CS310 : Automata Theory 2019

Lecture 5: ϵ-transitions

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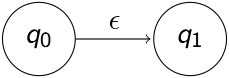
Topic 5.1

ϵ-transitions

ϵ-transitions

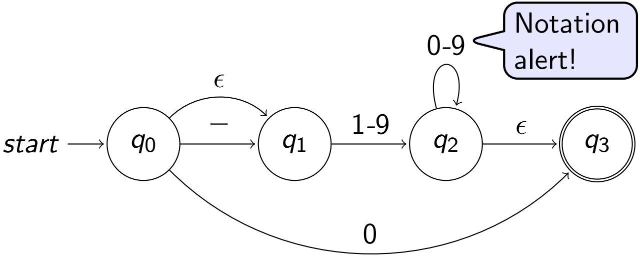
Let us add another feature to our automaton.

Let us allow it to jump states without reading inputs. We will call such moves ϵ-transitions.



Example 5.1

Consider the following automaton with ϵ-transitions that recognizes integers without leading zeros.



Notation altert: specifying range of symbols

'-' is used to specify range of symbols.

Example 5.2

0-9 *means* $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

k-z means $\{k, \ell, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

Does the new power brings new languages?

No!

ϵ-NFA

Definition 5.1

A ϵ-nondeterministic finite automaton(ϵ-NFA) A is a five-tuple

$$(Q, \Sigma, \delta, q_0, F)$$

where

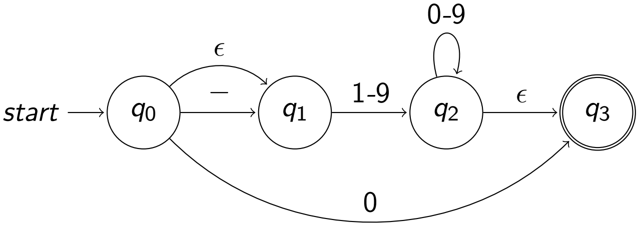
- *Q is a finite set of states,*
- *Σ is a finite set of input symbols,*
- *$\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow \mathfrak{p}(Q)$ is the transition function,*
- *$q_0 \in Q$ is the start/initial state, and*
- *$F \subseteq Q$ is a set of accepting states.*

ϵ closure

Definition 5.2

Let $(Q, \Sigma, \delta, q_0, F)$ be an ϵ-NFA. For each set $S \subseteq Q$, $ECLOSE(S)$ is the set of states reachable via ϵ-transitions from S.

Example 5.3



- $ECLOSE(\{q_0\}) = \{q_0, q_1\}$
- $ECLOSE(\{q_1\}) = \{q_1\}$
- $ECLOSE(\{q_2\}) = \{q_2, q_3\}$
- $ECLOSE(\{q_0, q_2\}) = \{q_0, q_1, q_2, q_3\}$

Extending the transition function to words

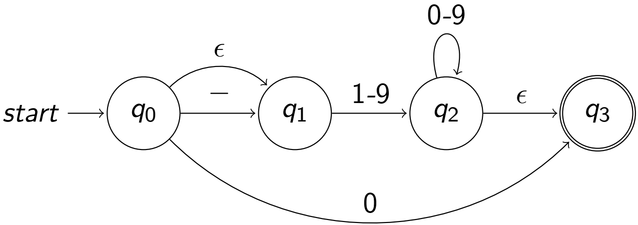
Definition 5.3

Let $A = (Q, \Sigma, \delta, q_0, F)$ *be an* ϵ*-NFA. Let* $\hat{\delta} : Q \times \Sigma^* \rightarrow \mathfrak{p}(Q)$ *be defined as follows.*

$$\hat{\delta}(q, \epsilon) \triangleq ECLOSE(\{q\})$$

$$\hat{\delta}(q, wa) \triangleq \bigcup_{q' \in \hat{\delta}(q,w)} ECLOSE(\delta(q', a))$$

Example 5.4



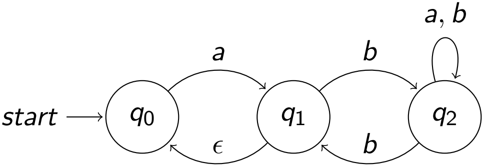
$\hat{\delta}(q_0, \epsilon) = \{q_0, q_1\}$ $\quad \hat{\delta}(q_0, 1) = \{q_2, q_3\}$

$\hat{\delta}(q_0, 10) = \bigcup_{q' \in \hat{\delta}(q_0,1)} \delta(q', 0) = ECLOSE(\delta(q_2, 1)) \cup ECLOSE(\delta(q_3, 1))$

$= ECLOSE(\{q_2\}) \cup ECLOSE(\emptyset) = \{q_2, q_3\} \cup \emptyset = \{q_2, q_3\}$

Exercise: extended transitions

Exercise 5.1



Give value of the following function applications

- $\hat{\delta}(q_2, ba) =$
- $\hat{\delta}(q_1, bb) =$
- $\hat{\delta}(q_0, a) =$
- $\hat{\delta}(q_0, bbb) =$

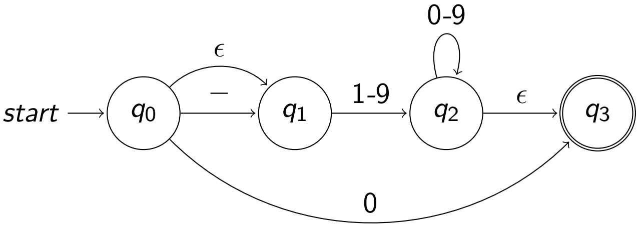
Accepted word

Definition 5.4

A word w is accepted by an ϵ-NFA $A = (Q, \Sigma, \delta, q_0, F)$ if $\hat{\delta}(q_0, w) \cap F \neq \emptyset$.

