CS228 : Logic for Computer Science

Module 1: Propositional Logic

Lecture 1: Syntax

Instructor: S. Akshay

IIT Bombay, India

Logistics

- Course webpage: `https://www.cse.iitb.ac.in/~akshayss/courses/cs228`

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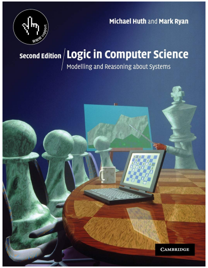
- Textbook: For the first part of the course, we will use Huth and Ryan's book on Logic in Computer Science.



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Topic 1.1

Propositional logic: Syntax

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  - Let p_A denote the statement: A is good. It is either true or false.
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- Step 2: Encode puzzle using the symbols: $\underbrace{(\neg p_A \vee p_B)}_{\text{Statement of A}} \leftrightarrow p_A$
- Step 3: Solve the puzzle: Assign truth values to p_A and p_B such that the above "formula" holds. (how? trial and error?)

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"I saw the man with the telescope."

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Exercise 1.2

- Give two examples of sentences that are not propositions.
- Give three examples of propositions.

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- Propositions are Boolean variables (as they take true or false as value!)

Topic 1.2

Propositional Formulas

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- We assume that the logical connectives are not in **AP**.
- Do we need all these connectives? Which can be derived from others?

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- if $p \in$ **AP** then $p \in$ **Prop**
- if $F \in$ **Prop** then $\neg F \in$ **Prop**
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Another way to write this definition

$F \in \mathbf{Prop}$ if

$$F ::= p \mid \top \mid \bot \mid \neg F \mid (F \vee F) \mid (F \wedge F) \mid (F \rightarrow F) \mid (F \leftrightarrow F) \mid (F \oplus F)$$

where $p \in \mathbf{AP}$.

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Another way to write this definition – Have you see this way of defining before?

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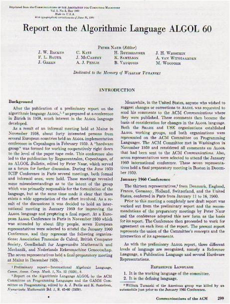
A bit of history

Named after John Backus and Peter Naur, who used it to formally specify the grammar of ALGOL 60 in 1963 – one of the earliest programming languages, and ancestor of Pascal, C, and more!



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Report on the Algorithmic Language ALGOL 60

Peter Naur (Editor)

J. W. Backus, C. Katz, H. Rutishauser, J. H. Wegstein, F. L. Bauer, J. McCarthy, K. Samelson, A. van Wijngaarden, J. Green, A. J. Perlis, B. Vauquois, M. Woodger

Dedicated to the Memory of William Turanski

INTRODUCTION

Background

After the publication of a preliminary report on the algorithmic language Algol,[1,2] as prepared at a conference in Zürich in 1958, much interest in the Algol language developed.

As a result of an informal meeting held at Mainz in November 1958, about forty interested persons from several European countries held an Algol implementation conference in Copenhagen in February 1959. A "hardware group" was formed for working cooperatively right down to the level of the paper tape code. This conference also led to the publication by Regnecentralen, Copenhagen, of an ALGOL *Bulletin*, edited by Peter Naur, which served as a forum for further discussion. During the June 1959 ICIP Conference in Paris several meetings, both formal and informal ones, were held. These meetings revealed some misunderstandings as to the intent of the group which was primarily responsible for the formulation of the language, but at the same time made it clear that there exists a wide appreciation of the effort involved. As a result of the discussions it was decided to hold an international meeting in January 1960 for improving the Algol language and preparing a final report. At a European Algol Conference in Paris in November 1959 which was attended by about fifty people, seven European representatives were selected to attend the January 1960 Conference, and they represent the following organizations: Association Francaise de Calcul, British Computer Society, Gesellschaft fur Angewandte Mathematik und Mechanik, and Nederlands Rekenmachine Genootschap. The seven representatives held a final preparatory meeting at Mainz in December 1959.

