Forward Analysis for WSTS, Part III: Karp-Miller Trees

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- co-written with Michael Blondin & Jean Goubault-Larrecq.

WSTS are very important for infinite-state models

- counter machines with reset-transfer-affine-ω extensions
- Lossy fifo systems and variants with time, data and priority
- Parameterized broadcast protocols and other
- CFG, graph rewriting
- Systems with pointers, graph memory
- Fragments of the π-calculus, depth bounded processes
- ...

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Theorem

Coverability is decidable for WSTS (will be defined...).

The proof can be made by a backward algorithm on upward closed sets or by a recent forward algorithm of ideals (= directed downward closed sets).

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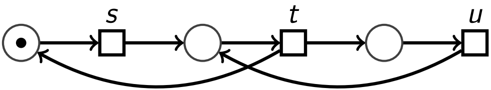
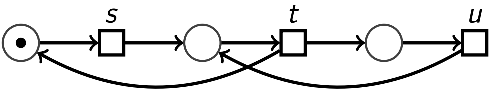
  - Karp-Miller trees are finite since strict-strong monotony of the completion and Ideals(X) has finitely many levels (new definition)
  - PN are positive (new definition)
  - The existence of a positive sequence in a PN is decidable.



Every infinite execution has infinitely many occurrences of s?

i.e. satisfies LTL formula $\Box\Diamond s$?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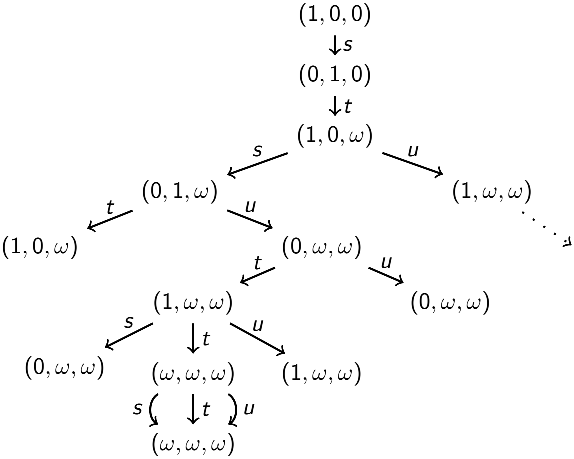
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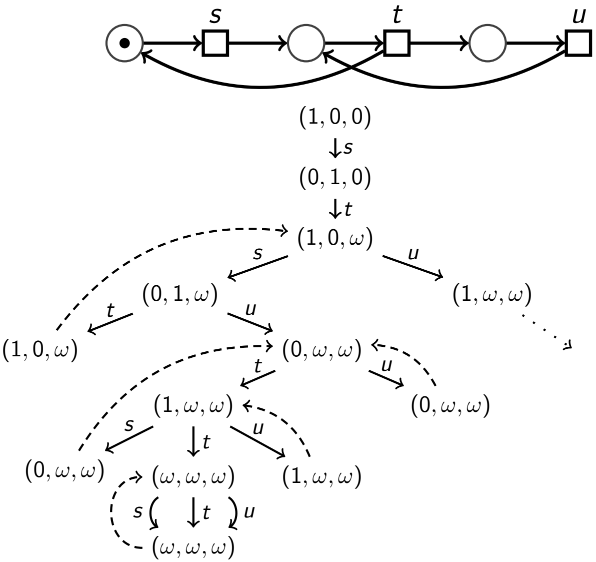
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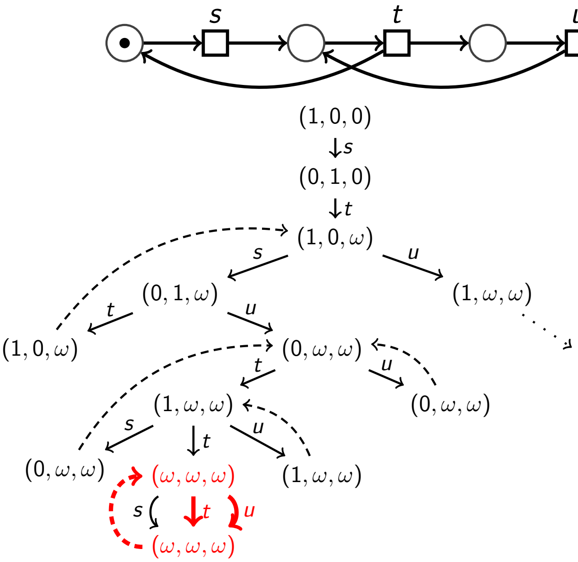
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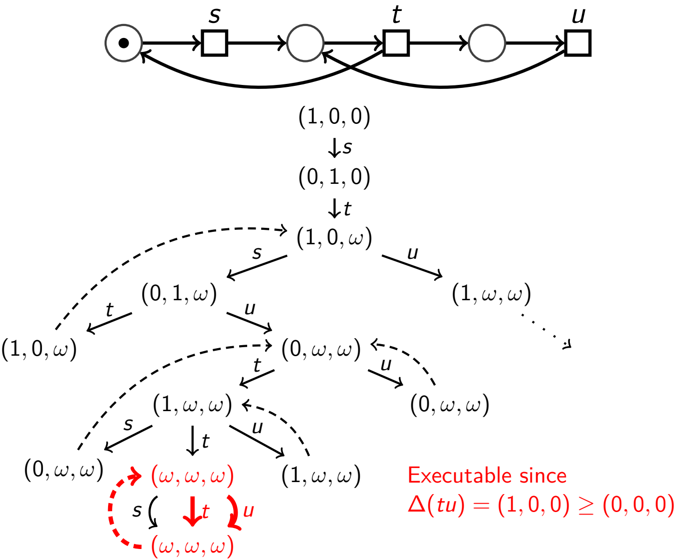
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Executable since
Δ(tu) = (1, 0, 0) ≥ (0, 0, 0)

