CS310 : Automata Theory 2019

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Tutorial sheet 1DFA, NFA, subset construction

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1. Give an NFA $(Q, \Sigma, \delta, q_0, F)$ with $\Sigma = \{0, 1\}$ that accepts the language of words ...
   (a) ... that begin and end with the same letter.
   (b) ... that contain 000 as a subword but not 001.
   (c) ... that contain at least two 0's and at most one 1.
   (d) ... that can be obtained by deleting letters from 00101
   (e) ... having occurrences of 0 multiples of three apart
2. Construct the DFA that accepts all the binary numbers whose integer equivalent is divisible by 4.
3. Give a language that cannot be recognized by a DFA with a single final state.
4. Prove/disprove a regular language can be recognized by an NFA with a single final state.
5. Given two regular languages L and L' over alphabet Σ, we define a new language L'' as follows:

   $$L'' = \{x_1 y_1 x_2 y_2 ... x_n y_n \in \Sigma^* | x_1 ... x_n \in L \text{ and } y_1 ... y_n \in L'\}$$

   Intuitively, you take any two words in L and L' of equal length and 'alternately concatenate' them. Show that L'' is regular by constructing a DFA for L'' using DFA/NFAs of L and L'.
6. Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Let *universal recognized language* $U(A)$ of A be defined as follows.

   $$U(A) = \{w \in \Sigma^* | \hat{\delta}(q_0, w) \subseteq F\}.$$

   a) Prove/Disprove $U(A) \subseteq L(A)$
   b) Let $F = \emptyset$, give an NFA A' such that $U(A) = L(A')$
   c) Prove/Disprove that universal recognized languages are regular languages.