On Petri nets with Hierarchical Special Arcs

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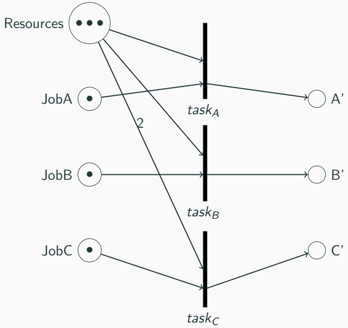
Preliminaries

Petri nets



- Petri net (PN) is a tuple (P, T, F, M_0),
  - P is set of *places*, T is set of *transitions*,
  - $M_0 : P \to \mathbb{N}$ is the *initial marking* and
  - $F : (P \times T) \cup (T \times P) \to \mathbb{N}$ is the *flow relation*.
- usual definitions: *marking* $M : P \to \mathbb{N}$, *firability*, *runs*...

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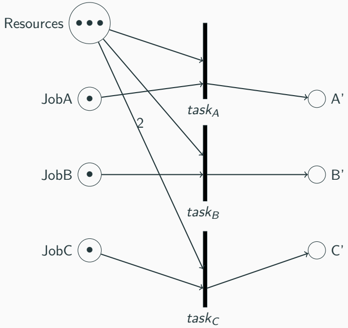

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- usual definitions: *marking* $M : P \to \mathbb{N}$, *firability*, *runs*...
- $\leq$ is component-wise order over markings

Decision Problems

Definition

Given a Petri net $N = (P, T, F, M_0)$,

- Termination (or $\textsc{Term}$): Does there exist an infinite run from marking M_0?
- Reachability (or $\textsc{Reach}$): Given a marking M, is there a run from M_0 which reaches M?
- Coverability (or $\textsc{Cover}$): Given a marking M, is there a marking $M' \geq M$ which is reachable from M_0?

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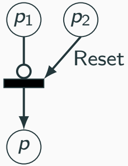
- Deadlock-freeness (or $\textsc{DLFree}$): Does there exist a marking M reachable from M_0, such that no transition is firable at M?
- (Place-)Boundedness: Does some (a given) place get unboundedly many tokens?

Special Arcs in Petri nets

- We can add a few special arcs into Petri nets.
    - Inhibitor arcs

Special Arcs in Petri nets



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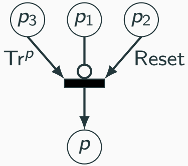

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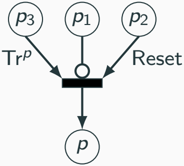

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  - Inhibitor arcs
  - Reset arcs
  - Transfer arcs
- Redefine flow
  $F : (P \times T) \cup (T \times P) \to \mathbb{N} \cup \{I, R\} \cup \{S_p \mid p \in P\}$

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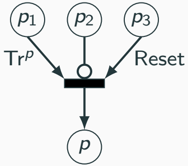

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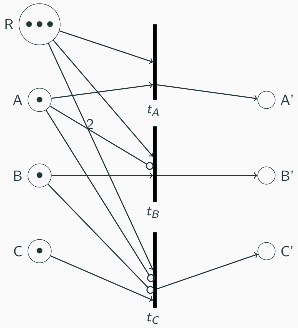

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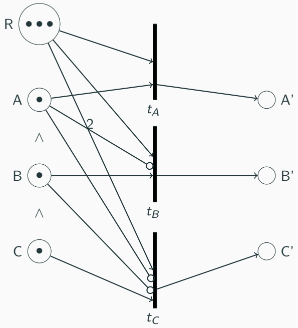

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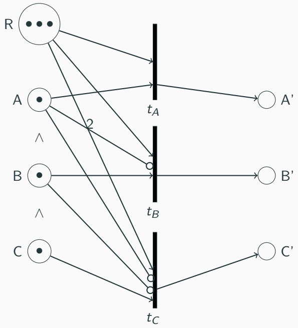

- The case of a single inhibitor arc/transition is an interesting and well-studied subcase!

State of the art: What is known about these problems?

