Translating LTL to Probabilistic Automata

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Linear Temporal Logic (LTL)

[Pnueli 1977]

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$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid$	Boolean operators
$X\varphi \mid \varphi \, U \, \varphi \mid \varphi \, R \, \varphi$	Temporal operators

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Theorem (Sistla-Vardi-Wolper 1985)

For every LTL formula φ, there is a nondeterministic Büchi automaton $\mathcal{M}$ of size $O(2^{|\varphi|})$ such that $\mathcal{L}(\mathcal{M}) = [\![\varphi]\!]$

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- Satisfiability and validity of LTL

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- Satisfiability and validity of LTL
- Verifying system designs

Why translate LTL to probabilistic automata?

Understanding the power of randomization

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  - What about from the perspective of memory/states?

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Translation from LTL to nondeterministic automata not good for certain applications

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- Solving games
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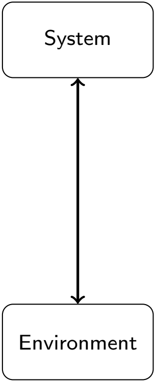

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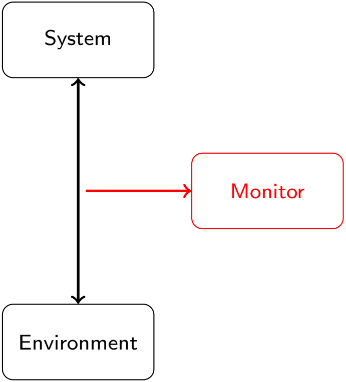

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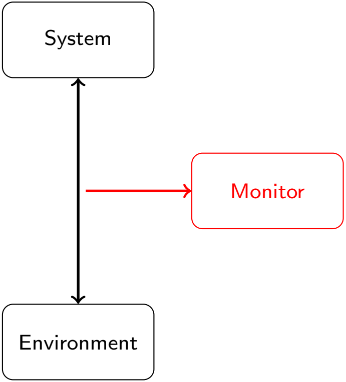

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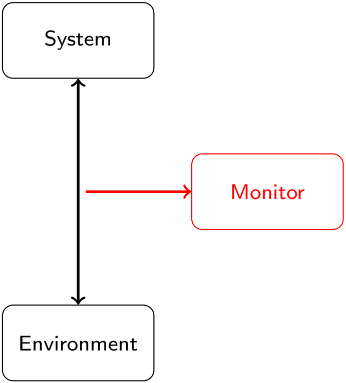

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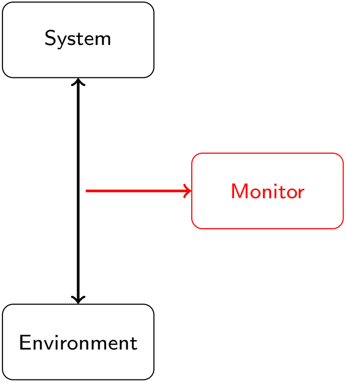

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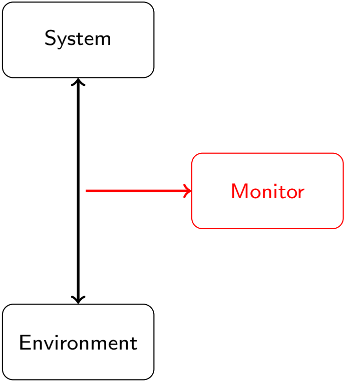

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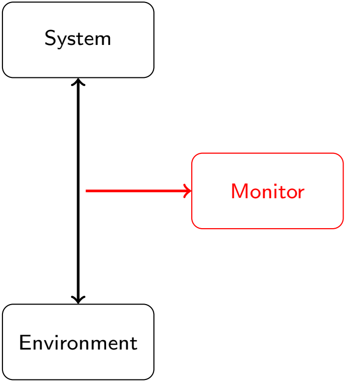

