CS310 : Automata Theory 2019
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Tutorial sheet 3 Pumping lemma and DFA minimization

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1. Prove or disprove that the following languages are regular
   (a) $\{0^n 1^m | n \neq m\}$
   (b) $\{0^{p^2} 1^{p^2} | p \geq 0\}$
   (c) $\{0^n 1^m | n = m \vee n = 2m\}$
   (d) $\{w 1^{|w|} | w \in \{0,1\}^*\}$
   (e) $\{w | \exists y.\ |y| = 2^{2^{|w|}} \wedge wy \in L\}$, where L is regular language
2. We know that pumping lemma fails to prove that the following language is not regular.
   $$L = \{ca^n b^n | n \geq 1\} \cup \{c^n w | n \neq 1 \text{ and } w \in \{a,b\}^*\}$$
   a) Using other means prove that the above language is not regular.
   b) Does the generalized version of the pumping lemma prove that the above language is not regular?
3. Determine the residuals of the following languages over $\Sigma = \{a, b\}$
   - $(ab + ba)^*$,
   - $(aa)^*$,
   - $\{a^n b^n c^n \mid n \geq 0\}$.
4. (a) Give two languages such that L_1 and L_2 are not regular, but $L_1 \cap L_2$ is regular.
   (b) Give two languages such that L_1 and L_2 are not regular, but $L_1 \cup L_2$ is regular.
   (c) Give two languages such that L_1 and L_2 are not regular, but $L_1 L_2$ is regular.
5. A DFA $A = (Q, \Sigma, \delta, q_0, F)$ is reversible if no letter can enter a state from two distinct states, i.e. for every $p, q \in Q$ and $\sigma \in \Sigma$, if $\delta(p, \sigma) = \delta(q, \sigma)$, then $p = q$.
   (a) Give a reversible DFA recognizing $L = \{ab, ba, bb\}$
   (b) Show that the minimal DFA recognizing L is not reversible
   (c) Is there a unique minimal reversible DFA recognizing L (up to isomorphism)? Justify your answer
   (d) Prove that the language $(a^* b a^*) + (b^* a b^*)$ is not recognized by any reversible DFA.
6. Design an efficient algorithm $Res(r, a)$, where r is a regular expression over an alphabet Σ and $a \in \Sigma$, that returns a regular expression satisfying $L(Res(r, a)) = L(r)^a$.