CS228 Logic for Computer Science 2020

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Tutorial sheet 1: Propositional logic, Formal proofs, and Normal forms

Ashutosh Gupta and S. Krishna

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1. What is wrong with the following inductive proof?

   claim: All horses have same color
   base case:
   One horse has a single color
   induction step:
   We assume that n horses have same colors.
   Take $n+1$ horses $h_1, .., h_{n+1}$.
   Due to the assumption, $h_1, .., h_n$ have same color.
   Due to the assumption, $h_2, .., h_{n+1}$ have same color.
   Therefore, $h_1, .., h_{n+1}$ have same color.

2. Convert the following argument into a propositional statement, i.e., $\Sigma \vdash F$.

   If the laws are good and their enforcement is strict, then crime will diminish. If strict enforcement of laws will make crime diminish, then our problem is a practical one. The laws are good. Therefore our problem is a practical one. (Hint: needed propositional variables G, S, D, P)
   (Source : Copi, Introduction of logic)

3. Write a formal proof proving the statement in the previous problem.

4. Let us suppose we only have connectives $\wedge$, $\vee$, or $\neg$ in our formulas. Consider a set Σ of formulas such that

   (a) for each $p \in \mathbf{Vars}$, $p \notin \Sigma$ or $\neg p \notin \Sigma$
   (b) if $\neg\neg F \in \Sigma$ then $F \in \Sigma$
   (c) if $(F \wedge G) \in \Sigma$ then $F \in \Sigma$ and $G \in \Sigma$
   (d) if $\neg(F \vee G) \in \Sigma$ then $\neg F \in \Sigma$ and $\neg G \in \Sigma$
   (e) if $(F \vee G) \in \Sigma$ then $F \in \Sigma$ or $G \in \Sigma$
   (f) if $\neg(F \wedge G) \in \Sigma$ then $\neg F \in \Sigma$ or $\neg G \in \Sigma$

   Show that Σ is satisfiable, i.e., there is an assignment that satisfies every formula in Σ.

5. Show $\Rightarrow$ alone can not express all the Boolean functions

6. Let us suppose we have access to the parse tree of a formula, which is represented as a directed acyclic graph (DAG) (*not as a tree*). Write an algorithm that produces negation normal form (NNF) of the formula in linear time in terms of the size of the DAG. You may assume the cost of reading and writing to a map data structure is constant time.