CS228 : Logic for Computer Science

Lecture 0: Introduction and logistics

Instructor: S. Akshay

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27-07-2026

Bugs in programs can be costly!

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June 4, 1996 - Ariane 5 Flight 501.
"Working code for the Ariane 4 rocket is reused in the Ariane 5, but the Ariane 5's faster engines trigger a bug in an arithmetic routine inside the rocket's flight computer. The error is in the code that converts a 64-bit floating-point number to a 16-bit signed integer. The faster engines cause the 64-bit numbers to be larger in the Ariane 5 than in the Ariane 4, triggering an overflow condition that results in the flight computer crashing."

Source: Simson Garfinkel, Wired Magazine, 2005: `https://www.wired.com/2005/11/historys-worst-software-bugs/`

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1993-Intel Pentium floating point divide.
"A silicon error causes Intel's highly promoted Pentium chip to make mistakes when dividing floating-point numbers that occur within a specific range. For example, dividing 4195835.0/3145727.0 yields 1.33374 instead of 1.33382, an error of 0.006 percent... The bug ultimately costs Intel $475 million."

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2005 - Toyota Prius Bug.
"Toyota announced a recall of 160,000 of its Prius hybrid vehicles following reports of vehicle warning lights illuminating for no reason, and cars' gasoline engines stalling unexpectedly. But unlike the large-scale auto recalls of years past, the root of the Prius issue wasn't a hardware problem - it was a programming error in the smart car's embedded code. The Prius had a software bug."

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How do you catch bugs?

- Software testing not enough always...
- Need mathematically formal and rigorous guarantees.
- How do we reason about software/programs?

CS228: Logic for Computer Science

This course in a nutshell

1. How to reason

CS228: Logic for Computer Science

This course in a nutshell

1. How to reason
   - about human thought

CS228: Logic for Computer Science

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OMG!: Machines reasoning about machines? Is this AI?!

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- Not quite: but as we will see, we can use these techniques to reason about AI too!

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What we need are

1. languages to formalize reasoning: symbolic logic
2. mathematical models of the computing objects: abstractions, transition systems
3. algorithms and tools to automate this whole process!

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- ... cover a specific topic in depth
- ... exhaustively cover all logics, automata, models

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The idea of this course is to...

- ... prepare students to have a background in formal logic.
- ... see where logic occurs in CS and how it can be used for CS.
- ... look at some logics, techniques in detail.
- ...

Who uses Logic in CS?

How many queries to a logic tool (SAT/SMT solver to be precise) does Amazon Web Services invoke per day?

- Option A: 1 thousand
- Option B: 100 thousand
- Option C: 1 Million
- Option D: 1 Billion

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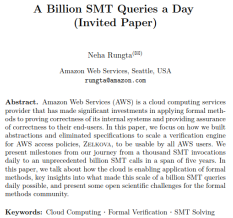

A Billion SMT Queries a Day
(Invited Paper)

Neha Rungta(✉)

Amazon Web Services, Seattle, USA
rungta@amazon.com

Abstract. Amazon Web Services (AWS) is a cloud computing services provider that has made significant investments in applying formal methods to proving correctness of its internal systems and providing assurance of correctness to their end-users. In this paper, we focus on how we built abstractions and eliminated specifications to scale a verification engine for AWS access policies, Zelkova, to be usable by all AWS users. We present milestones from our journey from a thousand SMT invocations daily to an unprecedented billion SMT calls in a span of five years. In this paper, we talk about how the cloud is enabling application of formal methods, key insights into what made this scale of a billion SMT queries daily possible, and present some open scientific challenges for the formal methods community.

Keywords: Cloud Computing · Formal Verification · SMT Solving

Who uses Logic in CS?

Widely used in industry

- Hardware and software verification
- Cyber-physical systems: Avionics, Automobiles, space!
- AI and databases
- Hot area: explainability in AI/ML!

Including Amazon, Intel, IBM, Microsoft Research, Google, Samsung, TCS, NASA, Mathworks ...

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- An interplay of mathematics and computer science, logic and coding, theory and practice.

