A survey on WSTS

Alain Finkel

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5*th* March 2018

- Based on joint works with Michael Blondin, Jean Goubault-Larrecq & Pierre McKenzie.

Exercise 1

- $T(w) =$ length of a longest computation starting from $w \in \Sigma^*$.
- $T(w) \in \mathbb{N}_\omega$.
- $w \leq_T w'$ if $T(w) \leq T(w')$.

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- $w \leq_T w'$ if $T(w) \leq T(w')$.

Prove the following theorem

Theorem

Turing machines are WSTS with strict and strong monotony wrt $\leq_T$.

Exercise 2

y is not coverable from x iff $y \notin \downarrow \mathrm{Post}^*(x)$.

Let $(S_i)_i$ be an enumeration of finite sets of ideals, $\downarrow \mathrm{Post}^*(x) = S_m$, for some m and $(F_i)_i$ an enumeration of finite sets $F_i \subseteq X$.

procedure 2: non coverability certificate of y from x

while $\neg(\downarrow \mathrm{Post}(S_i) \subseteq S_i$ and $x \in S_i$ and $y \notin S_i)$ **do**
  $i \leftarrow i + 1$
return *false*

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while $\neg(\mathrm{Pre}(\uparrow F_i) \subseteq \uparrow F_i)$ and $x \notin \uparrow F_i$ and $y \in \uparrow F_i)$ **do**
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- Find a picture for representing Pre^*-coverability semi-algorithm.
- Find a picture for representing $Post^*$-coverability semi-algorithm.

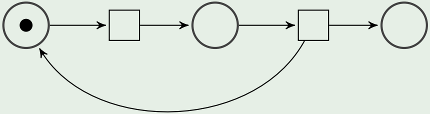
Motivation

Verification of infinite-state models

- counter machines with reset-transfer-affine-ω extensions
- Lossy fifo systems and variants with time, data and priority
- Parameterized broadcast protocols and other
- CFG, graph rewriting
- Systems with pointers, graph memory (Well-Structured Graph Transformation Systems (CONCUR 2014))
- Fragments of the π-calculus, depth bounded processes

Well Structured Transition Systems (WSTS) encompass a large number of infinite state systems (PN and reset-transfer-affine-ω extensions, lossy fifo systems, broadcast protocols, CFG, graph rewriting, depth bounded processes, fragments of the π-calculus,....)

Example of WSTS: Petri nets



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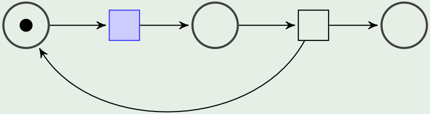

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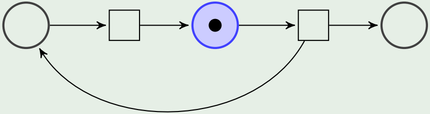

Multiple decidability results are known for (finitely branching) WSTS.

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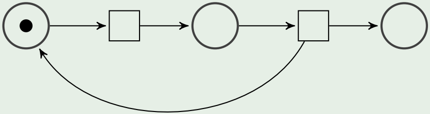
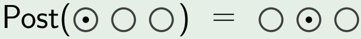
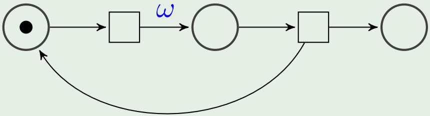




And also for (infinitely branching) WSTS such as systems with infinitely many initial states and parametric systems

Example of WSTS: ω–Petri nets (Geeraerts, Heußner, Praveen & Raskin PN'13)



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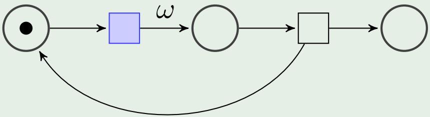

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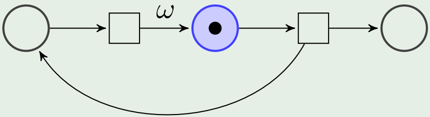

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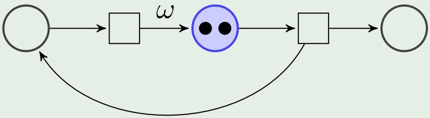

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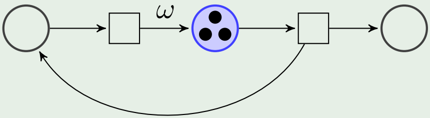

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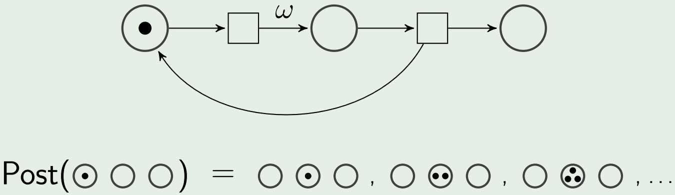
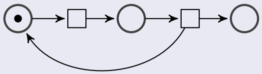


Well structured transition system (F, ICALP'87)

$S = (X, \rightarrow, \leq)$ where

- X set,
- $\rightarrow \subseteq X \times X$,
- monotony,
- well-quasi-ordered.