State of the art

	TERM	COVER	REACH	DLFREE
PN	✓ (see [FS01])	✓ (see [FS01])	✓ [May84, Ler12]	✓ [CEP95, Hac74]
R/T-PN	✓ (see [FS01])	✓ (see [FS01])	✗ [DFS98]	✗ [Red. from [DFS98]]
I-PN	✗ [Min67]	✗ [Min67]	✗ [Min67]	✗ [Min67]
HIPN	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	
R+HIPN			✗[[DFS98], Thm 4]	✗[Red.frm [DFS98],Thm 4]
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- Can we “weaken” the notion of Hierarchy?

Outline

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- (4.) "Weakening" Hierarchy in HIPN using resets and transfers.

Part 1: Termination in R+HIPN

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Idea

Modify the definition of FRT (specifically the subsumption condition), to allow inhibitor arcs.

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Theorem

Checking termination in R+HIPN is decidable.

Proof sketch/intuition:

- For any place $p \in P$, we define the *index of the place p (Index(p))* as the number of places $q \in P$ such that $q \sqsubseteq p$.

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- For any transition $t \in T$, its *index* is defined as

$$\mathit{Index}(t) = \max_{F(p,t)=I} \mathit{Index}(p)$$

By convention, if there is no such place, then $\mathit{Index}(t) = 0$.

Termination in R+HIPN

Definition (Modified subsumption)

Consider a run $M_2 \xrightarrow{\rho} M_1$. Let $t^* = \mathrm{argmax}_{t \in \rho} \, \mathit{Index}(t)$.

$$\mathit{Subsume}(M_2, M_1, \rho) = M_2 \leq M_1 \wedge \left(\mathit{Compat}_{\mathit{Index}(t^*)}(M_1, M_2) \right)$$

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From this and effectivity, we get our result.

Part 2: Moving on to transfer arcs

But first – A detour to program termination!

Termination of linear loop programs

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But now, let us consider a much simpler case:

An initialized homogeneous linear program

$\vec{x} := \vec{b};\;\; \textbf{while } (\vec{c}^T \vec{x} > \vec{0}) \;\; \{\vec{x} := A\vec{x}\}$

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– rewrite as $\forall n \geq 0$, is $\vec{c}^T \cdot A^n \cdot \vec{b} > 0$?

Decidability of the Positivity problem

- Decidability of Skolem/Positivity for 2,3,4... in 1981, '85, '05, '06, '09 by various authors.
- In 2014, Ouaknine and Worrell showed the best known result:
  - positivity of order ≤ 5 is decidable with complexity $coNP^{PP^{PP^{PP}}}$.
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Bottomline: The general problem is still wide open!

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Simulating a program

Consider the following while loop program
v = v0; while (v >= 0) v = Mv.

- Clearly, this program is non-terminating iff $M^k v_0 \geq 0$ for all k.
- We construct a net N which simulates the program, i.e., terminates iff the program does.

Reduction from Positivity to T+HIPN

Consider

$$M = \begin{bmatrix} 1 & -4 & 7 \\ 2 & -5 & -8 \\ -3 & -6 & 9 \end{bmatrix}$$

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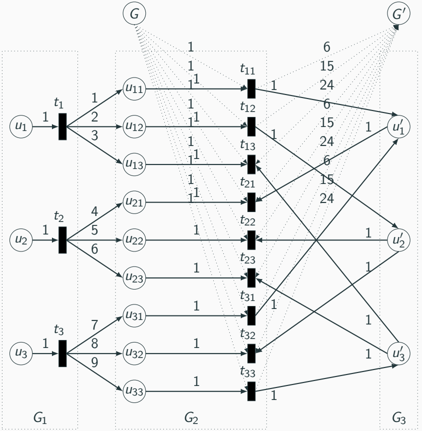

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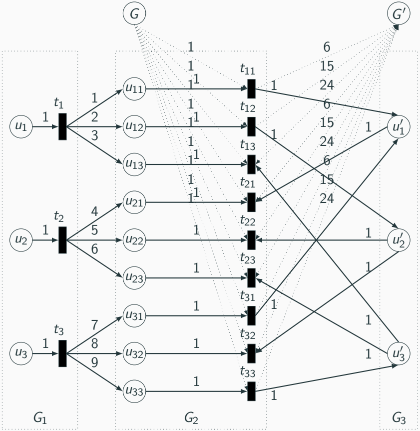
$$\begin{bmatrix} 1 & -4 & 7 \\ 2 & -5 & -8 \\ -3 & -6 & 9 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$