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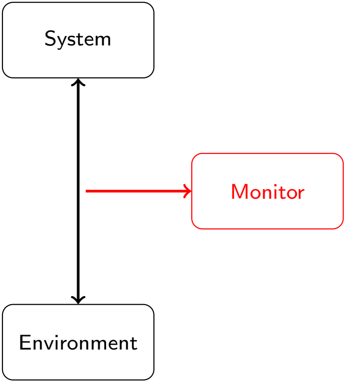
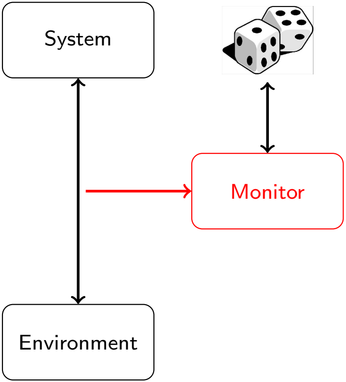
- Application: Discovery of errors and intrusions in deployed systems

Monitoring



Randomized Monitoring



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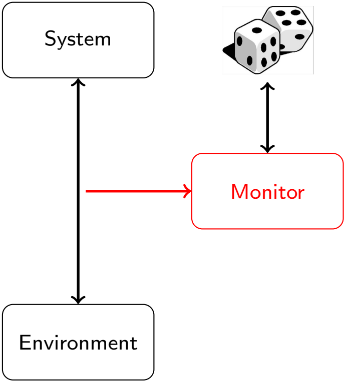

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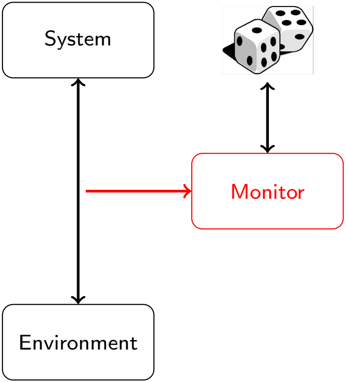
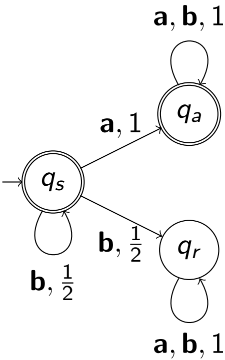


- The monitor has access to private source of randomness
- The system itself is not probabilistic

Finite State Probabilistic Monitors (FPM)

[Chadha-Sistla-V. 2008]



Definition

A FPM over alphabet Σ is $\mathcal{M} = (Q, q_s, q_r, \delta)$, where Q is a finite set of states, $q_s \in Q$ is the initial state, $q_r \in Q$ is the absorbing reject state, and $\delta : Q \times \Sigma \times Q \rightarrow [0, 1]$ is such that for any $q \in Q$ and $a \in \Sigma$, $\sum_{q' \in Q} \delta(q, a, q') = 1$.

Acceptance/Rejection Probability

For $\alpha \in \Sigma^\omega$, let $\alpha[0:j]$ denote the prefix of length $j+1$. The probability of rejecting and accepting α is defined as follows.

$$\mathrm{rej}(\alpha) = \lim_{j\to\infty} \delta_{\alpha[0:j]}(q_s, q_r)$$

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Given $\lambda \in [0,1]$, $\mathcal{L}_{>\lambda}(\mathcal{M})$ is the set of words α accepted with probability $> \lambda$.

Strong and Weak Monitors

Property L is monitorable

- strongly if there is an $\mathcal{M}$ such that $\mathcal{L}_{=1}(\mathcal{M}) = L$

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- weakly if there is an $\mathcal{M}$ such that $\mathcal{L}_{>0}(\mathcal{M}) = L$; no missed alarms

Expressive Power of Randomized Monitors

Deterministic Monitoring [Schneider]

Properties monitored deterministically are safety properties

- $L \subseteq \Sigma^\omega$ is a safety property if $\alpha \notin L$ iff there is a prefix $\alpha[0:i]$ such that $\alpha[0:i]\Sigma^\omega \subseteq \overline{L}$.

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Strong There is FPM $\mathcal{M}$ such that $L = \mathcal{L}_{=1}(\mathcal{M})$ iff L is a regular, safety property.

