I say, “I am lying.”

Am I?

Commentary: Consider the case if the statement is true. Then the statement claims that it is not the fact. On the other hand if the statement is false. Then, the negation of the statement is true and I am not lying. Therefore, the statement must be true. Therefore, both the cases leads to absurdity. For a similar fun, https://en.wikipedia.org/wiki/Paradox_of_the_Court

CS228 Logic for Computer Science 2020

Lecture 1: Introduction and logistics

Instructor: Ashutosh Gupta

IITB, India

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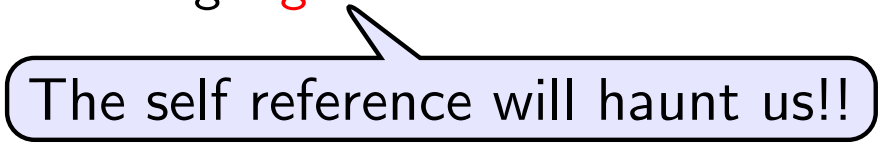
Topic 1.1

What is logic?

What is logic?

- Have you ever said to someone "be logical"?
  - whatever your intuition was that is logic
- Mathematization/Formalization of the intuition is mathematical logic
- Two streams of studying logic
  - use of logic : logic as a tool to study something else, e.g., math
  - properties of logic: since logic has mathematical structure, we may study its mathematical properties using logic



Why a CS student should study logic?

Differential equations
are the calculus of
Newtonian physics

Logic
is the calculus of
computer science

Logic provides tools to define/manipulate computational objects

Defining logic

Logic is about inferring conclusions from given premises

Example 1.1

1. *Humans are mortal*
2. *Socrates is a human*

Socrates is mortal ✓

1. *Apostles are twelve*
2. *Peter is an apostle*

Peter is twelve ✗

What went wrong here?

Intuitive Pattern:

1. α*s are* β
2. γ *is an* α

γ *is* β

where α *and* γ *are noun and* β *is adjective*

Very easy to ill-define.
Logic needs rigorous definitions!!

Commentary: Clearly understood formalization arrived in early 20th century. The above was one of the mistakes in the Aristotle's syllogism(inference rules), which dominated European thought until late middle ages.https://en.wikipedia.org/wiki/Syllogistic_fallacy

Topic 1.2

Course logistics and contents

Evaluation and website

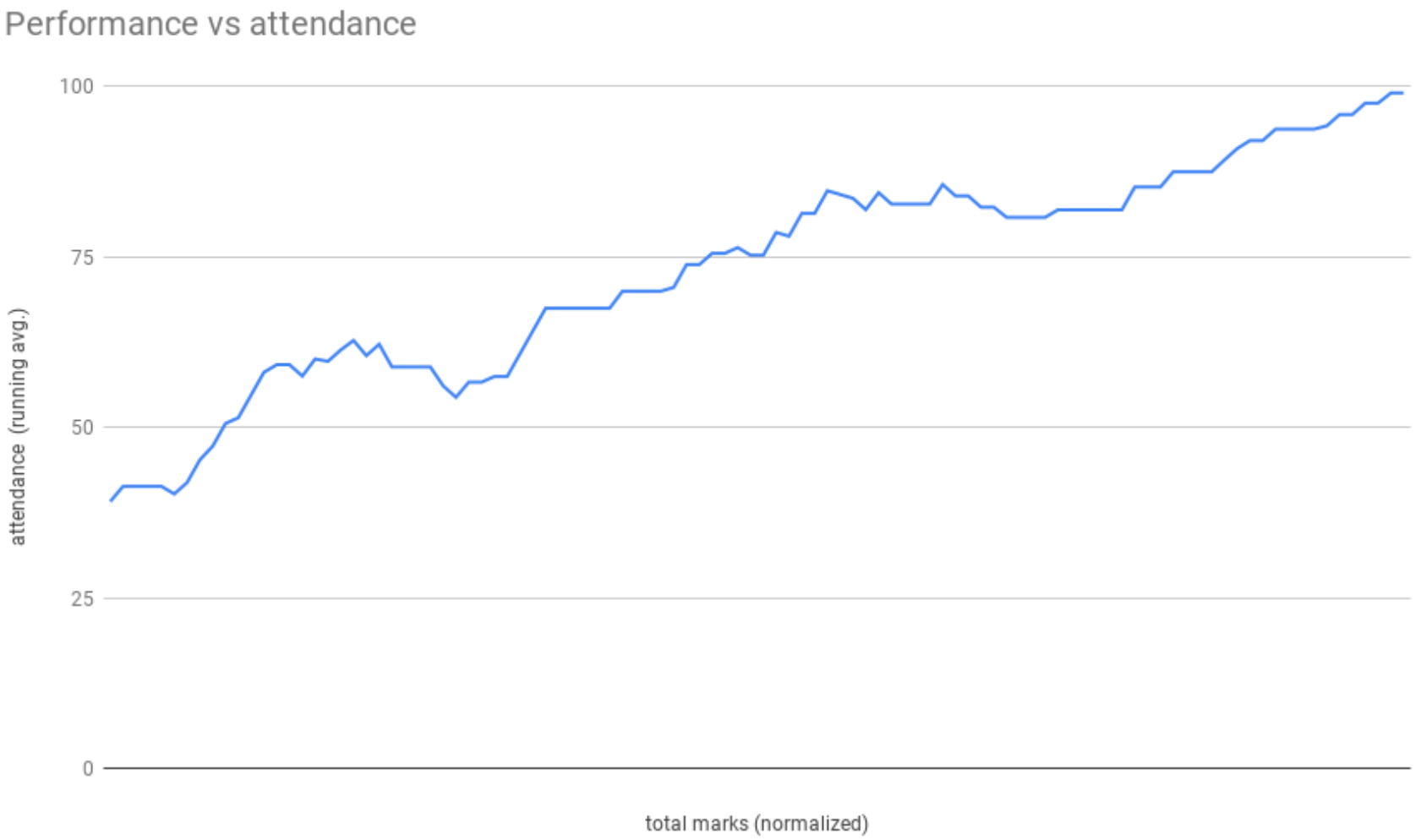
- Quizzes : 30% (3 quizzes + 1 lab quiz)
  - Quiz 1 : 29th Jan, Lab quiz: 14th Feb, ...
- Midterm : 25% (2 hours)
- Final : 40% (3 hours)
- Attendance : 5% (class quizzes)
  - Bring your smartphone in the class with a browser
  - Out of class attendance marking $\Rightarrow$ Fail

