CS766: Analysis of concurrent programs 2023

Lecture 2: Traces

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Compile date: 2023-01-28

Topic 2.1

Formal model of execution

Introducing parallel composition

We add parallel composition in our simple programming language.

We define interleaved semantics as follows

$$((v, c_1||c_2), (v', c_1'||c_2)) \in T \quad \text{if} \quad ((v, c_1), (v', c_1')) \in T$$

$$((v, c_1||c_2), (v', c_1||c_2')) \in T \quad \text{if} \quad ((v, c_2), (v', c_2')) \in T$$

$$((v, \texttt{skip}||\texttt{skip}), (v, \texttt{skip})) \in T$$

Interleaved semantics

Global variables

Definition 2.1

We call a variable global if two or more threads access the variable. Other variables are called local.

Topic 2.2

Understanding time in interleaved semantics

Instantaneous events

Events happen in threads

- are instantaneous
- No simultaneous events

Global instantaneous events

On global variables

- Writes
- Reads
- Read/Write

Synchronization

- fences

We write them as follows

R/W<VariableName>=<Value>|fence

Events on a timeline

```
                    init: counter := 0

thread0:                          thread1:
a1:tmp0 := counter          ||    b1:tmp1 := counter
a2:counter = tmp0 + 1             b2:counter = tmp1 + 1
```

Global event order in a possible execution:

```
thread0: Rcounter=1
thread1: Rcounter=1
thread1: Wcounter=2
thread0: Wcounter=2
```

Time is always totally ordered.

Intervals

- Often things are not instantaneous
- Takes time to finish such as function calls.

Definition 2.2
An interval

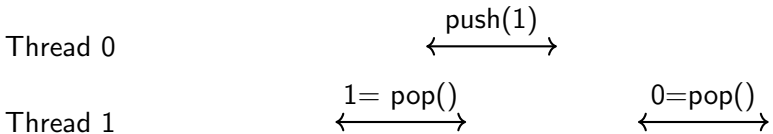
$$A = (e_1, e_2)$$

is time between events e_1 *and* e_2.

Intervals may or may not overlap

Example 2.1

Operations on stack:



Happens before relation (hb)

Definition 2.3

If A ends before A starts, we say A happens before B.

Strict partial order

Theorem 2.1

hb relation is a strict partial order.

Proof.

hb has the following properties of strict partial order.

- Antisymmetry: $hb(A, B) \Rightarrow \neg hb(B, A)$
- Transitivity: $hb(A, B) \wedge hb(B, C) \Rightarrow hb(A, C)$
- Irreflexive: $\neg hb(A, A)$

□

Exercise 2.1

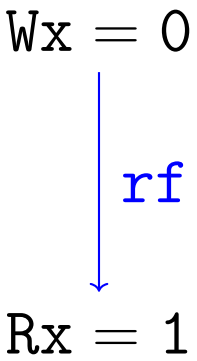
Which condition to add in partial orders conditions such that it becomes total order?

Orders that we know: program order(po)

po : events in a thread are ordered.

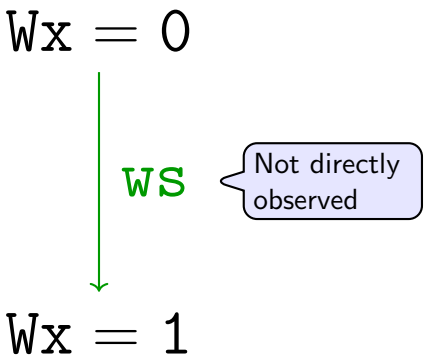
Orders that we know: read from(rf)

