CS310 : Automata Theory 2019
IITB, India
Tutorial sheet 5 Context-free Grammar and Pushdown automata

Ashutosh Gupta and S. Akshay

Compile date: 2019-02-18

1. Give a context-free grammar(CFG) for the following languages.

   (a) all binary words with both an even number of zeroes and an even number of ones
   (b) $\{0^n 1^\ell 0^m | n + m = \ell\}$
   (c) $\{a^m b^n \mid m \geq n\}$
   (d) $\{a^m b^n \mid m \neq n\}$
   (e) $\{a^m b^n \mid n > m\}$
   (f) $\{a^m b^n \mid m < 2n < 3m\}$
   (g) All propositional logic formulas in conjunctive normal form, where variables are denoted by x followed by a binary number.

   Also give unambiguous grammars, if possible.

2. For each of the above languages, give left most derivations of some word of length longer than 5.

3. Prove that any grammar $G = (N, T, P, S)$ that has only the following types of rules generates a regular language : $A \to aB$, $A \to a$, or $A \to \epsilon$, where $A,B \in N$ and $a \in T$.

4. Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be an PDA accepting language L. Show that there exists another NPDA $P' = (Q', \Sigma, \Gamma', \delta', q_0', Z_0, F')$ accepting the same language L such that $|\Gamma'| = 2$. Can we compare the size of P and P'?

5. Let us consider a derivation $S \overset{*}{\Rightarrow} w$ in grammar $G = (N, T, P, S)$ such that no word is repeated in the derivation. Can we bound the length of the derivations in terms of $|G|$ and $|w|$? If not, give a counterexample.

6. Let P be a PDA with empty-stack language $L = L^\epsilon(P)$, and suppose that ϵ is not in L. Modify P such that it accepts $L \cup \{\epsilon\}$ by empty stack.

7. Solve Exercise 6.2.5 of the text book.