SAT@Mandi 2019

Lecture 2: Encoding into reasoning problems

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Topic 2.1

Z3 solver

Solver basic interface

- Input : formula
- Output: sat/unsat

If satisfiable, we may ask for a satisfying assignment.

Exercise 2.1

What can we ask from a solver in case of unsatisfiability?

Z3: SMT solver

- Written in C++
- Provides API in C++ and Python
- We will initially use python interface for quick ramp up
- Later classes we will switch to C++ interface

Locally Installing Z3 (Linux)

- Download

```
https://github.com/Z3Prover/z3/releases/download/z3-4.7.1/z3-4.7.
1-x64-ubuntu-16.04.zip
```

- Unzip the file in some folder. Say

/path/z3-4.7.1-x64-ubuntu-16.04/

- Update the following environment variables

```
$export LD_LIBRARY_PATH=$LD_LIBRARY_PATH:/path/z3-4.7.1-x64-ubuntu-16.04/bin
$export PYTHONPATH=$PYTHONPATH:/path/z3-4.7.1-x64-ubuntu-16.04/bin/python
```

- After the setup the following call should throw no error

$python3 /path/z3-4.7.1-x64-ubuntu-16.04/bin/python/example.py

Steps of using Z3 via the python interface

```
from z3 import *          # load z3 library

p1 = Bool("p1")           # declare a Boolean variable
p2 = Bool("p2")
phi = Or( p1, p2 )        # construct the formula

s = Solver()              # allocate solver
s.add( phi )              # add formula to the solver
r = s.check()             # check satisfiability
if r == sat:
    print("sat")
else:
    print("unsat")
# save the script test.py
# run \$python3 test.py
```

Commentary: In python 3.0 onward one may have to write `print("sat")` instead of `print "sat"`

Get a model

```
r = s.check()
if r == sat:
    m = s.model()        # read model
    print(m)              # print model
else:
    print("unsat")
```

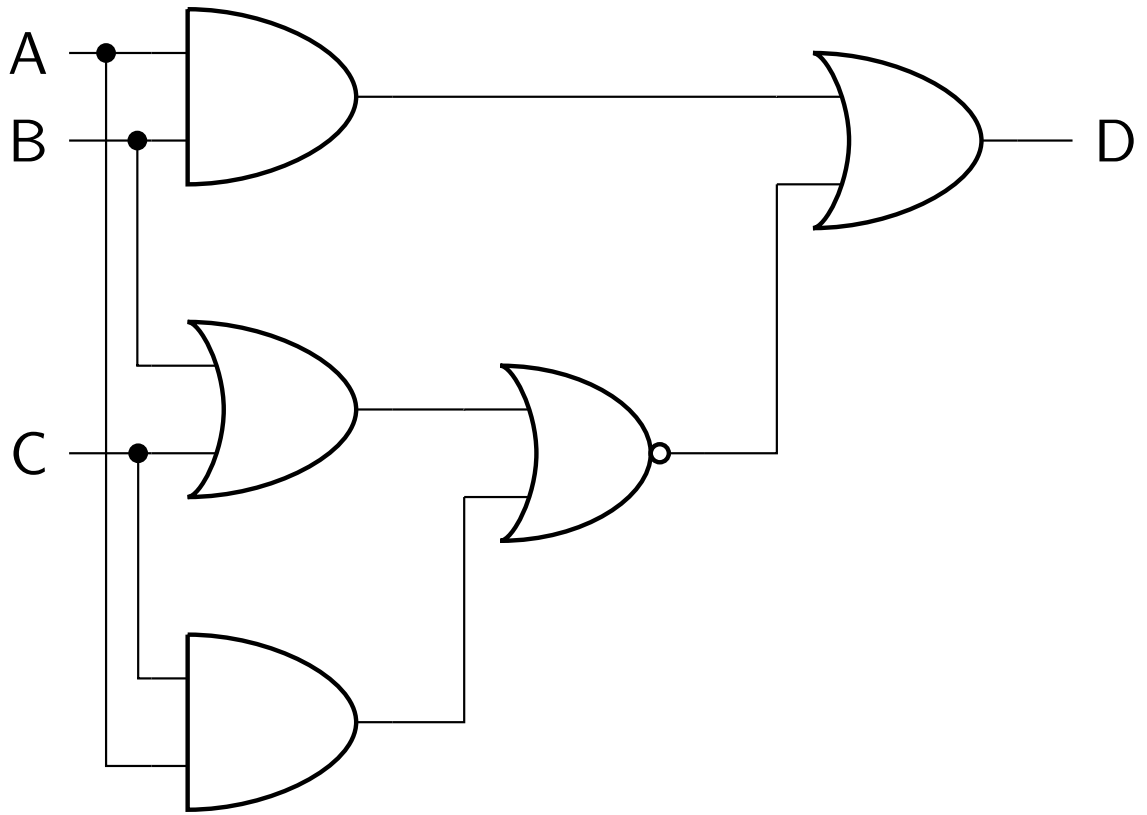
Solve and print model

```
from z3 import *

# packaging solving and model printing
def solve( phi ):
   s = Solver()
   s.add( phi )
   r = s.check()
   if r == sat:
        m = s.model()
        print(m)
   else:
        print("unsat")

 # we will use this function in later slides
```

Exercise: encoding Boolean circuit

Exercise 2.2

Using Z3, find the input values of A, B, and C such that output D is 1.