E.g: First entry of col vec $Mv = 5(1) + 6(-4) + 7(7)$

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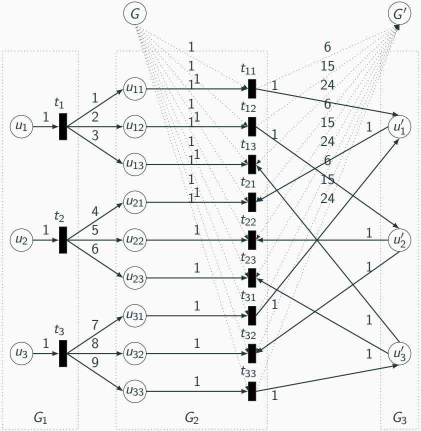

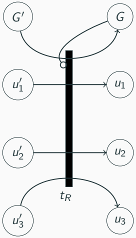

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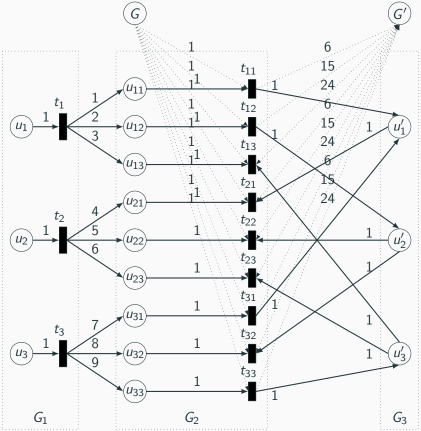

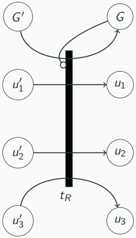

Initial marking assigns $(v_0)_i$, to place u_i, and $\sum_{1\leq i\leq n}(\sum_{1\leq j\leq n}|M_{ji}|)(v_0)_i$ tokens to G, all others 0.

Lemma: $\exists$ a non-term run in N iff $M^k v_0 \geq 0\ \forall k \in \mathbb{N}$.

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- Can we reduce positivity to termination of R+HIPN? (Open problem 2) :P
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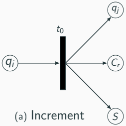
  What about coverability?

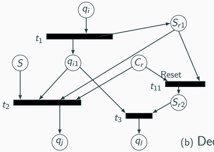
Coverability for R+HIPN

Theorem

Coverability is undecidable for Petri nets with 2 resets and 1 inhibitor arc.



(a) Increment

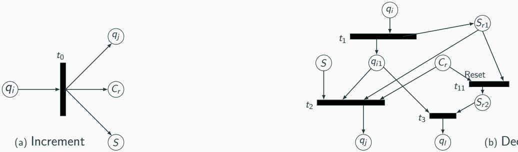


(b) Decrement

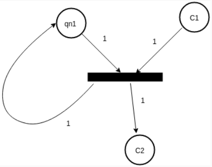
Coverability for R+HIPN

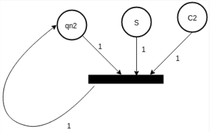
Theorem

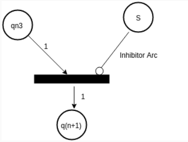
Coverability is undecidable for Petri nets with 2 resets and 1 inhibitor arc.



(a) Increment (b) Decrement







Summary till now

	TERM	COVER	REACH	DLFREE
PN	✓ (see [FS01])	✓ (see [FS01])	✓ [May84, Ler12]	✓ [CEP95, Hac74]
R/T-PN	✓ (see [FS01])	✓ (see [FS01])	✗ [DFS98]	✗ [Red. from [DFS98]]
I-PN	✗ [Min67]	✗ [Min67]	✗ [Min67]	✗ [Min67]
HIPN	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	(see paper!)
R+HIPN	✓	✗	✗[[DFS98], Thm 4]	✗[Red.frm [DFS98],Thm 4]
T+HIPN	Positivity-Hard	✗	✗[[DFS98], Thm 4]	✗[Red.frm [DFS98],Thm 4]

Part 3: “Weakening” Hierarchy? Adding resets/transfers within hierarcy

Adding resets and transfers within Hierarchy

Definition of HIPN

A total order $\sqsubset$ on P such that

$\forall (p, t) \in P \times T, \quad F(p, t) \in I \implies (\forall q \sqsubset p,\ F(q, t) \in I).$

Adding resets and transfers within Hierarchy

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$\forall (p,t) \in P \times T, \quad F(p,t) \in I \implies (\forall q \sqsubset p,\ F(q,t) \in I).$

What if we change this to:

Adding resets and transfers within Hierarchy

Definition of HIRPN : A seemingly larger class!