rf : every read reads from exactly one write



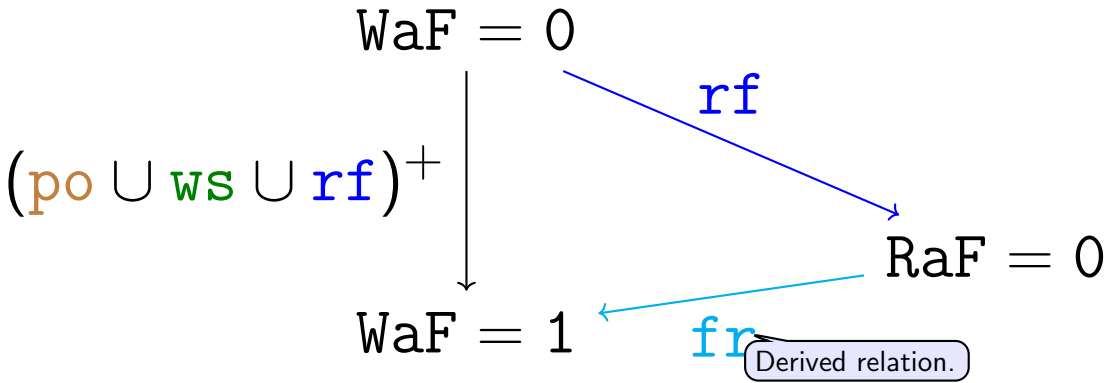
Orders that we know: write serialization(ws)

ws : all writes on a global are totally ordered



Orders that we know: from read(fr)

fr : no write comes between the pair in **rf**



Traces

Definition 2.4

A trace consists of the following relations.

- `po` : events in a thread are ordered.
- `rf` : every read reads from exactly one write
- `ws` : all writes on a global are totally ordered
- `fr` : no other write comes between the read write pairs in `rf`

Definition 2.5

A trace is valid if $(\texttt{po} \cup \texttt{rf} \cup \texttt{ws} \cup \texttt{fr})^+$ *is strict partial order.*

Theorem 2.2

An execution of a program induces only a valid trace.

Analyzing programs using traces

For analyzing a program,

1. We enumerate all (possibly infinite) execution paths in each thread.
2. Each path has a set of events and defines po.
3. We enumerate all rf by mapping each read to some write if values match.
4. We enumerate all ws among writes.
5. Using po, rf, and ws, we compute fr and check if trace is valid.

If no trace reaches a bad state, then program is safe.

Topic 2.3

Formal properties of mutual exclusion protocols

Structure of mutual exclusion protocol

In a typical mutual exclusion protocol, we have the following structure

- Global initialization
- Some code before critical section, called lock
- critical section
- Some code after critical section, called unlock

```
init: counter := 0
```

```
    while(true){
l0:   lock()
c0:   tmp0 := counter
      counter := tmp0 + 1
u0:   unlock()
    }
```

||

```
    while(true){
l1:   lock()
c1:   tmp1 := counter
      counter := tmp1 + 1
u1:   unlock()
    }
```

Properties: mutual exclusion

```
init: counter := 0
```

```
   while(true){                          while(true){
l0:  lock()                          l1:  lock()
c0:  tmp0 := counter                 c1:  tmp1 := counter
     counter := tmp0 + 1                   counter := tmp1 + 1
u0:  unlock()                        u1:  unlock()
   }                                     }
```

Let i th occurrence of critical section in thread 0 be interval $\texttt{CS0}^i = (\texttt{c0}, \texttt{u0})$.

For all i and j, $\quad \texttt{hb}(\texttt{CS0}^i, \texttt{CS1}^j) \vee \texttt{hb}(\texttt{CS1}^j, \texttt{CS0}^i)$

Properties: deadlock free

```
                    init: counter := 0

thread0:                                  thread1:
   while(true){                              while(true){
l0:  lock()                               l1:  lock()
c0:  tmp0 := counter          ||          c1:  tmp1 := counter
     counter := tmp0 + 1                       counter := tmp1 + 1
u0:  unlock()                             u1:  unlock()
   }                                         }
```

If thread0 visits `l0` and never reaches `u0`, then thread1 visits `u1` infinitely often.

And also symmetric condition. Assumes fair scheduling.

Properties: starvation free

```
                    init: counter := 0

thread0:                                  thread1:
   while(true){                              while(true){
l0:  lock()                               l1:  lock()
c0:  tmp0 := counter          ||          c1:  tmp1 := counter
     counter := tmp0 + 1                       counter := tmp1 + 1
u0:  unlock()                             u1:  unlock()
   }                                         }
```

If thread0 visits l0, it eventually reaches u0.