Design of solvers: context vs. solver

Any complex software usually has a context object.

The context consists of a formula store containing the constructed formulas.

Z3 Python interface instantiates a default context. Therefore, we do not see it explicitly.

A `Solver` is a solving instance. There can be multiple solvers in a context.

The `Solver` solves only the added formula.

Solving rational(real) arithmetic

```
x = Real('x')
y = Real('y')
phi = And(x + y > 5, x > 1, y > 1)
solve( phi )
```

For linear arithmetic
Real == rational

Solving integer arithmetic

```
x = Int('x')
y = Int('y')
phi = And(x + y > 5, x > 1, y > 1)
solve( phi )
```

Exercise: bounded model checking

Exercise 2.3

Using Z3, find the inputs x and y such that the assert fails.

```
int foo( int x, int y ) {
  int z = 3*x + 2*y - 3;
  if( y > 0 )
    assert( z != 0 );
}
```

Uninterpreted functions

```
x = Int('x')
y = Int('y')

# declaring Int -> Int function
h =  Function('h', IntSort(), IntSort() )

phi = And( h( x ) > 5, h( y ) < 2 )

solve( phi )
```

Exercise:

Exercise 2.4

Give a satisfying model of the following formula

$$g(x,y) < 0 \wedge g(y,x) > 0 \wedge y \approx x$$

Uninterpreted sorts

```
# declaring new sort
u = DeclareSort('U')

# declaring a constant of the sort
c =  Const('c', u )

# declaring a function of the sort
f =  Function('f', u, u )

# declaring a predicate of the sort
P =  Function('P', u, BoolSort() )

phi = And( f(c) == c,  P( f(c) ),  Not( P(c) ) )

solve( phi )
```

Commentary: Please try getting model after dropping the third atom. The solver also chooses domains for the uninterpreted sorts and the models of the functions are presented in terms of the domains.

Quantifiers

```
u = DeclareSort('U')
H = Function('Human', u, BoolSort() )
M = Function('Mortal', u, BoolSort() )

# Humans are mortals
x = Const('x', u )
all_mort = ForAll( x, Implies( H(x), M(x) ) )

s = Const('Socrates', u )
thm = Implies( And( H(s), all_mort ), M(s) )

solve( Not(thm) )   # negation of a valid theorem
                    # is unsatisfiable
```

Exercise: handling quantifiers

Exercise 2.5

Prove/disprove if the following statement is valid.

> *There is someone such that if the one drinks, then everyone drinks*

Topic 2.2

SMT2 format

API vs Input language

- Each solver has their own API
- We need a common input format for
  - interoperability and
  - database of problems

Standard format for SMT solvers

SMT2 is a standard input format for SMT solvers.

`http://smtlib.cs.uiowa.edu/language.shtml`

- Formulas are written in prefix notation (why?)

```
(>= (* 2 x) (+ y z))
```

- There is a simple type system. Similar to Z3 API.
- Solver interacts like a stack

File format

An SMT2 file has four distinct parts

1. Preamble declarations
2. Sort declarations
3. Variable declarations
4. Asserting formulas
5. Solving commands

Preamble declaration

- Set configurations of the solvers

```
; setting Theory/Logic
(set-logic QF_UFLIA)

; enable proof generation in case of unsat formula
(set-option :produce-proofs true)
```

Sort declarations

- Declare new sorts of the variables

```
(declare-sort symbol numeral)
```

Example 2.1

```
(declare-sort U 0)     ; new sort with no parameters
(declare-sort Arr 2)   ; new sort with two parameters
```

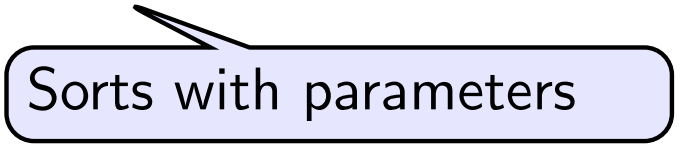
Variable declarations

- Declare variables and functions that may be used in the formulas

```
(declare-fun symbol (sort*) sort)
```