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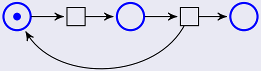

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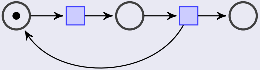

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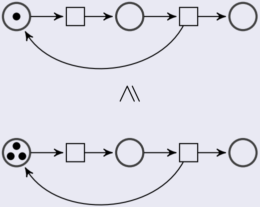

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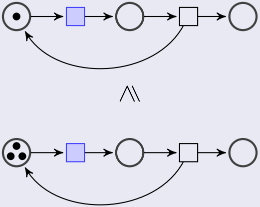

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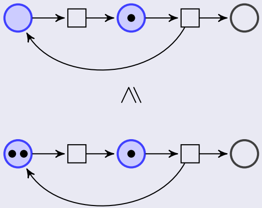

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\begin{array}{cccc}
\forall\ x & \rightarrow & y & \\
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x' & \xrightarrow{*} & y' & \exists
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$S = (X, \rightarrow, \leq)$ where

- X set,
- $\rightarrow \subseteq X \times X$,
- monotony,
- well-quasi-ordered:
  $\forall x_0, x_1, \ldots \; \exists i < j \;$ s.t. $\; x_i \leq x_j$.

The magical theorem of wqo

$(X, \leq)$ is a wqo if and only if every upward closed set $U = \uparrow U \subseteq X$ has a finite basis, i.e., it is equal to a finite union of elements $\uparrow u_i$ with $u_i \in U$.

Many caracterisations of wqo

$\leq$ is a wqo if and only if $\leq$ is FAC + WF.

WSTS Everywhere! (F, Schnoebelen LATIN'98, TCS'01)

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Theorem

Turing machines are WSTS with strict and strong monotony wrt $\leq_T$.

WSTS Everywhere!

- $\leq_T$ is not decidable.
- Hence TM are non-effective WSTS.
- This also proves that there is no (non-trivial) decidability result for non-effective WSTS (not surprising !).

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We want to study the usual reachability problems, e.g.,

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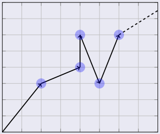

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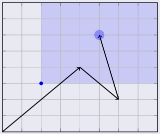
- Coverability (the most used property)



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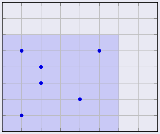

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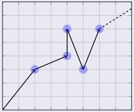
- Termination
- Coverability (the most used property)
- Boundedness
- And other properties like eventuality, simulation by finite automaton...

Termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \ldots ?$



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- Undecidable for non-effective finitely branching WSTS with strict and strong monotony (F-Schnoebelen, TCS'01), since every TM is a WSTS for $\leq_T$.

Proposition (2016)

Termination is undecidable for post-effective finitely branching WSTS with non-transitive monotony.

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Proof

We give a reduction from the halting problem.
Let M_i be a TM, and let $S_i = (\mathbb{N}, \rightarrow_i, \leq)$ defined by:
$x \rightarrow_i x + 1$ if M_i does not halt in $\leq x$ steps. Let $C = \{S_i \mid i \geq 0\}$.
S_i is finitely branching, post-effective, monotone but not transitive and $\leq$ is a wpo.

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S_i is finitely branching, post-effective, monotone but not transitive and $\leq$ is a wpo.

Now, $\exists$ infinite run $x_0 = 0 \rightarrow_i x_1 \rightarrow_i \ldots$ iff M_i does not halt.
Hence termination for C is undecidable. □

The survey for termination

Post-effective	Finitely branching	Transitive	Decidability
Yes	Yes	Yes	Decidable [F87]
non effective	Yes	Yes + strict-strong	Undecidable [FS01]
Yes	Yes	NO	Undecidable [BFM16]
Yes	NO	Yes + strict-strong	Undecidable [BFM14]

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- Undecidable for non-effective finitely branching WSTS (with wpo) with strict and strong monotony (F-Schnoebelen, TCS'01), since every TM is a WSTS for $\leq_T$.

