CS228 Logic for Computer Science 2020
IITB, India
Tutorial sheet 2: 2-CNF, Resolution, and SAT solving

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Compile date: 2020-02-05

1. An unsatisfiable core of an unsatisfiable CNF formula is a (preferably minimal) subset of the formula that is also unsatisfiable. Give an algorithm to compute a minimal unsatisfiable core of a 2-CNF formula.

2. In the Horn solving algorithm, we started with all false assignment and incrementally turned the variables true.
   a. Give an algorithm that starts with all true initial assignment and finds satisfying assignment for the Horn clauses.
   b. Can we also start with any starting assignment? If yes, how the algorithm needs to be modified?

3. Let us suppose we have a resolution proof deriving $\bot$. We have discussed that valid clauses should not be used for resolution. However, no one stops us in producing them and then further using them for the resolution. Let us suppose we have valid clauses occurring somewhere in the middle of our proof. Give a linear (or close to linear) time algorithm in terms of the size of the proof that removes the valid clauses from the proof.

4. Let F an unsatisfiable CNF formula with n variables. Show that there is a resolution proof of $\bot$ from F that is smaller than $2^{n+1} - 1$.

5. Run DPLL on the following 2-SAT problem

   $$\underbrace{(p_1 \vee p_2)}_{c1} \wedge \underbrace{(p_3 \vee p_4)}_{c2} \wedge \underbrace{(p_5 \vee p_6)}_{c3} \wedge \underbrace{(\neg p_1 \vee \neg p_3)}_{c4} \wedge \underbrace{(\neg p_1 \vee \neg p_5)}_{c5} \wedge \underbrace{(\neg p_3 \vee \neg p_5)}_{c6} \wedge \underbrace{(\neg p_2 \vee \neg p_4)}_{c7} \wedge \underbrace{(\neg p_4 \vee \neg p_6)}_{c8} \wedge \underbrace{(\neg p_6 \vee \neg p_2)}_{c9}$$

   Use the variable ordering $p_1, p_2, p_3, p_4, p_5, p_6$. Always use 0 as first choice for a decision. Annotate a unit propagation with clause number. Also annotate the conflict(s) with the clause number.

6. Write a SAT encoding of the following problem

   For a set of size n, find m subsets with size k such that any pair of the sets have exactly one common element.

7. Encode N-queens problem into a SAT problem.
   N-queens problem: Place n queens in $n \times n$ chess such that none of the queens threaten each other.

8. Write a function in Z3 python interface that finds the set of Boolean variables that occur only positively in a propositional logic formula.

   An occurrence of a variable is positive if there are even number of negations from the occurrence to the root of the formula. Assume there are no $\oplus$, $\Rightarrow$, or $\Leftrightarrow$ in the formula.

   Examples:

   Only q occurs positively in $p \wedge \neg(\neg q \wedge p)$.

   p occurs positively in $\neg\neg p$.

   p does not occur positively in $\neg p$.

   p and q occur positively in $(p \vee \neg r) \wedge (r \vee q)$.