CS 105: Department Introductory Course on Discrete Structures

Instructor : S. Akshay

Aug 7, 2023

Lecture 01 – Introduction

Welcome to CSE@IIT Bombay!

Logistics

Course hours: Slot 4;

Mon 11:35-12:30, Tue 08:30-09:25, Thu 09:30-10:25

Office hours: By appointment

Problem Solving/Help Session (Optional): One hour per week (Time and Venue to be decided)

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Course material, references will be posted at

- http://www.cse.iitb.ac.in/~akshayss/teaching.html
- Announcements and Online Discussion: Piazza (will be set up soon)
- Attendance: SAFE

More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

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Minimum requirements (Tentative)

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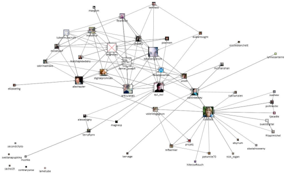
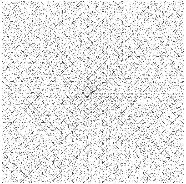
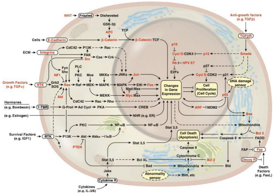
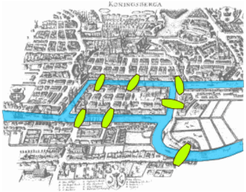
How to reach me after class?

- Send a message on piazza
- Drop by my office...
  - CS 507 (5th floor of New CSE/CC building)
  - Temporarily CC 313 (3rd floor!)

Goal




First things first...

- What are discrete structures?
- Why are we interested in them?

Course Outline

What we will broadly cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. If time permits: Introduction to some selected topics: e.g, abstract algebra and/or number theory

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What we don't cover

1. Logic : predicate, first-order logic– CS228
2. Discrete probability – IC102
3. Algorithms – CS218
4. Data structures – CS213 and CS293
5. Finite automata – CS310
6. Details and applications of everything above – rest of your (academic) life!

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Textbooks

- Discrete Mathematics and its Applications with Combinatorics and Graph Theory, by Kenneth H Rosen.
- Discrete Mathematics by Norman Biggs.
- More will be listed on webpage as we go along.

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Prerequisities

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- Logical mind and critical thinking

Chapter 1: Proofs and Structures

Outline of next few classes

- Propositions, statements
- What/why of proofs and some generic proof strategies
- Mathematical induction
- Notions and properties of sets, functions, relations

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n+1)(n-1) = (n^2 - 1)$ for any integer n

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What is common between these statements?

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Give an example of a statement that is not a proposition.

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Give an example of a statement that is not a proposition.

- $x + 1 = 8$

Propositional calculus

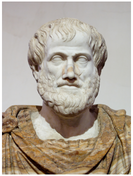

Figure: Aristotle (384 – 322 BCE)

- propositions are statements that are either true or false.
- Just as we use variables $x, y, \dots$ for numbers, we will use variables $p, q, \dots$ for propositions.
- "if it rains, it will be wet" : $p \to q$
- combining propositions: $\neg p, p \vee q, p \wedge q, p \to q$, p iff q.
- Can all mathematical statements be written this way?

Predicates and quantifiers

Consider again...

$$(n+1)(n-1) = (n^2 - 1)$$
$$x = y + 8$$

Predicates and quantifiers

Consider again...

- $\forall n \qquad (n+1)(n-1) = (n^2-1)$
- $\forall x, \exists y, \qquad x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n

Predicates and quantifiers

Consider again...

- $\forall n \in \mathbb{N}\ (n+1)(n-1) = (n^2 - 1)$
- $\forall x, \exists y, x, y \in \mathbb{Z}\ x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n
- $\in$ is the element of symbol
- $\mathbb{N}$ stands for all natural numbers
- $\mathbb{Z}$ stands for all integers
- $\mathbb{R}$, $\mathbb{Q}$, ...

Some propositions are not so easy to "determine"...
– e.g., $2^{67} - 1$ is not a prime.

Theorems and proofs

A theorem is a proposition which can be shown true

Classwork: Prove the following theorems.

1. For all $a, b, c \in \mathbb{R}^{\geq 0}$, if $a^2 + b^2 = c^2$, then $a + b \geq c$
2. If 6 is prime, then $6^2 = 30$.
3. For all $x \in \mathbb{Z}$, x is an even iff $x + x^2 - x^3$ is even.

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3. For all $x \in \mathbb{Z}$, x is an even iff $x + x^2 - x^3$ is even.

4. There are infinitely many prime numbers.
5. There exist irrational numbers x, y such that x^y is rational.
6. For all $n \in \mathbb{N}$, $n! \leq n^n$.
7. There does not exist a (input-free) program which will always determine whether an arbitrary (input-free) program will halt.

Theorems and proofs

Contrapositive and converse

- The contrapositive of "if A then B" is "if $\neg B$ then $\neg A$".
- A statement is logically equivalent to its contrapositive, i.e., it suffices to show one to imply the other.
- To show A iff B, you have to show A implies B and conversely, B implies A.
- Note the difference between contrapositive and converse.