Example 2.2

```
(declare-fun x () Int)
(declare-fun f (Int) Int)
(declare-fun g (Int Int) Int)
(declare-fun h ((Arr U Int) Int) Int)
```



Asserting formulas

- Formulas are asserted in a sequence

Example 2.3

```
(assert (>= (* 2 x) (+ y z)))
(assert (< (f x) (g x x)))
(assert (> (f y) (g x x)))
```

Commands

- Commands are the actions that solver needs to do

Example 2.4

```
(check-sat) ; checks if the conjunction of asserted formula
            ; is sat
(get-model) ; returns a model if the formulas are sat
```

Stack interaction

The standard is designed to be interactive

- Asserted formulas are pushed in the stack of the solver
- `(push)` command places marker on the stack
- `(pop)` removes the formulas upto the last marker

Example 2.5

```
(push)
(assert (= x y))
(check-sat)
(pop)
```

The full example

```
(set-logic QF_UFLIA)
(set-option :produce-proofs true)
(declare-fun x () Int)
(declare-fun y () Int)
(declare-fun z () Int)
(declare-fun f (Int) Int)
(declare-fun g (Int Int) Int)
(assert (>= (* 2 x) (+ y z)))
(assert (< (f x) (g x x)))
(assert (> (f y) (g x x)))
(check-sat)
(get-model)
(push)
(assert (= x y))
(check-sat)
(pop)
(exit)
```

Demo

http://rise4fun.com/z3

Topic 2.3

Problems

Exercise : Python programming

Exercise 2.6

Write a Python program that generates a random graph in a file `edges.txt` for n nodes and m edges, which are given as command line options.
Please store edges in `edges.txt` as the following sequence of tuples

```
10,12
30,50
....
```

Exercise 2.7

Write a program that reads a directed graph from `edges.txt` and finds the number of strongly connected components in the graph

Exercise 2.8

Write a program that reads a directed graph from `edges.txt` and finds the cliques of size k, which is given as a command line option.

Integer vs. Reals

Exercise 2.9

Consider the following constraints

$$3x - y \geq 2 \wedge 3y - z \geq 3 \wedge 3 \geq x + y$$

Solve the above constraints using SMT solver under the following theories

- *Reals (QF_LRA)*
- *Int (QF_LIA)*

Proving theorems

Exercise 2.10

Prove/disprove the following theorems

- *Sky is blue. Space is black. Therefore sky and space are blue or black.*
- *Hammer and chainsaw are professional tools. Professional tools and vehicles are rugged. Therefore, hammers are rugged.*