A total order $\sqsubset$ on P such that

$\forall (p,t) \in P \times T, \quad F(p,t) \in I \implies (\forall q \sqsubset p,\ F(q,t) \in (I \vee R))$.

Adding resets and transfers within Hierarchy

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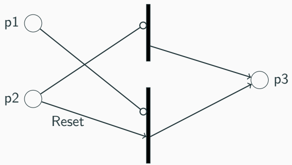

- This is not a HIPN (or a R+HIPN), but it is a HIRPN!

Adding resets and transfers within Hierarchy

Definition of HIRPN : A seemingly larger class!

A total order $\sqsubset$ on P such that

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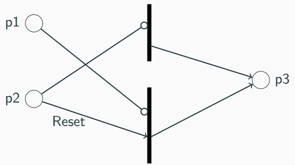

- This is not a HIPN (or a R+HIPN), but it is a HIRPN!
- A R+HIPN which is not a HIRPN.

Adding resets and transfers within Hierarchy

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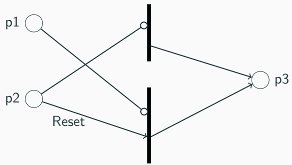

- This is not a HIPN (or a R+HIPN), but it is a HIRPN!
- A R+HIPN which is not a HIRPN.
- Can do the same with transfers...

Results on HIRPN and HITPN

Theorem

HIRPNs are still easy: Can reduce to HIPNs, which preserving reachability. Hence obtain decidability of properties.

Results on HIRPN and HITPN

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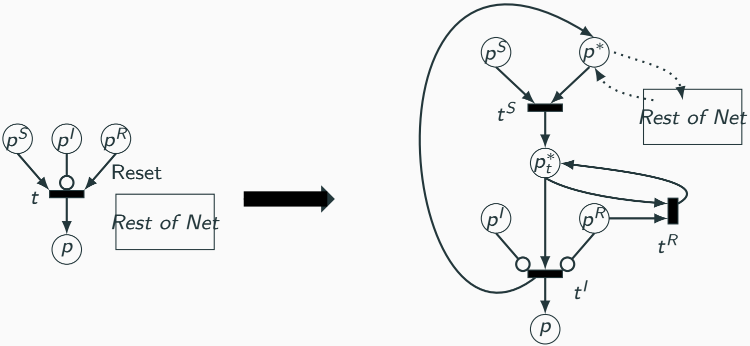

Results on HIRPN and HITPN

Theorem

HIRPNs are still easy: Can reduce to HIPNs, which preserving reachability. Hence obtain decidability of properties.

Theorem

Hierarchy is useless with transfers: i.e., HITPNs have same properties as T+HIPNs.

Conclusion

Results: Summary

	Term	Cover	Reach	DLFree
PN	✓ [FS01]	✓ [FS01]	✓ [May84, Ler12]	✓ [CEP95, Hac74]
R/T-PN	✓ [FS01]	✓ [FS01]	✗ [DFS98]	✗ [Red. from [DFS98]]
I-PN	✗ [Min67]	✗ [Min67]	✗ [Min67]	✗ [Min67]
HIPN	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	✓ [Rei08, Bon13]	✓
HTPN	✓ [FS01]	✓ [FS01]	✗	✗
HIRPN	✓	✓	✓	✓
HITPN	Positivity-Hard	✗	✗	✗
HIRcTPN	✓	✓	✓	✓
R+HIPN	✓	✗	✗[[DFS98]]	✗[Red.frm [DFS98]]
T+HIPN	Positivity-Hard	✗	✗[[DFS98]]	✗[Red.frm [DFS98]]
R+HIRPN	✓	✗	✗[[DFS98]]	✗[Red.frm [DFS98]]

Table 1: Results for all other extensions are subsumed by these results. Can add boundedness column too!

Work in progress and Open problems

- Reducing the number of counters.
- What about complexity?
- Coverability for Petri nets with 1 reset and 1 inhibitor arc (without hierarchy)?
- An approach towards the positivity/Skolem problem via WSTS?

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