Structure from Motion using Factorization

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Note: These slides are best seen with accompanying video

Problem Definition

Can we understand motion using a single camera?

Given 2D point tracks of landmark points from a *single view point*, recover 3D pose and orientation

Assumptions

- 2D tracks of major landmark points are provided
- Scaled-projective/orthographic projection model.

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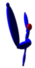

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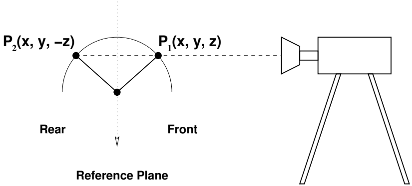
Why is this a hard problem?

The mapping between 2D tracked positions and 3D body pose is many-to-many[1]. This confounds standard regression algorithms.



1

SOATTO, S., AND BROCKETT, R.
1998.
Optimal structure from motion: Local ambiguites and global estimates.
IEEE Computer Society Conference on Computer Vision and Pattern Recognition.

Why this “may not” be such a hard problem after all?

- Human brain perform this *disambiguation* with very little ease.
- Psycho-physical and neuro-physiological imaging experiments have confirmed the fact that we can perceive structure even when we are presented with a video sequence containing only the point tracks of the major joints in the human body[2]

2

JOHANSSON, G.
1976.
Spatio temporal differentiation and integration in visual motion perception.
Psychological Research.

How can we mimic this ability?

Let's observe the trajectories of joint

(a) The top view trajectories of a few dofs plotted (b) One more dof added to the plot (c) All the dofs included

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How do we capture these structures?

Matrix Factorization

$$\mathbf{W}_{2F\times P} = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ y_{11} & \cdots & y_{1p} \\ \vdots & \vdots & \vdots \\ x_{f1} & \cdots & x_{fp} \\ y_{f1} & \cdots & y_{fp} \end{pmatrix}$$

*If the object in the scene is **rigid** this matrix **W** has a very small rank!!*

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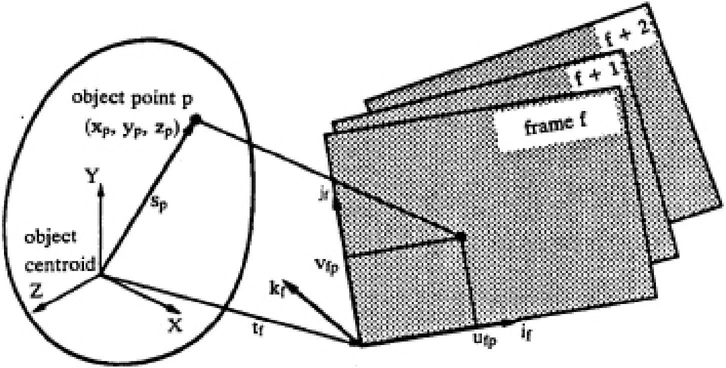
Rigid Body Geometry and Motion



- Object centroid based World Co-ordinate System (WCS)

Rank Theorem

Define $\tilde{x}_{ij} = x_{ij} - \bar{x}_i$ and $\tilde{y}_{ij} = y_{ij} - \bar{y}_i$ where the bar notation refers to the centroid of the points in the ith frame. We have the *measurement matrix*

$$\bar{\mathbf{W}}_{\mathbf{2F \times P}} = \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1p} \\ y_{11} & \cdots & y_{1p} \\ \vdots & \vdots & \vdots \\ \tilde{x}_{f1} & \cdots & \tilde{x}_{fp} \\ y_{f1} & \cdots & y_{fp} \end{pmatrix}$$

The matrix $\bar{\mathbf{W}}$ has rank 3

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Rank Theorem Proof

$$x_{ij} = \mathbf{i}_i^T(\mathbf{P_j} - \mathbf{T_i}), \quad \mathbf{y_{ij}} = \mathbf{j_i^T}(\mathbf{P_j} - \mathbf{T_i}), \quad \frac{\mathbf{1}}{\mathbf{n}}\sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{P_j} = \mathbf{0}$$

$$\tilde{x}_{ij} = \mathbf{i}_i^T(\mathbf{P_j} - \mathbf{T_i}) - \frac{\mathbf{1}}{\mathbf{n}}\sum_{\mathbf{m=1}}^{\mathbf{n}} \mathbf{i_i^T}(\mathbf{P_m} - \mathbf{T_i})$$

