CS228 Logic for Computer Science 2020
IITB, India
Tutorial sheet 1: Propositional logic, Formal proofs, and Normal forms

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1. Convert the following argument into a propositional statement

   If anything is expensive it is both valuable and rare. Whatever is valuable is both desirable and expensive. Therefore if anything is either valuable or expensive then it must be both valuable and expensive. (Use: $E(x)$: x is expensive; $V(x)$: x is valuable; $R(x)$: x is rare; $D(x)$: x is desirable) (Source : Copi, Introduction of logic)

2. Either prove the above statement holds. Or, show that the statement does not hold.

3. Let us suppose we do not have connectives $\oplus$, $\Leftrightarrow$, $\Rightarrow$, and $=$ in our formulas. Let $\mathbf{S}$ be a FOL signature. Consider a set Σ of FOL $\mathbf{S}$-sentences such that

   (a) for each atom $F \in A_{\mathbf{S}}$, $F \notin \Sigma$ or $\neg F \notin \Sigma$
   (b) if $\neg\neg F \in \Sigma$ then $F \in \Sigma$
   (c) if $(F \wedge G) \in \Sigma$ then $F \in \Sigma$ and $G \in \Sigma$
   (d) if $\neg(F \vee G) \in \Sigma$ then $\neg F \in \Sigma$ and $\neg G \in \Sigma$
   (e) if $(F \vee G) \in \Sigma$ then $F \in \Sigma$ or $G \in \Sigma$
   (f) if $\neg(F \wedge G) \in \Sigma$ then $\neg F \in \Sigma$ or $\neg G \in \Sigma$
   (g) if $\neg(\exists x.F(x)) \in \Sigma$ then $F(t) \in \Sigma$ for each ground term $t \in \hat{T}_{\mathbf{S}}$
   (h) if $\forall x.F(x) \in \Sigma$ then $F(t) \in \Sigma$ for each ground term $t \in \hat{T}_{\mathbf{S}}$
   (i) if $\neg(\forall x.F(x)) \in \Sigma$ then $F(t) \in \Sigma$ for some ground term $t \in \hat{T}_{\mathbf{S}}$
   (j) if $\exists x.F(x) \in \Sigma$ then $F(t) \in \Sigma$ for some ground term $t \in \hat{T}_{\mathbf{S}}$

   Show that Σ is satisfiable, i.e., there is a structure that satisfies every formula in Σ.

4. We may start with some set of sentences Σ and keep adding formulas due to the above definitions until we saturate the set, i.e., we cannot add formulas anymore.

   - Consider signature $\mathbf{S} = (\{a/0, b/0, c/0\}, \{P/1\})$. Saturate set $\{P(a) \wedge P(b) \wedge \exists x.\ \neg P(x)\}$ using the above process.
   - Consider signature $\mathbf{S} = (\{a/0, b/0\}, \{P/1\})$. Saturate set $\{P(a) \wedge P(b) \wedge \exists x.\ \neg P(x)\}$ using the above process.
   - Consider signature $\mathbf{S} = (\{\}, \{P/1\})$. Saturate set $\{\forall x.\ P(x) \wedge \exists x.\ \neg P(x)\}$ using the above process.

5. Which of the following formulas are not unifiable? $f/1, c/0, d/0$ are functions and x and y are variables.

- $f(x)$ and x
- $f(c)$ and x
- $f(y)$ and x
- $f(c)$ and d