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Weak There are FPMs $\mathcal{M}$ such that $\mathcal{L}_{>0}(\mathcal{M})$ is a non-regular, persistence property.

- L is a persistence property if it is a countable union of safety properties, i.e., "eventually always"-type properties

Safe LTL

[Sistla 1985]

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Strong Monitors for Safe LTL

Proposition (Kini-V. 2014)

For every Safe LTL formula φ, there is $\mathcal{M}_\varphi$ of size $O(2^{|\varphi|})$ such that $[\![\varphi]\!] = \mathcal{L}_{=1}(\mathcal{M})$.

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Proof.

- Construct nondeterministic Büchi automaton using [Vardi 1996]-method for $\neg\varphi$; the automaton has a single, absorbing accept state.
- Assign arbitrary probability to nondeterministic choices, and make accept state the unique reject state of FPM. □

Weak Monitors for Safe LTL

Theorem (Kini-V. 2014)

There are Safe LTL formulas φ such that the smallest FPM $\mathcal{M}$ with $\mathcal{L}_{>0}(\mathcal{M}) = [\![\varphi]\!]$ has at least doubly exponential states.

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Weak monitors are computationally more powerful than strong monitors but only as "efficient" as deterministic monitors for Safe LTL.

Weakly monitoring LTL(G)

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- [Kini-V. 2014] For every LTL(G) formula φ there is an FPM $\mathcal{M}$ such that $\mathcal{L}_{>0}(\mathcal{M}) = [\![\varphi]\!]$ and $\mathcal{M}$ has $O(2^{|\varphi|})$ states.

Communication Complexity
[Yao 1982]

Setup

Problem described by function $f : X \times Y \rightarrow \{0, 1\}$, where X, Y are finite sets.

- Alice is given input $x \in X$ and Bob is given input $y \in Y$
- Alice and Bob arbitrary computational devices and can toss coins
- Alice and Bob can send and receive messages

Goal

How bits need to be communicated for Bob to compute $f(x, y)$?

Set Membership

Problem

For a set S, take $X = 2^S$ and $Y = S$. Define $g^S : X \times Y \to \{0, 1\}$ such that $g^S(x, y) = 1$ iff $y \in x$.

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One Round Randomized Protocol

In this model, both Alice and Bob can toss coins, but Bob has to compute the answer based on single message sent by Alice.

- $R_\epsilon^{A \to B}(f)$ is the fewest number of bits that Alice needs to send to Bob so that Bob can compute f with error at most ϵ.

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Theorem (Kremer-Nisan-Ron 1995)

$R_\epsilon^{A \rightarrow B}(g^S) = \Omega(2^{|S|})$.

Hard Property to Weakly Monitor

For alphabet $\Sigma = \{0, 1, \#, \$\}$ define the following languages

$S_n = (\#(0+1)^n)^+\$(0+1)^n$	membership query
$R'_n = \{(\#(0+1)^n)^*(\#w)(\#(0+1)^n)^*\$w \mid w \in (0+1)^n\}$	positive query
$R_n - S_n \setminus R'_n$	negative query
$L_n = R_n^\omega + R_n^*(\#(0+1)^n)^\omega$	

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[Kupferman-Rosenberg 2010] There is φ_n such that $[\![\varphi_n]\!] = L_n$ and $|\varphi_n| = n \log n$

Protocol from Monitor

Lemma

For any ϵ and $\mathcal{M}_n$ such that $\mathcal{L}_{>0}(\mathcal{M}_n) = L_n$, there is a state q_ϵ, reachable through an input in R_n^* such that every $\beta \in L_n$ is accepted with probability $\geq 1 - \epsilon$ from q_ϵ.