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$$\tilde{x}_{ij} = \mathbf{i}_i^T\mathbf{P_j} \qquad \tilde{\mathbf{y}}_{\mathbf{ij}} = \mathbf{j_i^T}\mathbf{P_j}$$

$$\bar{\mathbf{W}} = \mathbf{RS}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{j}_1^T \\ \dots \\ \mathbf{i}_N^T \\ \mathbf{j}_N^T \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} \mathbf{P_1} & \mathbf{P_2} & \dots & \mathbf{P_N} \end{bmatrix}$$

Rigid Body Geometry and Motion

- Without noise $\overline{\mathbf{W}}$ is atmost of rank *three*
- Using SVD, $\overline{\mathbf{W}} = \mathbf{O_1} \Sigma \mathbf{O_2}$ where,
  $\mathbf{O_1}$, $\mathbf{O_2}$ are column orthogonal matrices and Σ is a diagonal matrix with singular values in non-decreasing order
- $\mathbf{O_1} \Sigma \mathbf{O_2} = \mathbf{O_1'} \Sigma' \mathbf{O_2'} + \mathbf{O_1''} \Sigma'' \mathbf{O_2''}$ where,
  $\mathbf{O_1'}$ has *first three* columns of $\mathbf{O_1}$, $\mathbf{O_2'}$ has *first three* rows of $\mathbf{O_2}$ and Σ' is 3×3 matrix with 3 largest non-singular values.
- The second term is completely due to noise and can be eliminated
- $\hat{\mathbf{R}} = \mathbf{O_1'} \left[\Sigma'\right]^{1/2}$ and $\hat{\mathbf{S}} = \left[\Sigma'\right]^{1/2} \mathbf{O_2'}$

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- Solution is not unique any invertible 3×3, $\mathbf{Q}$ matrix can be written as $\mathbf{R} = (\hat{\mathbf{R}}\mathbf{Q})$ and $\mathbf{S} = (\mathbf{Q}^{-1}\hat{\mathbf{S}})$
- $\hat{\mathbf{R}}$ is a linear transformation of $\mathbf{R}$, similarly $\hat{\mathbf{S}}$ is a linear transformation of $\mathbf{S}$.
- Using the following orthonormality constraints we can find $\mathbf{R}$ and $\mathbf{S}$

$$\hat{\mathbf{i}}_f^T \mathbf{Q}\mathbf{Q}^{\mathbf{T}} \hat{\mathbf{i}}_f = 1$$

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$$\hat{\mathbf{i}}_f^T \mathbf{Q}\mathbf{Q}^{\mathbf{T}} \hat{\mathbf{j}}_f = 0 \tag{1}$$

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\end{aligned} \tag{1}
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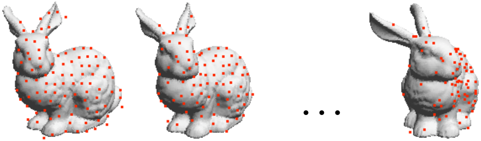
Tomasi Kanade Factorisation (Recap)



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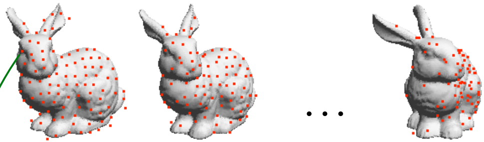

$$\begin{bmatrix} 27 & 61 & \cdots & 96 \\ 97 & 53 & \cdots & 122 \\ 28 & 62 & \cdots & 97 \\ 97 & 53 & \cdots & 122 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 94 & ? & \cdots & 131 \\ 109 & ? & \cdots & 135 \end{bmatrix}$$

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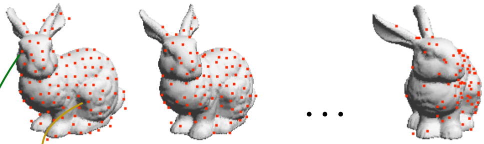

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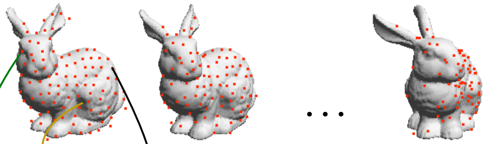

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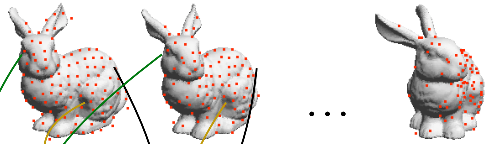