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For a more colorful video, see `https://www.youtube.com/watch?v=AM_gwEKjnGY`

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Topics to be covered in this course:

- Module 1: Symbolic Logics and formal reasoning

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There are advanced courses in each of these topics! CS433, CS615, CS 6104, CS713, CS735, CS738, CS739

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- Quizzes (2 or 3) : 30%
- Midsem: 25%
- Final exam: 40%
- Other: Pop quizzes, Class participation : 5%

Module 1: Logic, a language and calculus for reasoning

1. Men are mortal
2. Socrates is a man

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A symbolic logic zoo for reasoning

- Propositional logic
- Predicate logic
- Temporal logic
- Modal logic

Module 2: Satisfiability

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Theory behind powerful solvers

- SAT solvers
- SMT solvers

Module 3: Application to reasoning about programs

```
int foo(int n) {
    int k, j;
    k = 0; j = 1;
    while (k != n) {
        k = k + 1;
        j = 2*j;
    }
    return(j);
```

```
list * bar(list * i) {
    list *j, *k;
    j = NULL;
    while (i != NULL) {
        k = i→next;
        i→next = j;
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- If "foo" is called with $n > 0$, does it always return 2^n?
- If "bar" is called with i pointing to an acyclic list, does i always point to an acyclic list when "bar" returns?

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- The "art and science" of proving programs (in)correct – using logic and reasoning!

Module 4: Application to Model checking

System

satisfy?

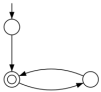
specification

System Model

model-checking

$\models$?

Spec Model



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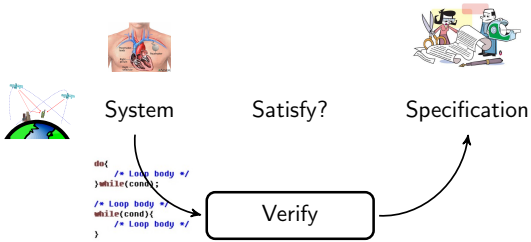
A Powerful Technique: Formal Verification

System Satisfy? Specification

```
do{
    /* Loop body */
}while(cond);

/* Loop body */
while(cond){
    /* Loop body */
}
```

A Powerful Technique: Formal Verification



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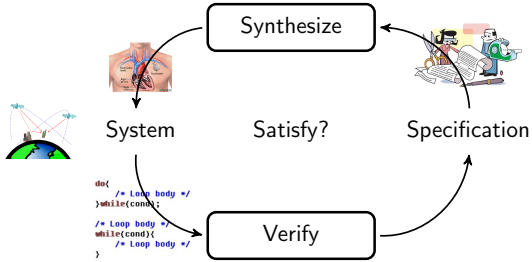

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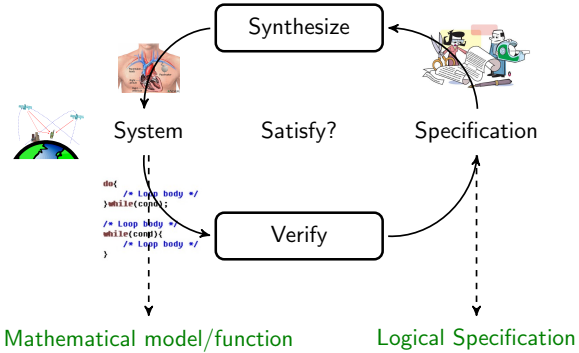

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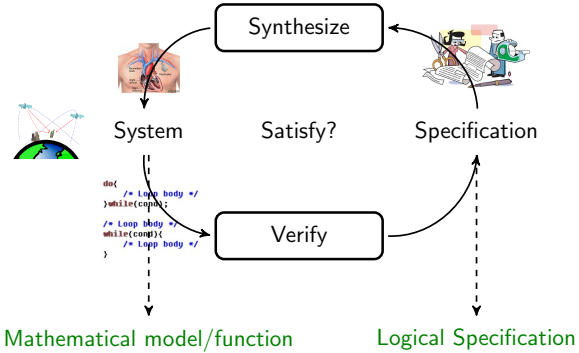

- Goal: Develop Efficient Algorithms to Verify and Synthesize
- Challenges: Expressivity of models, Scalability of Algorithms
- We stand on the Shoulders of Giants! Turing Awards in 1996 and 2007 were on this work.

Overlaps and links to other courses

- Unsurprisingly, there are overlaps with several courses

Undergrad courses

- Discrete structures (CS105) - reasoning ideas same, same but different!

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Other linked courses

- CS 433: Automated reasoning: advanced course more practical
- CS 713: Specialized course on Automata & logic connections
- CS 738: Model checking, cyber-physical systems
- CS 6104: Quantitative Verification
- CS 781: Formal methods and machine learning
- CS 766: Analysis of concurrent programs

Links to other domains

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- Cyber-physical, real-time systems, etc.

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- Visit the course webpage for all details!
  `http://www.cse.iitb.ac.in/~akshayss/courses/cs228`