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1. Let w_x be input corresponding to Alice's input x. Alice runs $\mathcal{M}_n$ on w_x from q_ϵ and sends the state q reached to Bob.
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Bits communicated $= \log |\mathcal{M}_n| \geq 2^n$

Fragments of LTL

LTL(F, G)

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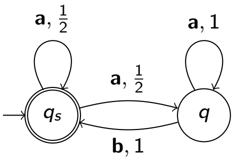
LTL $\setminus$ GU [Kretinsky-Esparza 2012]

$$\psi ::= \varphi \mid \psi \wedge \psi \mid \psi \vee \psi \mid \qquad \varphi \in \text{LTL}(F, G)$$
$$X\psi \mid \psi\ U\ \psi \qquad U \text{ above } G$$

Probabilistic Büchi Automata

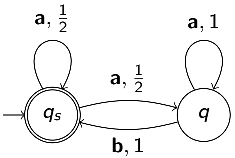
[Baier-Größer 2005]



A PBA is like an FPM except that it does not have a reject state and instead as final states.

Probabilistic Büchi Automata

[Baier-Größer 2005]



A PBA is like an FPM except that it does not have a reject state and instead as final states.

- An execution is accepting if it visits some final state infinitely often
- The acceptance probability of a word α, $\mathrm{acp}(\alpha)$, is the measure of all accepting executions on α.

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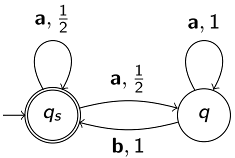

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- An execution is accepting if it visits some final state infinitely often
- The acceptance probability of a word α, $\mathrm{acp}(\alpha)$, is the measure of all accepting executions on α.
- $\mathcal{L}_{>0}(\mathcal{M})$ and $\mathcal{L}_{=1}(\mathcal{M})$ defined similarly.

PBA for LTL \ *GU*

Theorem (Kini-V. 2015)

For every φ in LTL \ GU there is a PBA $\mathcal{M}_\varphi$ such that $\mathcal{M}_\varphi$ has $O(2^{|\varphi|})$ states and $\mathcal{L}_{>0}(\mathcal{M}) = [\![\varphi]\!]$.

Simplifying Assumptions

- Focus on LTL(F, G)

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- Also, assume there are no X operators

Limit Deterministic Automata

Courcoubetis-Yannakakis 1995

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Let $\mathcal{N}$ be a nondeterministic Büchi automaton. Let $\mathcal{M}$ be the PBA obtained assigning some probabilities to the nondeterministic choices. Then $\mathcal{L}_{>0}(\mathcal{M}) = \mathcal{L}(\mathcal{N})$.

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We will construct a limit deterministic automaton for LTL $\setminus$ GU.

"Standard" LTL to NBA translation

Idea: Automaton guesses which temporal subformulas are true at each step.

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$$q_0 : \{\varphi, Fb\}$$

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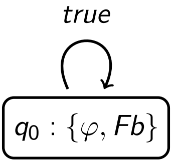

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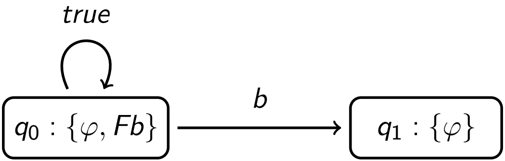

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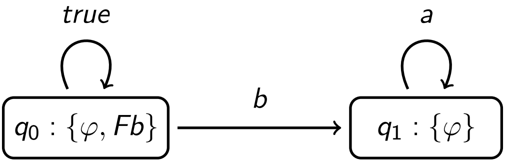

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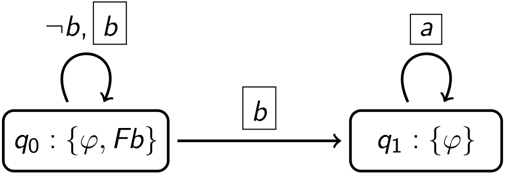

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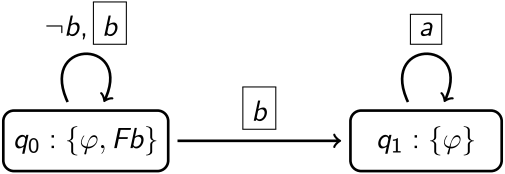

Automaton is not limit deterministic!