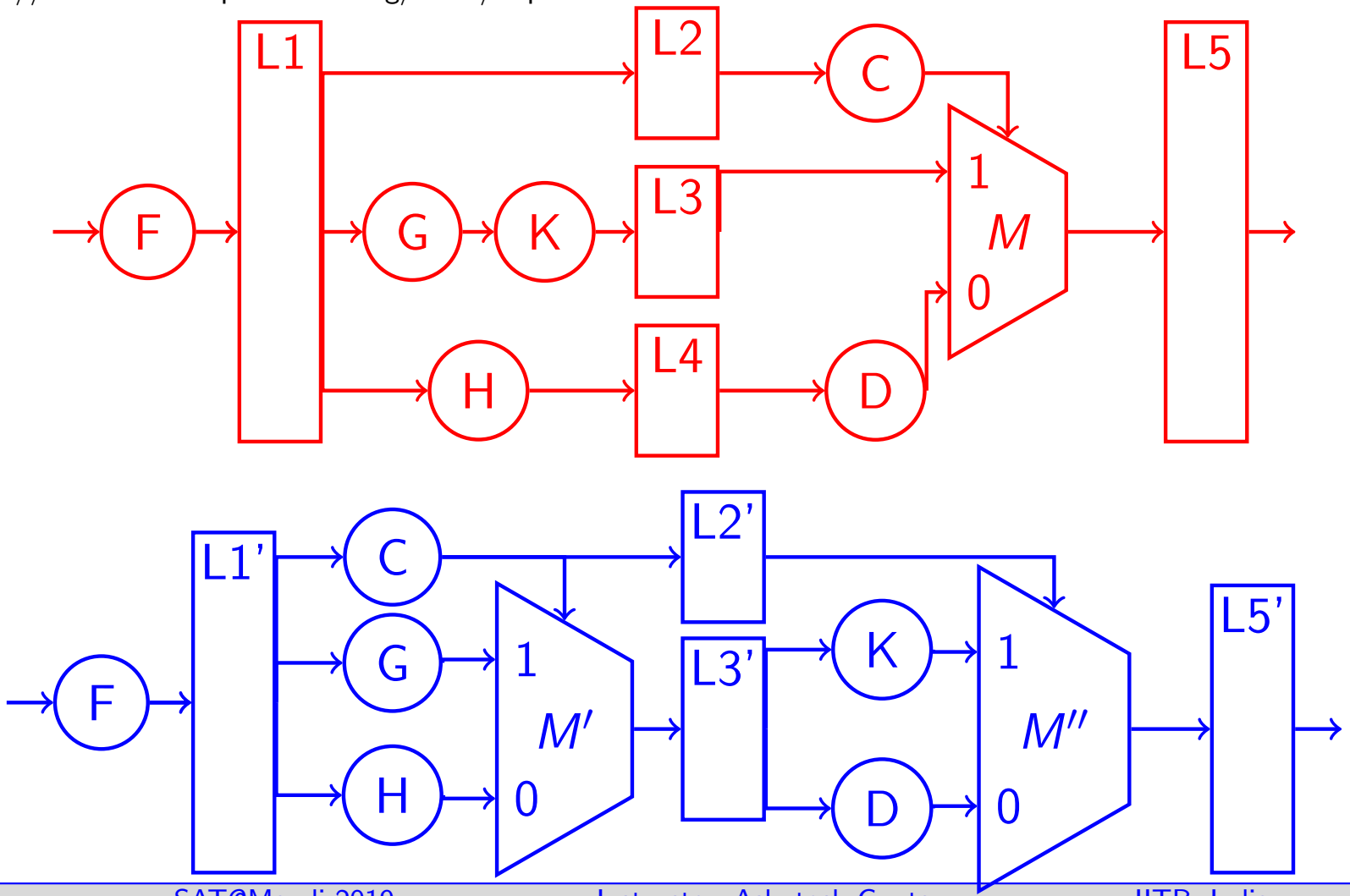
Exercise: translation validation

Exercise 2.11

Show that the following two circuits are equivalent.

Ls are latches, circles are Boolean circuts, and Ms are multiplexers.

Source: http://www.decision-procedures.org/slides/uf.pdf



Write a function: compute linear coefficient

Exercise 2.12

Find coefficient of each variable in a linear term. If the term is non-linear, throw an exception.

Examples:

$x - 2x + y + 4$ *should return* $[4, -1, 1]$ *if variables are ordered* $[x, y]$.

$x - x + 4y - 2(2y)$ *should return* $[0, 0, 0]$ *if variables are ordered* $[x, y]$.

$(x + 1) * y$ *should throw an exception*

Write a function: find positive variables

Exercise 2.13

Find the set of Boolean variables that occur only positively in a propositional logic formula.

An occurrence of a variable is positive if there are even number of negations from the occurrence to the root of the formula.

Examples:

Only q occurs positively in $p \wedge \neg(\neg q \wedge p)$.

p occurs positively in $\neg\neg p$.

p does not occur positively in $\neg p$.

p and q occur positively in $(p \vee \neg r) \wedge (r \vee q)$.

Write a function: find quantifier alternation depth

Exercise 2.14

Compute quantifier alternations depth of a sentence.

Maximum number of quantifier type switches is any path from an atom to the root of the sentence.

Examples: quantifier alternations depth of
$\forall x.\ \exists y.\ E(x,y)$ *is 1.*

$\forall x.\ \exists y.\ \forall z.\ E(x,y,z)$ *is 2.*

$\forall x.\ \forall y.\ E(x,y)$ *is 0.*

$\forall x.\ (\ (\ \exists y.\ H(x,y)\ ) \Rightarrow G(x))$ *is 0.* *($\exists$ under negation is $\forall$ and vice-versa)*

$\forall x.\ (\ (\ \exists y.\ H(x,y)\ ) \Rightarrow \exists z.\ G(x,z))$ *is 1.*

Write a function: find unrelated constraints

Exercise 2.15

Consider a formula F consists of only a conjunction of atoms. Find the subsets of F that have disjoint set of uninterpreted symbols.

Examples:

$x \approx y \wedge x \approx z \wedge P(u)$ *has two unrelated subsets* $\{x \approx y, x \approx z\}$ *and* $\{P(u)\}$

$x + y = 3 \wedge z + u \geq 10$ *has two unrelated subsets* $\{x + y = 3\}$ *and* $\{z + u \geq 10\}$, *while they have a common interpreted symbol* $+$.

Write a function: find maximum occurring symbol

Exercise 2.16

Consider a formula F. Find the uninterpreted symbol in F that occurs most often.

Examples:

x occurs most often in $g(g(x,x),g(x,x))$.

f occurs most often in $f(x,y) \approx f(x,b) \wedge f(2,3) > 10$.

D occurs most often in $\exists x.(D(x) \Rightarrow D(x+1))$. *quantified variables are not counted.*

Write a function: foreign function interface

End of Lecture 2