The survey for boundedness

Post-effective	Finitely branching	Strict monotony	wpo	Decidability
Yes	Yes	Yes	Yes	D [F87]
non effective	Yes	Yes + strong	Yes	U [FS01]
Yes	Yes	NO but strong	Yes	U [ICALP'98]
Yes	NO	Yes	Yes	D [BFM'16]
Yes	Yes	Yes	wqo	???
Yes	NO	Yes	wqo	???

Exercise: Is the boundedness problem decidable for WSTS with strict monotony ?

A survey on WSTS

Alain Finkel

LSV, ENS Paris-Saclay (ex ENS Cachan)

IIT Mumbai, India
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- Based on joint works with Michael Blondin, Jean Goubault-Larrecq & Pierre McKenzie.

Coming back with exercises

- Say that a sequence $x_0, x_1, \ldots$ is bad if there are no i, j s.t. $i < j$ and $x_i \leq x_j$

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- Let us prove that $\forall x_0, x_1, \ldots \ \exists i < j$ s.t. $x_i \leq x_j$ implies $\forall x_0, x_1, \ldots \ \exists i_1 < i_2 < \ldots < i_n < ..$ s.t. $x_{i_1} \leq x_{i_2} \leq \ldots \leq x_{i_n} \leq .$

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- PROOF: Define the set $A = \{i \mid \forall j > i; x_i \not\leq x_j\}$. A is finite else contradiction; let k the largest index of x_k in A, hence for all $i > k$, one may construct an infinite non-decreasing sequence from x_i.

A quick story of coverability in WSTS

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For monotone transition systems, y is coverable from x if

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Remark

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Execute two procedures in parallel, one looking for a coverability certificate and one looking for a non coverability certificate.

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Enumeration of upward closed sets by their finite basis is a consequence of $(X, \leq)$ is WQO.

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- Non-coverability is also semi-decidable:
  - $\neg(\exists x' \geq y,\ x \xrightarrow{*} x')$ iff $x \notin Pre^*(\uparrow y) = \uparrow J_m$ for some m.
  - One enumerates all the finite sets (*) $J \subseteq X$ such that $y \in \uparrow J$ and $Pre(\uparrow J) \subseteq \uparrow J$ (hence $Pre^*(\uparrow J) = \uparrow J$) and $x \notin \uparrow J$, hence $Pre^*(\uparrow y) = \uparrow J_m \subseteq \uparrow J = Pre^*(\uparrow J)$.

Enumeration of upward closed sets by their finite basis is a consequence of $(X, \leq)$ is WQO.

A conceptual coverability algorithm, not the original

Execute two procedures in parallel, one looking for a coverability certificate and one looking for a non coverability certificate.

- Coverability is semi-decidable:
  - if $\exists x' \geq y$, $x \xrightarrow{*} x'$, one finally will find x'.
- Non-coverability is also semi-decidable:
  - $\neg(\exists x' \geq y, x \xrightarrow{*} x')$ iff $x \notin Pre^*(\uparrow y) = \uparrow J_m$ for some m.
  - One enumerates all the finite sets (*) $J \subseteq X$ such that $y \in \uparrow J$ and $Pre(\uparrow J) \subseteq \uparrow J$ (hence $Pre^*(\uparrow J) = \uparrow J$) and $x \notin \uparrow J$, hence $Pre^*(\uparrow y) = \uparrow J_m \subseteq \uparrow J = Pre^*(\uparrow J)$.
  - Since we are sure that at least one J exists (J_m !), one finally will find one. May be we find a large J_p s.t. $\uparrow J_m = Pre^*(\uparrow y) \subsetneq \uparrow J_p$ but $x \notin \uparrow J_p \implies x \notin Pre^*(\uparrow y)$.

    Enumeration of upward closed sets by their finite basis is a consequence of $(X, \leq)$ is WQO.

The story of the backward coverability algorithm

- 1978: coverability for reset VAS is decidable (Arnold and Latteux published in French in CALCOLO'78). Their algorithm is an instance of the backward algorithm (LICS'96).
- 1993: decidability of coverability for LCS (Abdulla, Cerans, Jonsson, Tsay, LICS'93)
- 1996: decidability of coverability for strong WSTS assuming $\text{Pre}(\uparrow x)$ is computable (Abdulla, Cerans, Jonsson, Tsay, LICS'96)
- 1998: decidability of coverability for WSTS assuming $\uparrow\text{Pre}(\uparrow x)$ is computable (F., Schnoebelen LATIN'98)

Remarks on the backward coverability algorithm

- It computes $\mathrm{Pre}^*(\uparrow y)$ that is more than solving coverability.
- It is often but not always computable, ex: depth-bounded processes (Wies, Zufferey, Henzinger, FOSSACS'10)
- Backward algorithms are often less efficient than forward algorithms.