Meanwhile, in the United States, anyone who wished to suggest changes or corrections to Algol was requested to send his comments to the ACM *Communications* where they were published. These comments then became the basis of consideration for changes in the Algol language. Both the Share and USE organizations established Algol working groups, and both organizations were represented on the ACM Committee on Programming Languages. The ACM Committee met in Washington in November 1959 and considered all comments on Algol that had been sent to the ACM *Communications*. Also, seven representatives were selected to attend the January 1960 international conference. These seven representatives held a final preparatory meeting in Boston in December 1959.

January 1960 Conference

The thirteen representatives,[3] from Denmark, England, France, Germany, Holland, Switzerland, and the United States, conferred in Paris from January 11 to 16, 1960.

Prior to this meeting a completely new draft report was worked out from the preliminary report and the recommendations of the preparatory meetings by Peter Naur and the conference adopted this new form as the basis for its report. The Conference then proceeded to work for agreement on each item of the report. The present report represents the union of the Committee's concepts and the intersection of its agreements.

As with the preliminary Algol report, three different levels of language are recognized, namely a Reference Language, a Publication Language and several Hardware Representations.

Reference Language

1. It is the working language of the committee.
2. It is the defining language.

[1] Preliminary report—International Algebraic Language, *Comm. Assoc. Comp. Mach.* 1, No. 12 (1958), 8.

[2] Report on the Algorithmic Language ALGOL by the ACM Committee on Programming Languages and the GAMM Committee on Programming, edited by A. J. Perlis and K. Samelson, *Numerische Mathematik* Bd. 1, S. 41-60 (1959).

[3] William Turanski of the American group was killed by an automobile just prior to the January 1960 Conference.

Communications of the ACM 299

Examples of propositional formulas

Exercise 1.3 Which of the following belong to **Prop** (i.e., are well-formed)?

1. $\top \to \bot$
2. $(\top \to \bot)$
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4. (p_1)
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How can we argue that a string does or does not belong to **Prop**?

The Lady Or the Tiger

A prisoner must choose between three rooms. One room has a Lady and other two have tigers. If the prisoner chooses the room with the Lady, he wins! If he chooses a wrong room, he (probably) gets eaten by the tiger!
Each room has a sign in front of it. BUT Only one of the signs is true; the other two are false.

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Which room has the Lady?!

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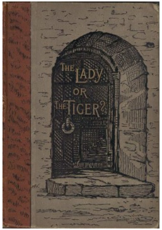
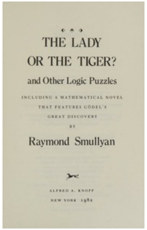
- Story from book by Frank Stockton.
- Puzzle from book by Raymond M. Smullyan





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(**Room3**)	*The Lady is in Room 2*

Which room has the Lady?!

Exercise 1.4

1. Solve the above puzzle!
2. Write each of the above statements in propositional logic.
3. Reduce the problem to a question in this logic!

Formalizing Exercise 1.4

- Propositions $S1, S2, S3$ represent whether the sign of room 1, 2, 3 (resp.) is true.
- Propositions $L1, L2, L3$ represent Lady in room 1, 2, 3 resp.

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- Is there a way to give truth values to propositions such that all these constraints are satisfied?
- **Bonus:** Can you encode this using only 3 propositions instead of 6?

Bonus: Encoding with only 3 propositions

Intuition

$S1, S2, S3$ were never really needed – they're just names for what the signs say.

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- exactly one of $\neg L1,\ \neg L2,\ L2$ holds (exactly one sign is true)

$$(\neg L1 \land L2 \land \neg L2) \lor (L1 \land \neg L2 \land \neg L2) \lor (L1 \land L2 \land L2)$$

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- exactly one of $L1, L2, L3$ holds (Lady is in exactly one room)

$$(L1 \wedge \neg L2 \wedge \neg L3) \vee (L2 \wedge \neg L1 \wedge \neg L3) \vee (L3 \wedge \neg L1 \wedge \neg L2)$$

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Another Bonus: Can we encode it with just two propositions?