And also symmetric condition. Assumes fair scheduling for th

Precise statements

Topic 2.4

Analyzing mutual exclusion protocols

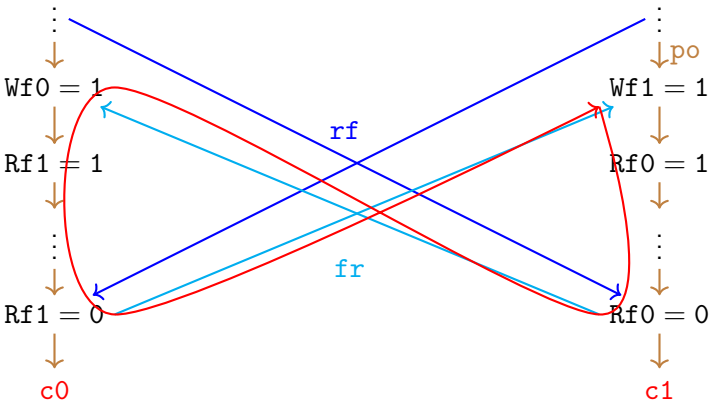
Example: Flag

```
init: f0 := false, f1 := false
```

```
thread0:                          thread1:
   while(true){                      while(true){
l0:   f0 := true                  l1:   f1 := true
      while(f1);          ||            while(f0);
c0:   // critical section         c1:   // critical section
u0:   f0 := false                 u1:   f1 := false
   }                                 }
```

Example: proving mutual exclusion for Flag

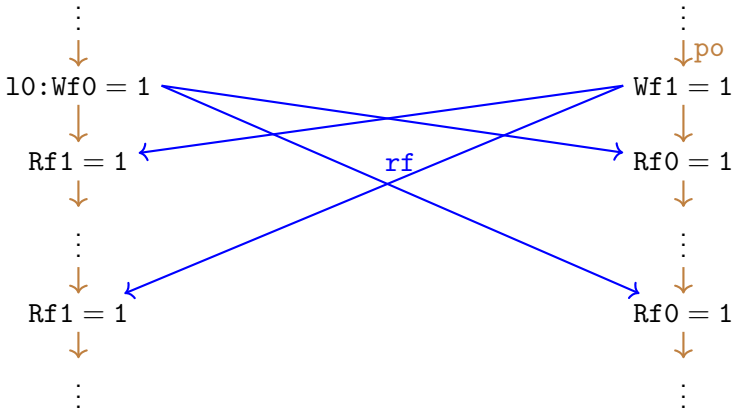
Violating execution: Assume threads reached c0 and c1 at the same time



The above drawing proves that there will be no violation of mutual exclusion.

Commentary: Not all `rf` edges are drawn. We need to check for all of them.

Example: Flag can deadlock



Assume never reaches `u0`

Does not visit `u1` infinitely often

Trace does not have cycles and does not violate fair scheduling. Therefore, execution possible.

Exercise 2.2 *Under our definition, does deadlock ⇒ starvation?*

Flag: an observation

- Concurrent execution may deadlock
- Sequential execution does not

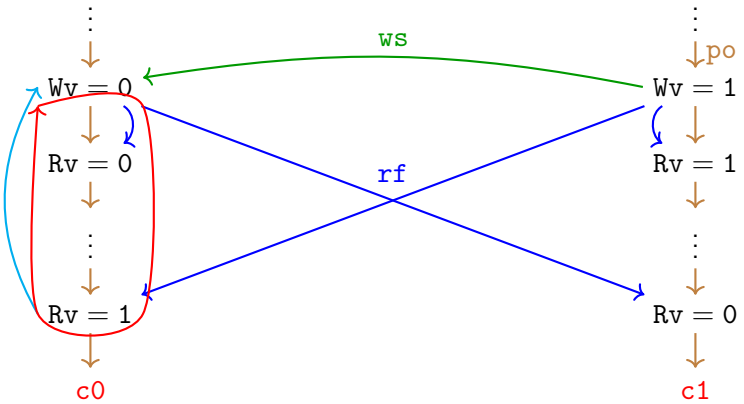
Example: LucknowNawab

```
                    init: v := random()

thread0:                                  thread1:
   while(true){                              while(true){
l0:   v := 0                              l1:   v := 1
      while( v == 0);          ||               while( v == 1);
c0:   // critical section                 c1:   // critical section
u0:   skip                                u1:   skip
   }                                         }
```