For the further information

`http://www.cse.iitb.ac.in/~akg/courses/2020-logic`

All the content will be posted on the website.

Why attend? – better attendance $\Rightarrow$ better grade



X-axis : students sorted by their marks

Y-axis: running average of attendance of 20 students

The course

We will study the following topics

- Propositional logic
- First-order logic

} Foundations

Midterm

- Logic for verification
- MSO logic
- Temporal logic

} Applications in computer science

Propositional logic

Propositional logic

- deals with propositions,
  - Example: the shirt is cheap, the shoe is expensive
- only infers from the structure over the propositions, and
  - Example: the shirt is cheap and the shoe is expensive
- does not look inside the propositions.
  - Example: the shirt is cheap and the shirt is made of gold

Example: Propositional argument

Example 1.2

Is the following argument valid?

If the seed catalog is correct then if seeds are planted in April then the flowers bloom in July. The flowers do not bloom in July. Therefore, if seeds are planted in April then the seed catalog is not correct.

Let us identify the propositions in the argument

- c = *the seed catalogue is correct*
- s = *seeds are planted in April*
- f = *the flowers bloom in July*

If we replace the proposition with symbols, we obtain

$$\textit{If } c \textit{ then if } s \textit{ then } f\textit{. not } f\textit{. Therefore, if } s \textit{ then not } c\textit{.}$$

Propositional logic (PL) topics

We will study

- Week1: definition of PL and meaning (For a philosopher view)
- Week2: Formal proofs (For a mathematician view)
- Week3: Proof systems for PL and their properties (For a computer scientist view)
- Week4: PL solvers *aka* SAT solvers (For a hacker view)

Commentary: PL has limited expressive power. However, there are a lot of real world problems that can be *encoded* using PL. SAT solver is an effective tool to solve the problems.

First-order logic (FOL)

First-order logic

- looks inside the propositions,
- deals with parameterized propositions, and quantifiers, and
- can express a lot of interesting math.

Example 1.3

Is the following argument valid?
Humans are mortal. Socrates is a human. Therefore, Socrates is mortal.

The parametric propositions in the argument.

- $H(x) = x$ *is a human*
- $M(x) = x$ *is mortal*
- $s =$ *Socrates*

If we replace the parametric propositions with symbols, we obtain

For all x *if* $H(x)$ *then* $M(x)$. $H(s)$. *Therefore,* $M(s)$.

First-order logic (FOL) topics

We will study

- Week 5: definition of FOL and syntactic properties (For a philosopher view)
- Week 6: proof systems for FOL, etc (For a computer scientist view)
- Week 7: first-order theorem provers (For a hacker view)

Topic 1.3

Problems

Mistake

Exercise 1.1

What is wrong with the following argument?
All supermen can fly. Therefore, there is a superman.

Commentary: One of the errors in Aristotle's logic https://en.wikipedia.org/wiki/Existential_fallacy

Does God exist?

Exercise 1.2

Is there a logical problem with the following argument aka *Ontological argument?*

1. *God is the greatest possible being that can be imagined.*
2. *God exists as an idea in the mind.*
3. *A being that exists as an idea in the mind and in reality is, other things being equal, greater than a being that exists only as an idea in the mind.*
4. *Thus, if God exists only as an idea in the mind, then we can imagine something that is greater than God.*
5. *But we cannot imagine something that is greater than God.*
6. *Therefore, God exists.*

(text source Wikipedia)
Fun side of the argument: `https://xkcd.com/1505/`

A puzzle from internet

Exercise 1.3

Sanjay and Salman are new friends with Madhuri, and they want to know her birthday. Madhuri gives them a list of possible dates.
March 14, March 15, March 18,
April 16, April 17,
May 13, May 15,
June 13, June 14, June 16

Madhuri then tells Sanjay and Salman separately the month and the day of her birthday respectively.

Sanjay: I don't know the date, but I know that Salman doesn't know too.
Salman: At first I didn't know the date, but I know now.
Sanjay: Then I also know the date.

So when is Madhuri's birthday?

End of Lecture 1