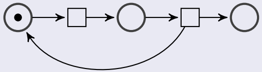
Plan

- Introduction
- WSTS
- IKM: Ideal Karp-Miller algorithm
- Very-WSTS
- From IKM to LTL
- Conclusion

Well structured transition system (F, ICALP'87)

$\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ where

- X set,
- $\xrightarrow{\Sigma} \subseteq X \times \Sigma \times X$,
- monotony,
- well-quasi-ordered.



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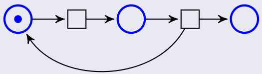

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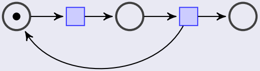

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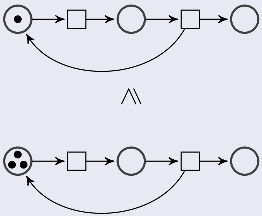

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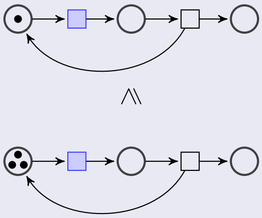

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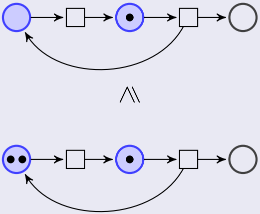

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$$
\begin{array}{ccccc}
\forall & x & \xrightarrow{a} & y & \\
 & \rotatebox{90}{$\leq$} & & \rotatebox{90}{$\leq$} & \\
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\end{array}
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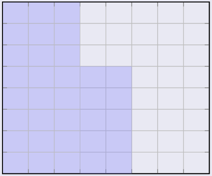
- X set,
- $\xrightarrow{\Sigma} \subseteq X \times \Sigma \times X$,
- monotony,
- well-quasi-ordered:
  $\forall x_0, x_1, \ldots \; \exists i < j \;$ s.t. $\; x_i \leq x_j$.