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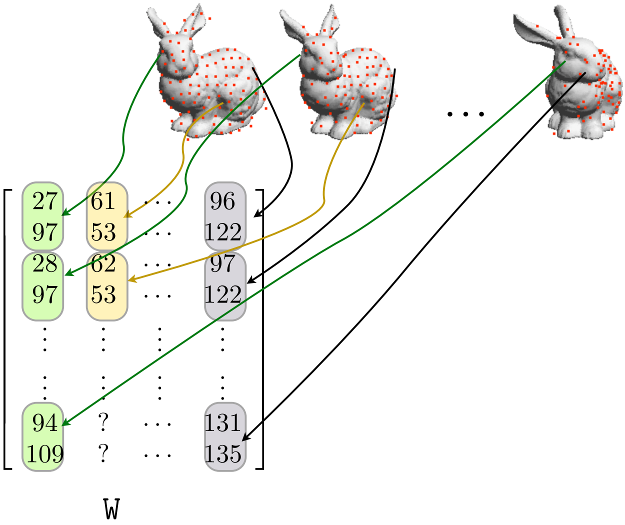

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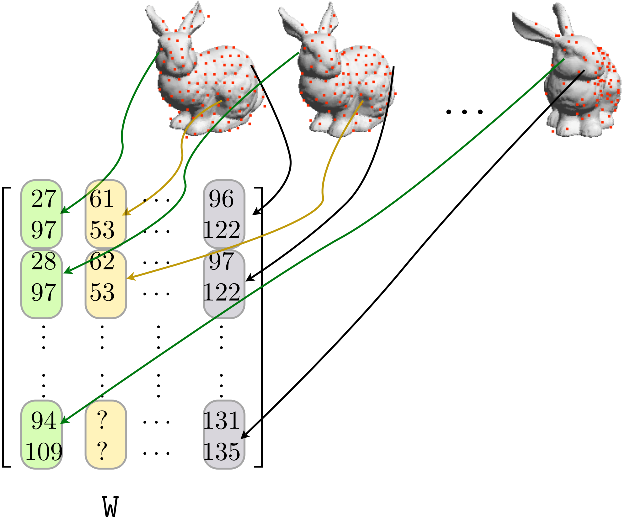

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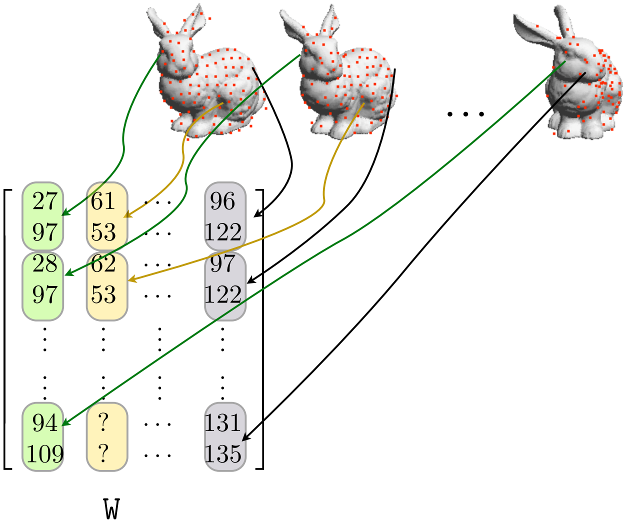

Central Observation: This matrix is rank-limited.
If the object motion is rigid the observation matrix (discounting noise) will have a maximum rank of 4

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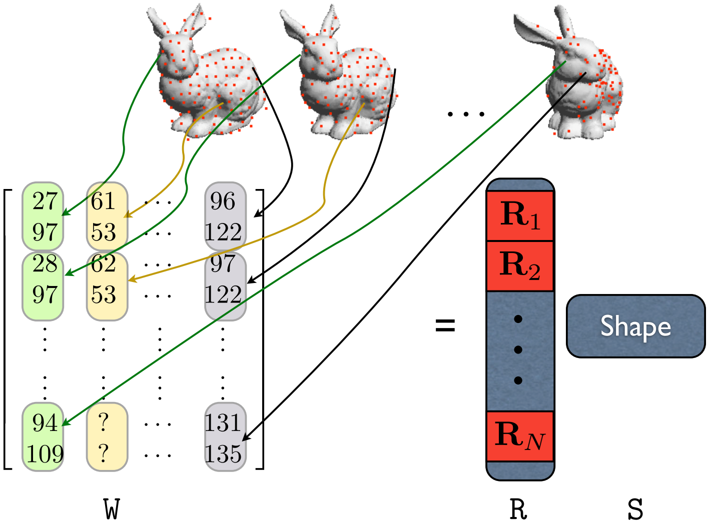

