

#### CS783: Theoretical Foundations of Cryptography

Lecture 1 (30/Jul/24)

Instructor: Chethan Kamath

#### Administrivia

- Timing and Venue: Slot 10 (14:00–15:25, Tuesdays and Fridays) in CC105
- Contact hours: after lectures, or appointment by e-mail
  Teaching assistants:
  - Sarthak Sharma (23M0789) and Nivesh Aggarwal (22b0912)

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- Resources
  - Slides and other resources will be posted on my website
    - cse.iitb.ac.in/~ckamath/courses/2024/CS783.html
  - Announcements/online discussion on Moodle:
    - moodle.iitb.ac.in/course/view.php?id=4702

#### Administrivia...

#### ■ Grading Scheme

• Six ungraded assignments to help with quizzes and exams

Weightage	Towards
35%	Final
25%	Mid-term
20%	Two quizzes, one each after Modules I and III
15%	Group project/chalk-talk
5%	Class participation, pop-quiz

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- Will scrap group project/chalk talk if number of creditors > 30 and grade will be redistributed
- Attendance is not mandatory (but encouraged)
- Any volunteers for class rep?



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Proton Mail	
Secure communication	
Secure Bod Reset Bod Security to Factory Defat Delete all Security Boot Keys	○ 合 ≓ (https://en.wikipedia.org/wiki/Main_Page)
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Using laptop/phone	Using internet



Science of carrying out *tasks* securely in an adversarial setting

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- An analogy: secret communication

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  - Bengaluru metro (understand Kannada, English and Hindi)
  - Mumbai local (understand Marathi, English and Hindi)





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- Graduate-level *theoretical* cryptography course
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    - Security Goal: What does it mean to be secure?

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- Soft prerequisites: discrete mathematics, probability theory, some familiarity with formal proofs

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 But if you are interested in coding, I will point you to relevant cryptographic libraries and real-world applications

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- Wait for the next iteration of *CS409: Introduction to Cryptography* (previously CS406)

#### 1 Module I: Secure Communication in Shared-Key Setting

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■ Roughly corresponds to four "eras"


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MOWLY I (Shared keys) For a large part of history



🔸 🔨 Present

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MODULEI

(Shared keys)

For a large part of history



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MODULE 2 (Public heys)

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### An Overview of the Course

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Credit for images: Wikipedia

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Credit for images: Wikipedia

Credit: Tekniska Museet

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■ Threat model: *Perfect secrecy* 

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- Adversary: All powerful eavesdropper Eve (but doesn't know K)
- Security goal: Eve learns *no* information about the message *M*
- What we will learn:
  - Classical ciphers, and why they are not perfectly secure
  - One-time pad (OTP), and why it is perfectly secret
  - Shannon's impossibility: for perfect secrecy,  $|K| \ge |M|$



■ How to overcome Shannon's impossibility?



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Restrict/bound the adversary's computational capabilities



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■ Threat model: *Computational* secrecy

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- Security goal: Eve learns *"no"* information about the message *M*

#### ■ What we will learn:

- How to model computationally-bounded adversaries?
- Hardness assumption: pseudo-random generator, one-way func.
- Secret communication with |K| < |M| assuming PRG

- What if Eve *also* has control over the messages?
- What we will learn: *chosen-plaintext attack* (CPA) and CPA-secure scheme from pseudo-random functions

Size 510 GB (5,10,10,91,55,328 bytes) Contents LUKS Encryption (version 2) — Unlocked

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### Advent of Internet and the Scaling Problem





Limitation of shared-key encryption: requires prior meeting

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Credit for images: (\*Wikipedia User: DARPA) (\*\*icour.fr) (\*\*\*cs.miami.edu (Rosenberg))

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Threat model

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■ Setting: Alice has published a public key PK, and Bob wants to send a secret message *M* to her



- Public-key encryption (PKE) from trapdoor functions
- Equivalence between PKE and key exchange
- Learning with errors, PKE in presence of quantum adversary

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### Ubiquity of Computing and Need for Privacy



Credit for images: Wikipedia (\*\*User:Saliko)