Ideals

$I \subseteq X$ is an *ideal* if

- downward closed: $I = \downarrow I$,



Ideals

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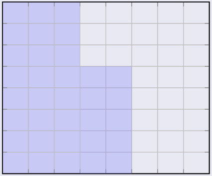

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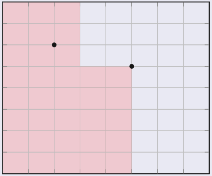

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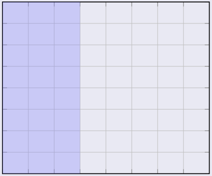
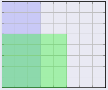


A very interesting but unknown theorem for the verification community (recall WQO = FAC + WF).

Theorem (Erdös & Tarski'43, Bonnet'75, Fraïsse'86,...)

$(X, \leq)$ has non infinite antichain (FAC) $\iff$ for all $D = \downarrow D \subseteq X$, $\exists I_1, I_2, \ldots, I_n \in \text{Ideals}(X)$ s.t. $D = I_1 \cup I_2 \cup \cdots \cup I_n$



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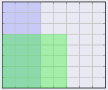

Corollary

Every downward closed set decomposes canonically as the union of its $\subseteq$-maximal ideals. We note $\mathit{IdealDecomp}(D) = \{I_1, I_2, \ldots, I_n\}$.

Completion of WSTS

Definition

Let $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ be a labeled WSTS. The *completion* of $\mathcal{S}$ is the labeled transition system $\widehat{\mathcal{S}} = (\mathrm{Ideals}(X), \xrightarrow{\Sigma}, \subseteq)$ s.t. $I \xrightarrow{a} J$ if, and only if,

$$J \in \mathit{IdealDecomp}(\downarrow \mathrm{Post}_S(I, a)).$$

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Remark

- *$\widehat{\mathcal{S}}$ is finitely branching and strongly monotone.*
- *$\widehat{\mathcal{S}}$ is a WSTS iff $(\mathit{Ideals}(X), \subseteq)$ is wqo*

We say that an infinite sequence of ideals $I_0, I_1, \ldots \in \text{Ideals}(X)$ is an *acceleration candidate* if $I_0 \subset I_1 \subset \cdots$ is strictly increasing.

Definition (also in F87 with another completion)

The n^{th} *level* of Ideals(X) is defined as

$$Acc_n(X) = \begin{cases} \text{Ideals}(X) & \text{if } n = 0, \\ \{\bigcup_{i \in \mathbb{N}} I_i : I_0, I_1, \ldots \in Acc_{n-1}(X)\} & \text{if } n > 0. \end{cases}$$

where $I_0, I_1, \ldots \in Acc_{n-1}(X)$ is an acceleration candidate

$\text{Ideals}(\mathbb{N}^d) = \mathbb{N}^d_\omega$.
$Acc_n(\mathbb{N}^d) = \{I \in \mathbb{N}^d_\omega : I \text{ has at least } n \text{ occurrences of } \omega\}$.

Definition

Ideals(X) has *finitely many levels* if there exists $n \in \mathbb{N}$ s.t. $Acc_n(X) = \emptyset$. For example, $Acc_{d+1}(\mathbb{N}^d) = \emptyset$.

A characterization of acceleration levels

Let Z be a well-founded po.
For $z \in Z$, $\mathrm{rk}\, z \stackrel{\text{def}}{=} \sup\{\mathrm{rk}\, y + 1 : y < z\}$, where $\sup(\emptyset) \stackrel{\text{def}}{=} 0$.
$\mathrm{rk}\, Z \stackrel{\text{def}}{=} \sup\{\mathrm{rk}\, z + 1 : z \in Z\}$.
$\mathrm{rk}\, \{0, 1, 2, 3\} = 4$, $\mathrm{rk}\, \mathbb{N}_\omega = \omega + 1$, $\mathrm{rk}\, \mathbb{N}_\omega^2 = \omega * 2 + 1$.

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Theorem

Let X be a countable wqo s.t. $(\mathit{Ideals}(X), \subseteq)$ is a wqo. Then: $\mathit{Ideals}(X)$ has finitely many levels if and only if $\mathrm{rk}\, \mathit{Ideals}(X) < \omega^2$.