Example: proving mutual exclusion for LucknowNawab

Violating execution: Assume threads reached c0 and c1 at the same time



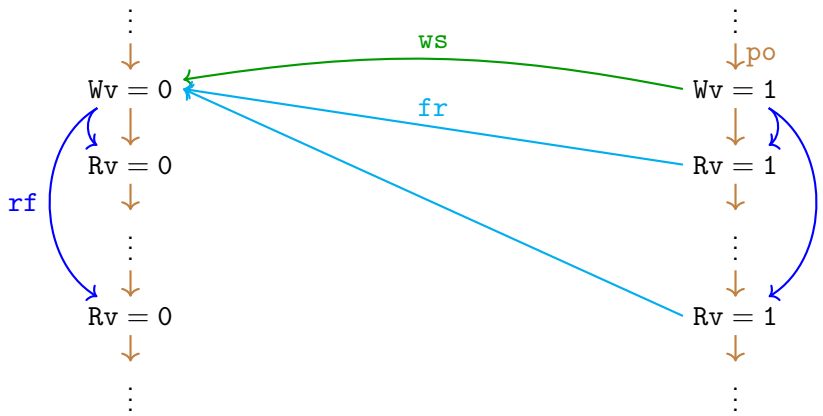
The above drawing proves that there will be no violation of mutual exclusion.

Exercise 2.3

If the last reads, read from some earlier writes, will the drawing continue to work?

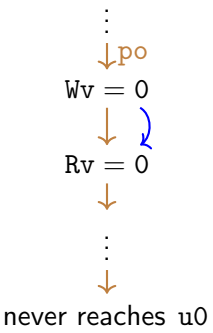
Example: proving deadlock freedom for LucknowNawab

Violating execution: Assume thread 0 and thread 1 are stuck at inner loops.



Wv = 0 has to wait for infinite time, which violates fair scheduling assumption. Therefore, we cannot have deadlock.

Example: starvation execution in LucknowNawab



Starvation in sequential execution! Odd!

Exercise 2.4

Does LucknowNawab really starve under the formal definition? Is the fair scheduling assumption real?

Example: Peterson (Nawab meets Flag)

```
init: f0 := false, f1 := false, v:= random()
```

```
thread0:
   while(true){
l0:  f0 := true
     v := 0
     while(f1 && v==0);
c0:  // critical section
u0:  f0 := false
   }
```

||

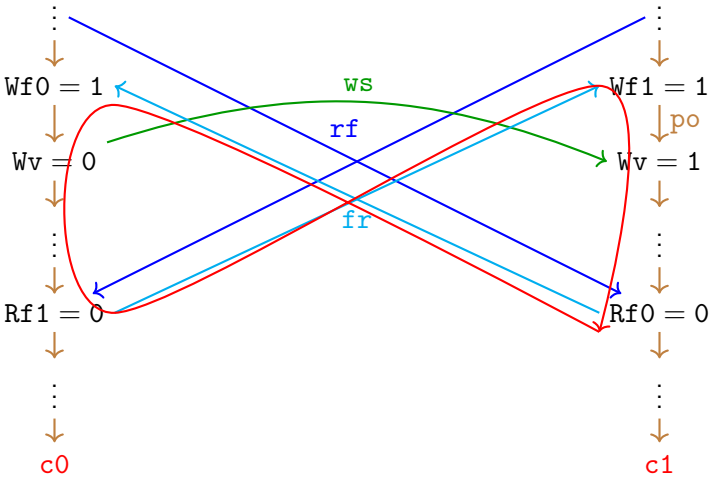
```
thread1:
   while(true){
l1:  f1 := true
     v := 1
     while(f0 && v==1);
c1:  // critical section
u1:  f1 := false
   }
```

The inner loop has two conditions. If any of them fails, the thread enters in the critical section.

There are four (2x2) scenarios that lead to violation.