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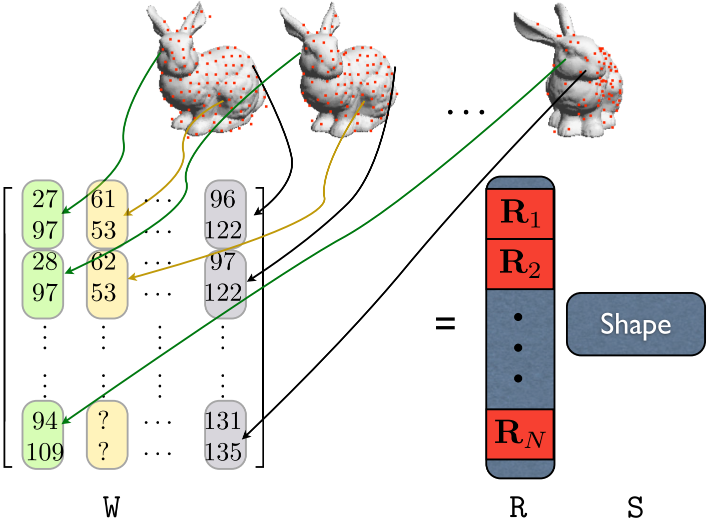

Assumptions

Orthographic Camera Model

Single Object in FOV of camera

Object undergoes rigid motion

All the points are visible throughout the sequence

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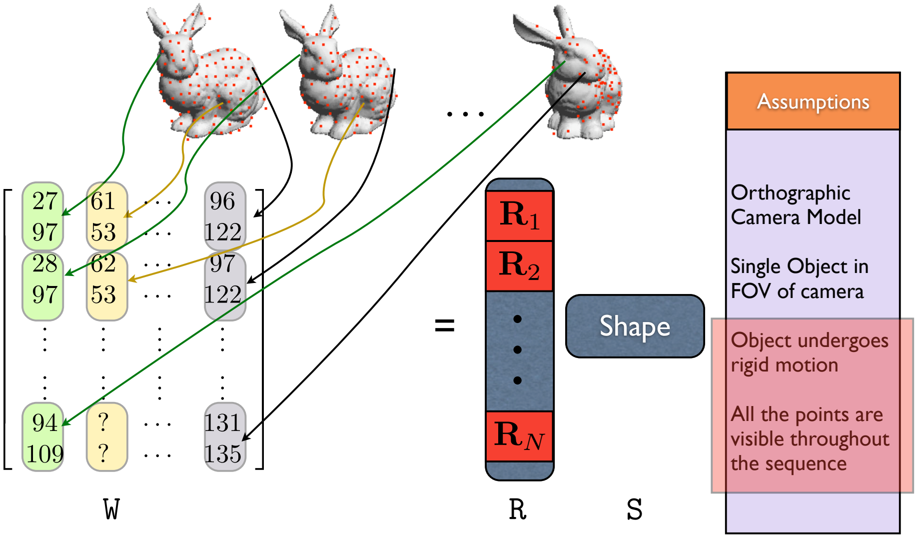

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For Further Reading I

G. Golub and A. Loan
Matrix Computations
John Hopkins U. Press, 1996

C. Tomasi and T. Kanade
Shape and motion from image stream: A factorization method
Image of Science: Science of Images, 90:9795–9802,1993

J. Xiao and J. Chai and T. Kanade
A Closed-Form Solution to Non-Rigid Shape and Motion Recovery
ECCV 2004

For Further Reading II

C. Bregler and A. Hertzmann and H. Biermann
Recovering Non-Rigid 3D Shape from Image Streams
CVPR, 2000

M. Brand
Morphable 3D Models from Video
CVPR, 2001

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NORDIA, *CVPRW*, 2011

M. Kilian, N. Mitra and H. Pottmann. Geometric Modeling in Shape Space. Siggraph, 2008.