Remark: $\mathrm{rk}\, \mathrm{Ideals}(X) < \omega^2$ is equivalent to $\mathrm{rk}\, \mathrm{Ideals}(X) \leq \omega \cdot n$ for some $n \in \mathbb{N}$.

Accelerations in WSTS

Let $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ be a WSTS s.t. $\widehat{\mathcal{S}}$ is deterministic.
Let $w \in \Sigma^+$ and $I \in \text{Ideals}(X)$.

The *acceleration of I under w* is defined as:

$$w^\infty(I) \stackrel{\text{def}}{=} \begin{cases} \bigcup_{k \in \mathbb{N}} w^k(I) & \text{if } I \subset w(I), \\ I & \text{otherwise.} \end{cases}$$

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Remark

- *$w^\infty(I)$ is an ideal.*
- *The sequence $(w^k(I))_k$ is non decreasing (resp. strictly increasing if moreover $\widehat{\mathcal{S}}$ is strict-strong monotone) .*

Computable accelerations in WSTS

- PN, reset/transfer PN, post self-modifying nets, ω-PN
- Non decreasing ω-recursive nets
- cover-flattable ω^2-WSTS.
- trace-bounded ω^2-WSTS (ex: flat machines)
- Lossy fifo systems
- ν-PN, unordered PN
- Depth-bounded π-calculus
- Open: Priority LCS ? Data PN ?
- for non WSTS, accelerations could be computable (def ?) from a configuration: counter machines (semi-linear sets), PN+one zero-test, perfect fifo systems (CQDD), Turing machines ?

Algorithm 4.1: Ideal Karp-Miller algorithm.

```
initialize a tree 𝒯 with root r: ⟨I_0, 0⟩
while 𝒯 contains an unmarked node c: ⟨I, n⟩ do
    if c has an ancestor c′: ⟨I′, n′⟩ s.t. I′ = I then  mark c
    else
        if c has an ancestor c′: ⟨I′, n′⟩ s.t. I′ ⊂ I
             and n′ = n /* no acceleration occurred between c′ and c */ then
            w ← sequence of labels from c′ to c
            replace c: ⟨I, n⟩ by c: ⟨w^∞(I), n + 1⟩
        for a ∈ Σ do
            if a(I) is defined then
                add arc labeled by a from c to a new child d: ⟨a(I), n⟩
        mark c
return 𝒯
```

Very WSTS

Definition

A *very-WSTS* is a labeled WSTS $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ such that:

- $\mathcal{S}$ has strong monotonicity
- $\widehat{\mathcal{S}}$ is a deterministic WSTS with strong-strict monotonicity
- $\operatorname{rk} \mathsf{Ideals}(X) < \omega^2$ (i.e. $\mathsf{Ideals}(X)$ has finitely many levels).

VAS, Petri nets, ω-Petri nets [GHPR15], post self-modifying nets [Valk78] and strongly increasing ω-recursive nets [FMP05] are very-WSTS.

Theorem

Ideal Karp-Miller algorithm terminates for very-WSTS.

(1) at line 2, all unmarked nodes of $\mathcal{T}$ are leaves.
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$c_0 :< I_0, n_0 >$, $c_1 :< I_1, n_1 >$, $\ldots$, $c_k :< I_k, n_k >$, $\ldots$,
Since Ideals(X) has finitely many levels, the numbers n_k assume only finitely many values. Let N be the largest of those values. Using (2), there is a $k_0 \in \mathbb{N}$ such that $n_k = N$ for every $k \geq k_0$. Since $\widehat{\mathcal{S}}$ is a WSTS, there are two indices i, j with $k_0 \leq i < j$ and s.t. $I_i \subseteq I_j$.

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If $I_i = I_j$, then line 3 would have stopped the path.
Hence $I_i \subset I_j$, but then line 8 would have replaced $numaccel(c_j) = N$ by $N + 1$, contradiction.