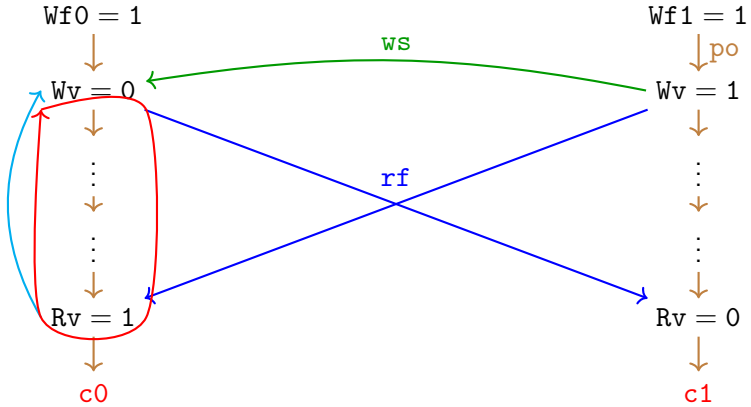
Example: proving mutual exclusion for Peterson (scenario 1: flag proof)

Scenario 1: Assume conditions `f0` and `f1` failed at the same time



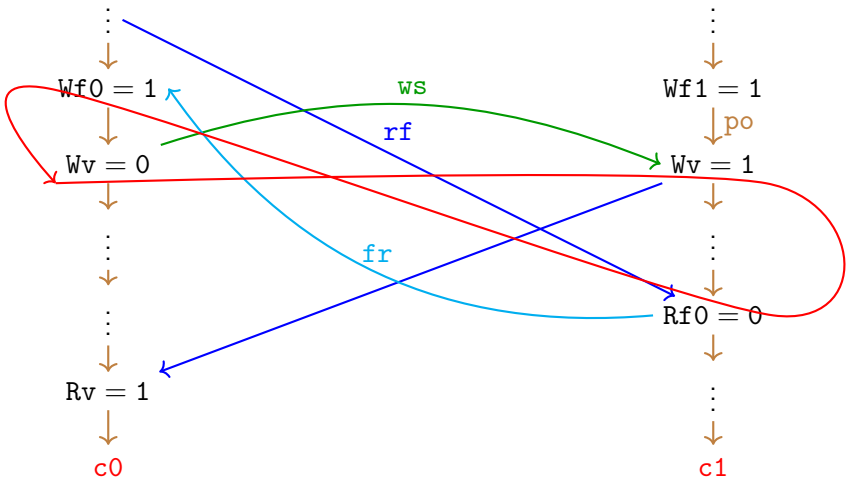
Example: proving mutual exclusion for Peterson (scenario 2: Nawab proof)

Scenario 2: Assume conditions `v == 0` and `v == 1` failed at the same time



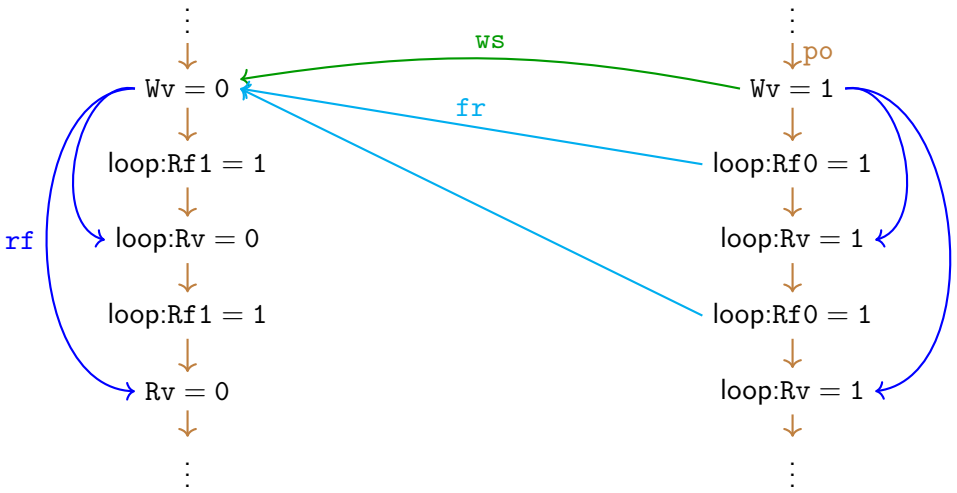
Example: proving mutual exclusion for Peterson (scenario 3/4)

Scenario 3: Assume conditions `v == 0` and `f0` failed at the same time



The drawings for the four scenarios prove that there will be no violation of mutual exclusion.

Example: proving deadlock freedom for Peterson: Like LucknowNawab



Wv = 0 has to wait for infinite time, which violates fair scheduling assumption. Therefore, we cannot have deadlock.

End of Lecture 2