The downward approach for coverability

- Initially presented by Geeraerts, Raskin, and Van Begin (FSTTCS'04) for strongly monotone WSTS with Adequate Domain of Limits (ADL).

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- Still simplified and extended with Blondin, McKenzie: WQO is not necessary. Decidable for more than WSTS. (arxiv, august 2016, in LMCS'2017).

y is not coverable from x iff $y \notin \downarrow \mathrm{Post}^*(x)$.

Let $(D_i)_i$ be an enumeration of dcs, hence $\downarrow \mathrm{Post}^*(x) = D_m$, for some m.

procedure 2: enumerates dcs to find non coverability certificate of y from x

$i \leftarrow 0$;
while $\neg(\downarrow \mathrm{Post}(D_i) \subseteq D_i$ and $x \in D_i$ and $y \notin D_i)$ **do**
  $i \leftarrow i + 1$
return *false*

Effective hypotheses

- dcs are recursive.
- Union of dcs is computable
- $\downarrow \mathrm{Post}(D)$ is computable.
- Inclusion between dcs is decidable.
- Works for post effective infinitely branching systems.

Theorem

Let $S = (X, \rightarrow, \leq)$ be a monotone transition system + there exists an enumeration of downward closed sets of X, and let $x, y \in X$.

1. y is coverable from x iff Procedure 1 terminates.
2. y is not coverable from x iff Procedure 2 terminates.

This theorem does not provide an algorithm.

Remark

WSTS, hence WQO implies possible enumeration of downward closed sets (by minimal elements of upward closed sets) but the converse is false: $(\mathbb{Z}, \leq)$ is not WQO but one may enumerate the D_i as follows: $D_i = \downarrow x_i$ for $x_i \in \mathbb{Z}$ or $D_i = \mathbb{Z}$.

Question

How to enumerate downward closed sets ?

Answer

By enumerating ideals ! (come to the next seminar tomorrow)

With the 2^{nd} magical theorem of wqo

If $\leq$ is a wqo then every downward closed set $D = \downarrow D$ has a finite basis, i.e., it is equal to a finite union of ideals.
(ideal = downward closed set + directed).

Remark

It is an if then but not an if and only if.

We will see a more magical theorem of FAC = "half wqo"

Come tomorrow !

We are tomorrow !

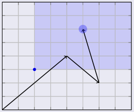
$\leq$ is FAC if and only if every downward closed set $D = \downarrow D$ has a finite basis, i.e., it is equal to a finite union of ideals.

The proof is in the paper WBTS in LMCS'2017.

Coverability

Input: $(X, \to, \leq)$ a WSTS, $x, y \in X$.

Question: $x \xrightarrow{*} x' \geq y$?



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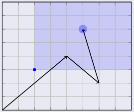

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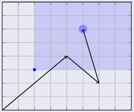

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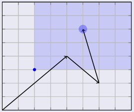

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Coverability:

- Enumerate executions $\downarrow x \xrightarrow{*} D$,
- Accept if $y \in D$.

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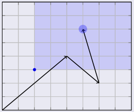

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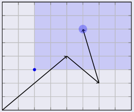

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- Enumerate $D \subseteq X$ downward closed, $x \in D$ and $\downarrow \text{Post}(D) \subseteq D$
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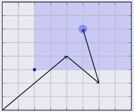

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Coverability:

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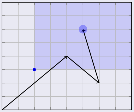

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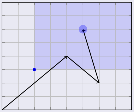

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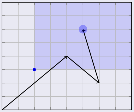

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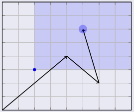

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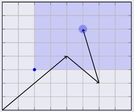

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The survey/story of coverability for WSTS