(Finite) Karp-Miller tree in WSTS

- PN, VASS, double PN, reset/transfer PN, post self-modifying nets, self-modifying nets, ω-PN
- strongly increasing ω-recursive nets (includes post self-modifying nets)
- cover-flattable ω^2-WSTS.
- trace-bounded ω^2-WSTS (ex: flat machines)
- Lossy fifo systems
- ν-PN, unordered PN
- Depth-bounded π-calculus
- BVASS
- name-bounded π-calculus processes
- PN+one zero-test. Remark: they are not WSTS.

Decidability of LTL

Theorem

Let $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ be a positive very-WSTS, and let $x, y \in X$. Let $A_{\downarrow x}$ be the IKM automaton. State y is repeatedly coverable from x iff there is a circuit $c \xrightarrow{w} c$ in the IKM automaton $A_{\downarrow x}$ with $w \in \Sigma^+$ s.t. w is positive and $y \in Ideal(c)$.

Positivity

- Let $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$ be a WSTS and let $x \in X$.
- $w \in \Sigma^*$ is *positive for x* if $\exists y \in X$ s.t. $x \xrightarrow{w} y$ and $x \leq y$.

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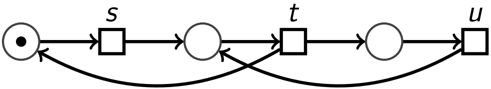
- $w \in \Sigma^*$ is *positive* if w is positive for every $x \in X$ s.t. $Post(x, w) \neq \emptyset$.
- A WSTS $\mathcal{S} = (X, \xrightarrow{\Sigma} \leq)$ is *positive* if for every $w \in \Sigma^*$, w is positive for some $x \in X$ if and only if w is positive.

Remark: PN and ω-PN are positive.

Positive WSTS and decidability

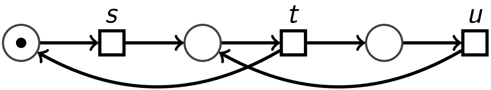
- Which WSTS models below are positive ?
- Given a positive WSTS $\mathcal{S}$ and a FA A, does there exist a positive sequence in $L(S) \cap L(A)$?

- YES: PN, ω-PN (new)
- OPEN
  - strongly increasing ω-recursive nets ?
  - cover-flattable ω^2-WSTS ?
  - trace-bounded ω^2-WSTS (ex: flat machines) ?
  - BVASS ?
  - double PN, post self-modifying nets ?
  - unordered PN ?
  - PN+one zero-test ?
  - Priority LCS ?



Every infinite execution has infinitely many occurrences of s?