- Threat model:
  - Adversary: honest-but-curious ("give-me-all-your-data") Bob
  - Security goal: Bob *does not* learn the dataset or the model

■ Setting: Alice has a weak laptop and wants to use Bob's cluster to train an Al model on her dataset *D* 



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- Security goal: Alice learns no *non-trivial* info. about Bob's input

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■ Threat model:

- Adversary: honest-but-curious Alice (Bob)
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(Aluce's)
# Task 4: Private Two-Party Computation

- Setting: Alice and Bob have *private* inputs  $x_A$  and  $x_B$  and want to compute  $f(x_A, x_B)$  for a *public* function f
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- Threat model:
  - Adversary: honest-but-curious Alice (Bob)
  - Security goal: Alice learns no *non-trivial* info. about Bob's input
- What we will learn:
  - How to model the security goal (via simulators)
  - Zero knowledge (ZK) proof, several ZK protocols
  - Yao's garbling, other two-party computation protocols

(Aluce's)

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#define F (getchar()&15)
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#define Z while(
#define P return y=-y,
#define \_ ;if(
char\*l="dbcefcbddahcdd haeWAB+ +BAW- +-48HLSU?A6J57IKJT576,";B,y,
b,I[149];main(w,c,h,e,S,g) int t,o,L,E,d,O=\*1,N=-1e9,p,\*m=1,d;r,rx=10\_\*1)[y=-y;
Z:-o220](O=I[D=O: getCripeO)[q4=(q<2)\*y,t=q("51#/++"],Eq("9543/31");do[T=I[D"+Te1]])[q2=28(89×P]30>9)75Ay;o;L=(q>176-q71[pX-1])[q2=(28(89×P]30>9)75Ay;o;L=(q>176-q71[pX-1])-1[(0X-1]]-q2=(02)[q2=(q>29)75Ay;o;L=(q>176-q71[pX-1])-1[(0X-1]]-q2=(02)[q2=(q>29)75Ay;o;L=(q>176-q71[pX-1])-1[(0X-1]]-q2=(02)[q2=(q>20)75Ay;o;L=(q>176-q71[pX-1])-1[(0X-1]]-q2=(02)[q2=(q>10)75Ay;L=ssh[]]==hEa5443\*h;O(I]=0,p[I]=q2=(28(89×P]30>9)75Ay;o;L=(q>176-q71[pX-1])-1[(0X-1]]-q2=(02)[q2=(q>10)](q1=(q>1)[q2=(q>1)]=(q=1)[q2=(q>1)](q1=(q>1)[q2=(q>1)]=(q=1)[q2=(q>1)](q1=(q>1)[q2=(q>1)]=(q=1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)](q1=(q>1)[q2=(q>1)[q1=(q>1)[q2=(q>1)[q1=(q>1)[q2=(q>1)[q1=(q>1)[q2=(q>1)[q1=(q>1)[q2=(q>1)[q

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char\*l="dbcefcbddabcdd=ba~WAB+ +BAW~ +-48HLSU?A6J57IKJT576,";B,v,  $Z = -0 \ge 20$  fo=I[p=0]  $g=e^{+y}$ , g>0 fa==(a<2) \* y, t=a["51#/+++"], E=a["95+3/33"]; do fr=I[p] $1 \times 1 \times 1$ ,  $+B < 120 - 2 + m < 9 + 1 > 30 [m] = 1, 90 [m] = -(20 [m] = +1 + x7), 80 [m] = -2; 2 p = 19) {Z + p < 0}$ 

■ It's a primitive chess engine! (nanochess.org/chess3.html)

■ Setting: Alice discovers algorithm A, which Bob wants to use





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 Example: Fast deterministic primality test





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#### ■ Threat model:

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#### ■ What we will learn:

- How to model the security goal
- Why program obfuscation is a powerful tool!
  - Can be used to carry out all the tasks we have seen so far

# Other Advanced Tasks

- If time permits:
  - Advanced notions of PKE (identity-based encryption, proxy re-encryption, functional encryption)
  - Secure messaging



Digital currency without central authorities (cryptocurrency)



### Next Lecture



Credit for images: Wikipedia

### Next Lecture



Credit for images: Wikipedia

### Next Lecture



More Questions?