Year	**Authors**	**Mathematical hyp.**	**Effectivity hyp.**	**back/forward**
1978	Arnold & Latteux	reset VAS	YES	backward
1987	F.	very WSTS (strong+strict, ω^2-wqo,...)	effective very WSTS	forward
1996	Abdulla & CJT	strong monotony	$\mathrm{Pre}_S(\uparrow x)$ comp.	backward
1998	F. Schnoebelen	monotony	$\uparrow \mathrm{Pre}_S(\uparrow x)$ comp.	backward
2004	Geeraerts & RV	strong monotony, ADL	effective ADL	forward
2006	Geeraerts & RV	monotony, ADL	effective ADL	forward
2009	F. & Goubault-Larrecq	strong monotony, weak ADL, flattable	effective WADL	forward
2009	F. & Goubault-Larrecq	strong monotony, flattable	ideally effective	forward
2014	Blondin & FM	monotony,	ideally effective	forward
2016	Blondin & FM	monotony, no wqo but FAC	ideally effective	forward
2017	Trivial	no monotony, wqo (Minsky machines)	ideally effective	Undec.
2017	Sutre	monotony, no wqo but WF	ideally effective	Undec

A survey (to complete) of KM algorithms for WSTS

Year	Authors	Model	Termination
1969	Karp & Miller	VASS	YES
1978	Valk	post self-modifying PN	YES
1978	Valk	self-modifying PN	NO
1994	Abdulla & Jonsson	LCS	NO
1998	Dufourd & F. & Schnoebelen	3-dim reset/transfer VASS	NO
1998	Emerson & Namjoshi	WSTS model checking	NO
1999	Esparza & F. & Mayr	broadcast protocols & transfer PN	NO
2000	F. & Sutre	2-dim reset/transfer VASS	YES
2004	F. & McKenzie & Picaronny	strongly increasing ω-resursive nets	YES
2004	Raskin & Van Begin	PN+NBA	NO
2005	Goubault-Larrecq & Verma	BVASS	YES
2009	F. & Goubault-Larrecq	ω^2-WSTS, cover-flattable	YES
2010	F. & Sangnier	PN+0-test	YES
2011	Acciai, Boreale, Henzinger, Meyer,...	depth-bounded processes, ν-PN	NO
2011	Chambard & F. & Schmitz	trace-bounded ω^2-WSTS	YES
2013	Geeraerts & Heußner & Praveen & Raskin	ω-PN	YES
2013	Hüchting & Majumdar & Meyer	name-bounded π-calculus processes	YES
2016	Hofman & Lasota & Lazic & Leroux & ST	unordered PN	YES

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  - WSTS definitions
  - decidability of termination
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- LICS'98 (Emerson, Namjoshi), LICS'99 (Esparza, F, Mayr)
  - broadcast protocols are WSTS
  - model checking of WSTS (with procedures)
- WSTS everywhere, TCS'01 (F, Schnoebelen)

- FSTTCS'04 (Geeraerts, Raskin and Van Begin):
  - The first forward coverability algorithm for WSTS (with ADL).
- STACS'09, ICALP'09 (F, Goubault-Larrecq), ICALP'14 (Blondin, F, McKenzie)
  - ADL is not an hypothesis.
  - Ideal completion of any WSTS
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- 2015-2016: Use of ideals decomposition in:
  - RP'15: The Ideal View on Rackoff's Coverability Technique (Lazić, Schmitz)
  - LICS'15: Demystifying Reachability in Vector Addition Systems (Leroux, Schmitz).
  - FOSSACS'16: Coverability Trees for Petri Nets with Unordered Data (Schmitz and a lot of authors...)
  - LICS'16: ν-Petri nets (Lazić, Schmitz).
  - ...

WSTS Everywhere!

- $S = (\mathbb{N}^k, \leq)$.
  - Petri nets: WSTS with strict and strong monotony.
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- $S = (Q \times \Sigma^{*k}, = \times \sqsubseteq^k)$.
  - LCS: WSTS with non-strict monotony.

WSTS still verywhere!

- Data nets: $S = (Q \times \mathbb{N}^k)^*$
  - Lazic, Newcomb, Ouaknine, Roscoe, Worrell (PN'07)
  - Hofman, Lasota, Lazić, Leroux, Schmitz, Totzke (FOSSACS'16).
  - Lasota (PN'16)
- ν-Petri nets: $S = (Q \times \mathbb{N}^k)^{\oplus}$.
  - Rosa-Velardo, de Frutos-Escrig (PN'07)
  - Lazić and Schmitz (LICS'16).
- Pi-calculus: Depth-Bounded Processes (trees).
  - Wies, Zufferey, Henzinger (FOSSACS'10, VMCAI'12).
- Timed Petri nets: $Regions = ((Q \times \mathbb{N}^k)^{\oplus})^*$
  - Bonnet, F, Haddad, Rosa-Velardo (FOSSACS'10)
  - Haddad, Schmitz, Schnoebelen (LICS'12).
- Process algebra (BPP,...).

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- Make the first efficient prototype for reachability for Petri nets.

Thank you!