Construction for LTL(F, G)

Intuition

Observation

For any formula φ over propositions P, any word $w \in (2^P)^\omega$ satisfies exactly one of the following

$$G\varphi \quad F\varphi \wedge \neg G\varphi \quad \neg F\varphi$$

What does this mean for F, G subformulas?

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Overview

- A state is a guess about how often each F, G subformula holds.

example

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Overview

- A state is a guess about how often each F, G subformula holds.
- The automaton checks if the guess is sound
  - A guess is sound if every $G\psi \in \pi_A$ is true and every $F\psi \notin \pi_A$ is true.

example

Construction for LTL(F, G)

Evaluation

	A	B	C
$G\psi$	$G\psi$	$\neg G\psi \wedge FG\psi$	$\neg FG\psi$
$F\psi$	$\neg F\psi$	$F\psi \wedge \neg GF\psi$	$GF\psi$

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The evaluation of φ denoted by $[\varphi]^{\pi}_{\nu}$ is the truth of φ at present with respect to the guess π and input $\nu \in 2^P$.

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The evaluation of φ denoted by $[\varphi]^{\pi}_{\nu}$ is the truth of φ at present with respect to the guess π and input $\nu \in 2^P$.

- truth of propositions obtained from input ν
- boolean connectives evaluated using their semantics
- $[G\psi]^{\pi}_{\nu}$ is true iff $G\psi \in \pi_A$ and $[F\psi]^{\pi}_{\nu}$ is true iff $F\psi \notin \pi_A$.

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$$\pi \xrightarrow{\quad \nu \quad} \rho$$

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π	$\xrightarrow{\nu}$	ρ
$G\psi \in \pi_A$		$G\psi \in \rho_A$

Ensure ψ is true by evaluating it

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$G\psi \in \pi_B$		$G\psi \in \rho_A \cup \rho_B$

No need to check $G\psi$ is false

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π	$\xrightarrow{\nu}$	ρ
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If $F\psi$ moves to A ensure ψ is true

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Check that ψ holds infinitely often: use a counter!

Construction for LTL(F, G)

States

An automaton state is a pair (π, n)

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- $n \in \{0, 1, \ldots k\}$ where k is the number of F formulas in π_C

Construction for LTL(F, G)
Transitions

Transitions should help check if the guess is sound

$$(\pi, m) \quad \xrightarrow{\nu} \quad (\rho, n)$$

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- increment counter if $m = 0$ or the m^{th} F-formula in π_C evaluates to true

Construction for LTL(F, G)

Acceptance Condition

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Together they ensure that every guess in an accepting run is sound.

Construction for LTL(F, G)

Initial Conditions

A transition $(\pi, 0) \xrightarrow{\nu} (\rho, n)$ is initial if $[\varphi]^{\pi}_{\nu}$ is true.
Since initial guess is sound in an accepting run, the truth of φ is ensured.

Construction for LTL(F, g)

Limit Determinism

Limit determinism is ensured because

- Once π_B becomes empty, the guess π cannot change across transitions
- Counter is incremented deterministically

Example

Consider $\varphi = G(a \vee Fb)$

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$$q_0 : \langle\ \varphi \,|\, Fb \,|\, \text{-}\ \rangle,\ \mathbf{0}$$

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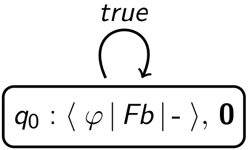

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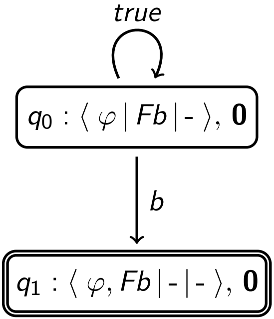

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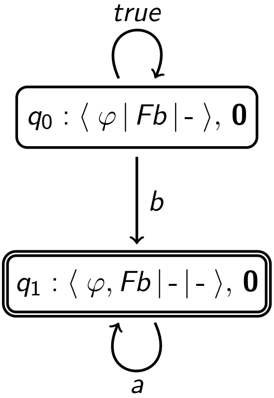