Example 5.5

Consider the following NFA



Since $\hat{\delta}(q_0, 10) = \{q_2, q_3\}$, 10 is accepted by the above ϵ-NFA.

Eliminating ϵ transitions

Theorem 5.1

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an ϵ-NFA. There is an NFA A' such that $L(A) = L(A')$.

Proof.

We construct $A' = (Q, \Sigma, \delta', q_0, F)$, where for each $a \in \Sigma$ and $q \in Q$

$$\delta'(q, a) \triangleq ECLOSE(\delta(q, a)).$$

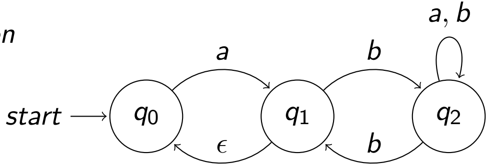
Intuitively, the above construction works. □

Commentary: The above construction adds only new transitions. No new state.

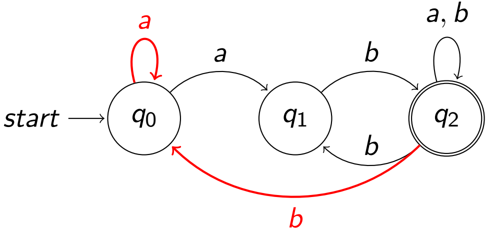
Example: eliminating ϵ transitions

Example 5.6

Consider automaton



After removing ϵ transition, we obtain the black transitions.

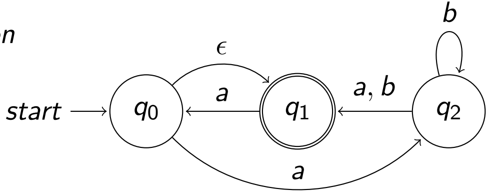


The above red transitions are added after applying ϵ closure.

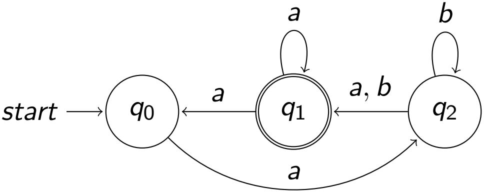
Example II: eliminating ϵ transitions

Example 5.7

Consider automaton



After removing ϵ transition and applying ϵ closure, we obtain



Exercise 5.2

Is the above construction correct?

Correct construction for eliminating ϵ transitions

Earlier construction was handling the base case incorrectly!

We need cases between initial visit of q_0 and later repeated visits of q_0.

Correct proof for theorem 5.1.

A fresh copy of the initial state!

We construct $A' = (Q \cup \{q'_0\}, \Sigma, \delta', q'_0, F')$, where

- $\delta'(q'_0, a) \triangleq ECLOSE(\delta(ECLOSE(\{q_0\}), a))$,

  For initial visit of q_0

- $\delta'(q, a) \triangleq ECLOSE(\delta(q, a))$ for each $a \in \Sigma$ and $q \in Q$,

- $F' \triangleq \begin{cases} F \cup \{q'_0\} & ECLOSE(\{q_0\}) \cap F \neq \emptyset \\ F & \text{otherwise.} \end{cases}$

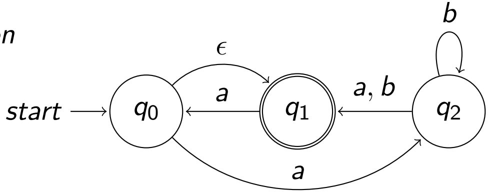
  q_0 is included here for later visits.

Commentary: In the book, the DFA construction is presented. Here we only translating to NFA, because we already know how to convert from NFA to DFA. Please be mindful of this difference.

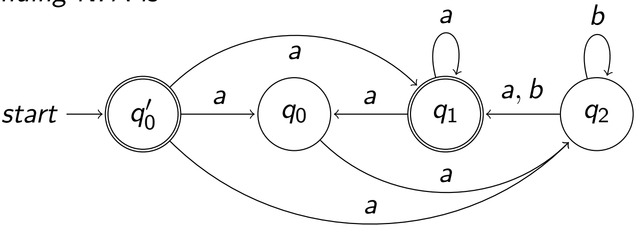
Example II: eliminating ϵ transitions (corrected)

Example 5.8

Consider automaton



The corresponding NFA is



$$\delta'(q_0', a) \triangleq ECLOSE(\delta(ECLOSE(\{q_0\}), a))$$
$$= ECLOSE(\delta(\{q_0, q_1\}, a)) = ECLOSE(\{q_2, q_0\})) = \{q_0, q_1, q_2\}$$

ϵ-NFAs

ϵ-NFAs still recognize regular languages.

ϵ-NFAs, why are they useful?

To be continued...

End of Lecture 5