CS228 Logic for Computer Science 2021

Lecture 13: Encoding into reasoning problems

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Topic 13.1

Z3 solver

Solver basic interface

- Input : formula
- Output: sat/unsat

If satisfiable, we may ask for a satisfying assignment.

Exercise 13.1
What can we ask from a solver in case of unsatisfiability?

Z3: SMT solver

- Written in C++
- Provides API in C++ and Python
- We will initially use python interface for quick ramp up
- Later classes we will switch to C++ interface

Installing Z3 (Ubuntu-18.04)

```
$sudo apt-get install z3
```

Not tested on 20.04

Locally Installing a version of Z3 (Linux)

Let us install `z3-4.7.1`. You may choose another version.

- Download

`https://github.com/Z3Prover/z3/releases/download/z3-4.7.1/z3-4.7.1-x64-ubuntu-16.04.zip`

- Unzip the file in some folder. Say

/path/z3-4.7.1-x64-ubuntu-16.04/

- Update the following environment variables

```
$export LD_LIBRARY_PATH=$LD_LIBRARY_PATH:/path/z3-4.7.1-x64-ubuntu-16.04/bin
$export PYTHONPATH=$PYTHONPATH:/path/z3-4.7.1-x64-ubuntu-16.04/bin/python
```

- After the setup the following call should throw no error

$python3 /path/z3-4.7.1-x64-ubuntu-16.04/bin/python/example.py

Topic 13.2

Using solver

Steps of using Z3 via python interface

```
from z3 import *          # load z3 library

p1 = Bool("p1")           # declare a Boolean variable
p2 = Bool("p2")

phi = Or( p1, p2 )        # construct the formula
print(phi)                # printing the formula

s = Solver()              # allocate solver
s.add( phi )              # add formula to the solver
r = s.check()             # check satisfiability
if r == sat:
    print("sat")
else:
    print("unsat")        # save the script test.py
                          # run \$python3 test.py
```

Get a model

```
r = s.check()
if r == sat:
    m = s.model()      # read model
    print(m)           # print model
else:
    print("unsat")
```

Exercise 13.2

What happens if we run `m = s.model()` in the unsat case?

Solve and print model

```
from z3 import *

# packaging solving and model printing
def solve( phi ):
  s = Solver()
  s.add( phi )
  r = s.check()
  if r == sat:
      m = s.model()
      print(m)
  else:
      print("unsat")

 # we will use this function in later slides
```

Pointer and variable

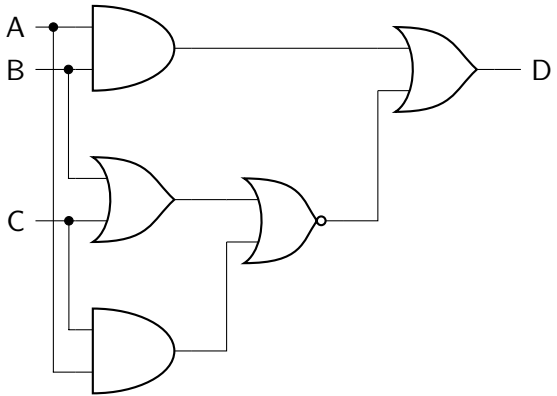
There is a distinction between the Python variable name and the propositional variable it holds.

```
from z3 import *      # load z3 library

x = Bool("y")         # creates Propositional variable y

z = x                 # python pointer z also holds variable y
```

Exercise: encoding Boolean circuit

Exercise 13.3

Using Z3, find the input values of A, B, and C such that output D is 1.



We know you can do it! Please do not shout the answer. Please make computer find it.

Topic 13.3

Solver engineering

Design of solvers: context vs. solver

Any complex software usually has a context object.

The context consists of a formula store containing the constructed formulas.

Z3 Python interface instantiates a default context. Therefore, we do not see it explicitly.

A `Solver` is a solving instance. There can be multiple solvers in a context.

The `Solver` solves only the added formula.

Formula handling

```
a = Bool('a')
b = Bool('b')
ab = And( a, b )

# accessing sub-formulas
print(ab.arg(0))
print(ab.arg(1))

# accessing the symbol at the head
ab_decl = ab.decl()
name = ab_decl.name()
if name == "and":
    print("Found an And")
```

Topic 13.4

Problems

Exercise : Python programming

Exercise 13.4

Write a Python program that generates a random graph in a file `edges.txt` for n nodes and m edges, which are given as command line options.
Please store edges in `edges.txt` as the following sequence of tuples

```
10,12
30,50
....
```

Exercise 13.5

Write a program that reads a directed graph from `edges.txt` and finds the number of strongly connected components in the graph

Exercise 13.6

Write a program that reads a directed graph from `edges.txt` and finds the cliques of size k, which is given as a command line option.

Proving theorems

Exercise 13.7

Prove/disprove the following theorems using a solver

- *Sky is blue. Space is black. Therefore sky and space are blue or black.*
- *Hammer and chainsaw are professional tools. Professional tools and vehicles are rugged. Therefore, hammers are rugged.*

Write a function: find positive variables

Exercise 13.8

Find the set of Boolean variables that occur only positively in a propositional logic formula.

An occurrence of a variable is positive if there are even number of negations from the occurrence to the root of the formula.

Examples:

Only q occurs positively in $p \wedge \neg(\neg q \wedge p)$.

p occurs positively in $\neg\neg p$.

p does not occur positively in $\neg p$.

p and q occur positively in $(p \vee \neg r) \wedge (r \vee q)$.

End of Lecture 13