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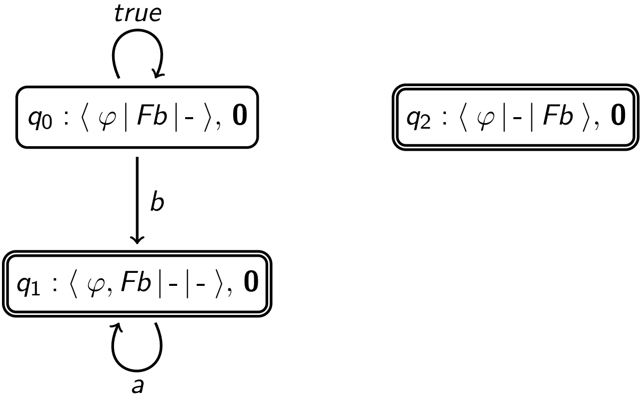

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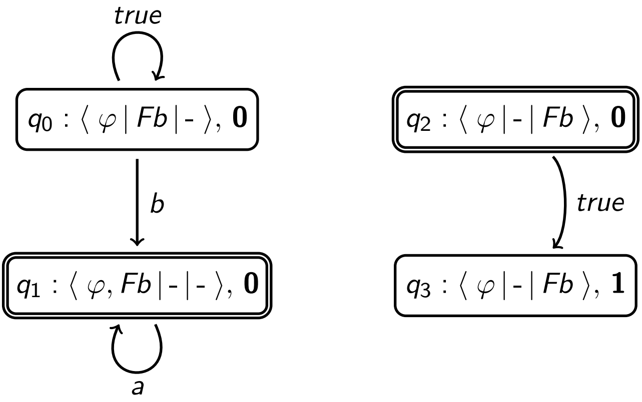

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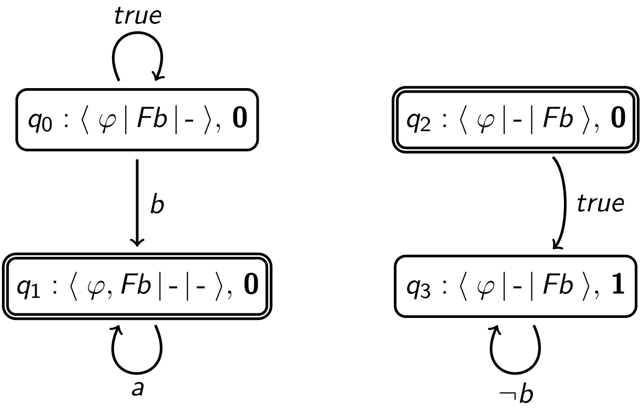

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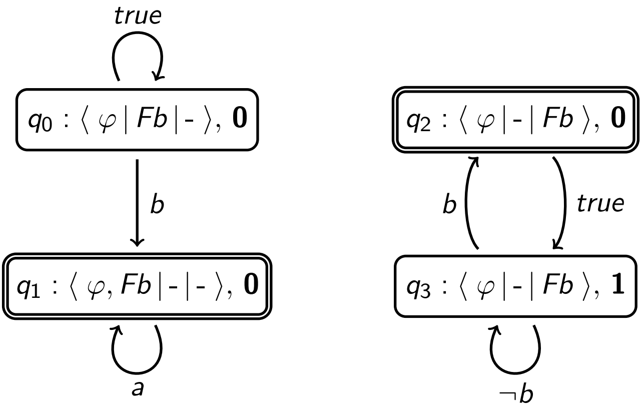

Markov Decision Processes

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- Models (closed) concurrent, stochastic programs
- Nondeterminism resolved by a scheduler

Model Checking Problem

Given and MDP A and LTL formula φ, is there a scheduler S such that the set of executions of A^S that satisfy φ has probability > 0?

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- Upper bound relies on analyzing the cross-product of the MDP with a limit deterministic automaton for φ.
- [Kini-V. 2015] The problem is EXPTIME-complete for LTL $\setminus$ GU specs

Wrapup

- Ideas can be generalized to construct limit deterministic automata for full LTL

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- Implementation of translation
  `http://web.engr.illinois.edu/ kini2/buchifier/`