i.e. satisfies LTL formula $\Box \Diamond s$?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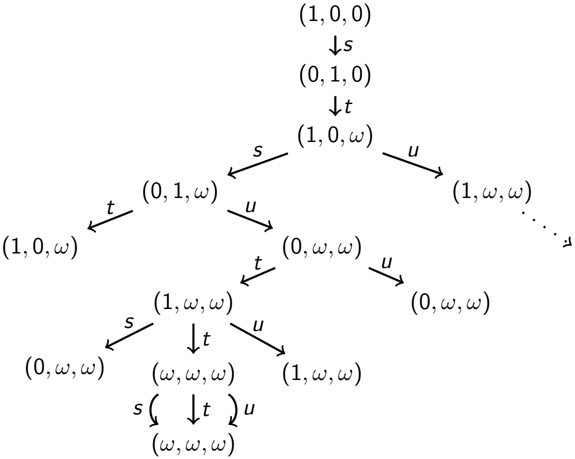
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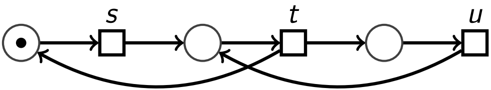
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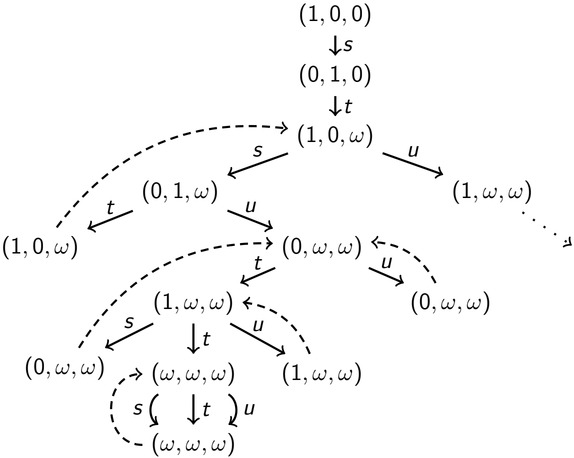
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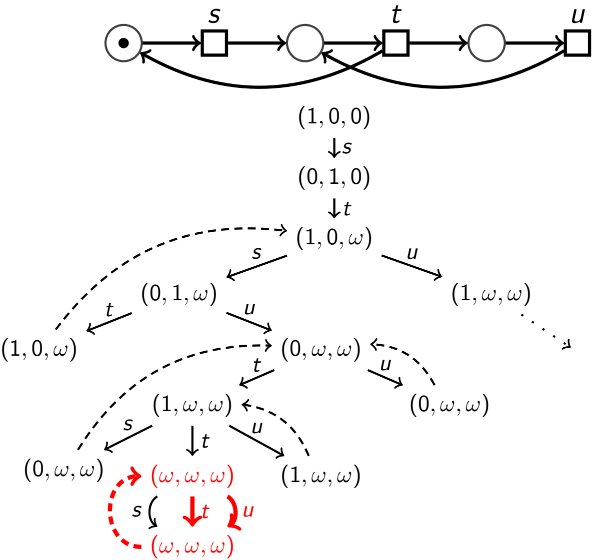
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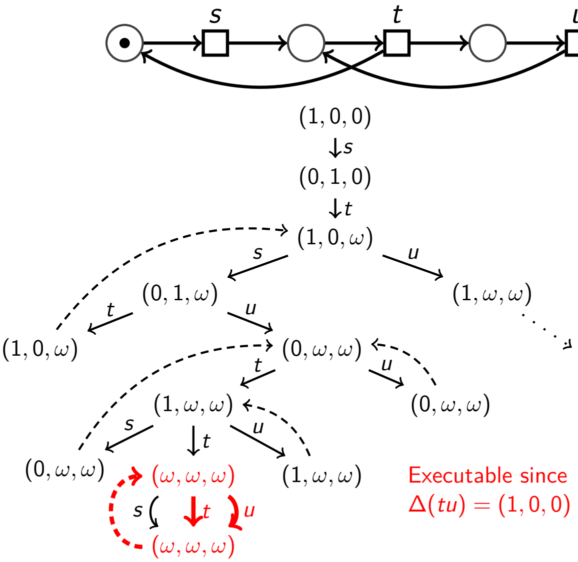

Executable since
$\Delta(tu) = (1, 0, 0) \geq (0, 0, 0)$

Part I to III

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  - Go to model checking LTL

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- Forward Analysis for WSTS, Part IV: I don't know...

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- Explore downward closed (sets and languages) for very-WSTS.

Thank you!

Completion of WSTS

Proposition (Blondin, F., McKenzie ICALP'14)

For every WSTS $\mathcal{S} = (X, \xrightarrow{\Sigma}, \leq)$*:*

1. *For all* $x, y \in X$ *and* $w \in \Sigma^*$*, if* $x \xrightarrow{w} y$*, then for every ideal* $I \supseteq \downarrow x$*, there exists an ideal* $J \supseteq \downarrow y$ *such that* $I \xrightarrow{w} J$*.*
2. *For all* $I, J \in Ideals(X)$ *and* $w \in \Sigma^*$*, if* $I \xrightarrow{w} J$*, then for every* $y \in J$*, there exist* $x \in I, y' \in X$ *and* $w' \in \Sigma^*$ *s.t.* $x \xrightarrow{w'} y'$ *and* $y' \geq y$*.*
   If S *has strong monotonicity, then* $w' = w$*.*