n_0 > n_1 > n_2 > \cdots > n_{l-2} > n_{l-1} > n_l$ is a lexicographically smaller bad sequence, a contradiction. Therefore there is no bad sequence and reverse lexicographic breadth-first search is a perfect elimination ordering.