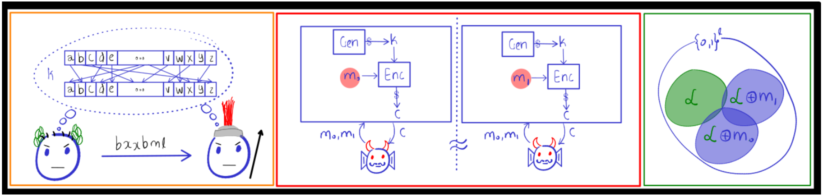



# CS783: Theoretical Foundations of Cryptography

Lecture 2 (02/Aug/24)

Instructor: Chethan Kamath

# Recall from Last Lecture...

MODULE 1 (Shared keys)

MODULE 2 (Public keys)

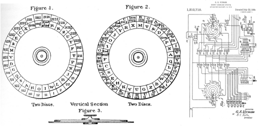
MODULE 3 (Secure comp.)

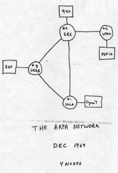
MODULE 4 (Adv. tasks)

For a large part of history

Advent of internet

Ubiquity of computing

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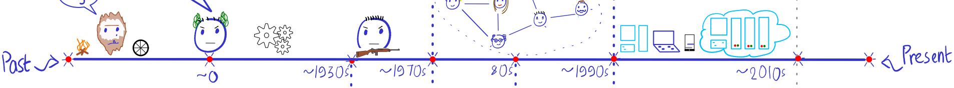
ooga

Ego quoque sum conscius secreti




Birth of information theory & CS

Foundations of modern cryptography

Birth of "provable security"

Credit for images: Wikipedia (*User:DARPA)(**User:Saliko)

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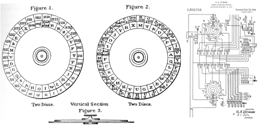


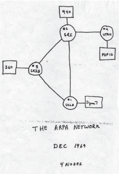


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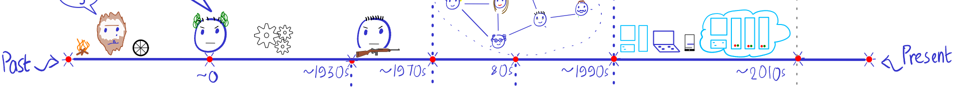


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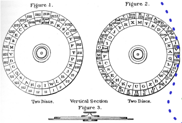


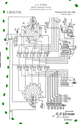

Some historical ciphers

Perfect secrecy and one-time pad

Ego quoque sum conscius secreti

ooga

Past

~0

~1930s

~1970s

Birth of information theory & CS

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# Recall from Last Lecture...

General *template*:

1. Identify the task
2. Come up with precise threat model $M$ (a.k.a security model)
   - Adversary/Attack: What are the adversary's capabilities?
   - Security Goal: What does it mean to be secure?
3. Construct a scheme $\Pi$
4. Formally prove that $\Pi$ in secure in model $M$

# Plan for this Lecture...

General *template*: secret communication with shared keys

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   Perfect secrecy
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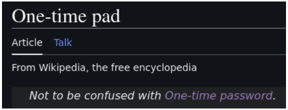
# Plan for this Lecture...

1 Syntax of Shared/Symmetric-Key Encryption (SKE)

2 Classical ciphers




+First proof

3 Perfect Secrecy and One-Time Pad (OTP)

# Plan for this Lecture

1. Syntax of Shared/Symmetric-Key Encryption (SKE)

2. Classical ciphers

3. Perfect Secrecy and One-Time Pad (OTP)

# Some Notation and Conventions

- Sets:
  - Denoted using calligraphic font: e.g., $\mathcal{M}$, $\mathcal{C}$
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- Probability notation:
  - For a distribution $M$ over a set $\mathcal{M}$ and element $m \in \mathcal{M}$, $m = M$ denotes the *event*: 'a random sample from $M$ equals $m$"
  - Following denotes probability that $\mathsf{A}(x) = 1$ when $x \leftarrow \{0,1\}^n$:

$$\Pr_{x \leftarrow \{0,1\}^n}[\mathsf{A}(x) = 1]$$

# Syntax of Shared/Symmetric-Key Encryption

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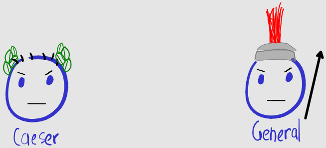

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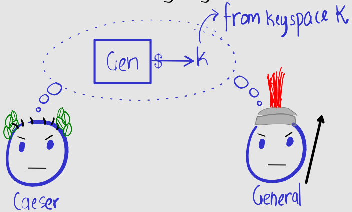

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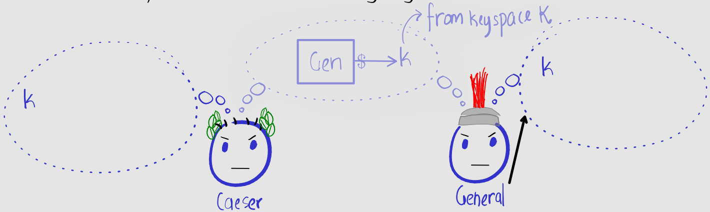

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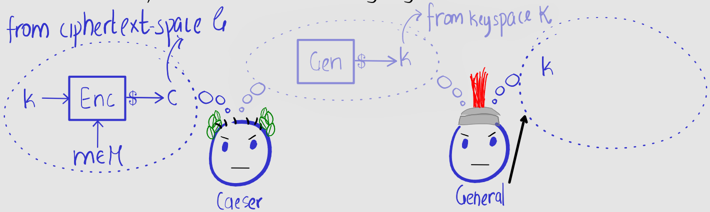

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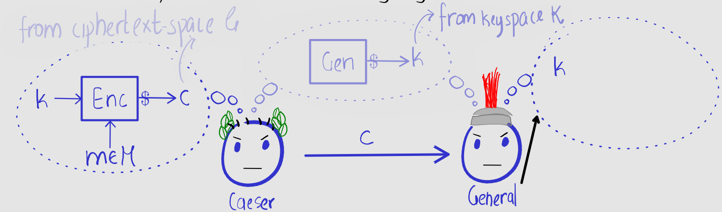

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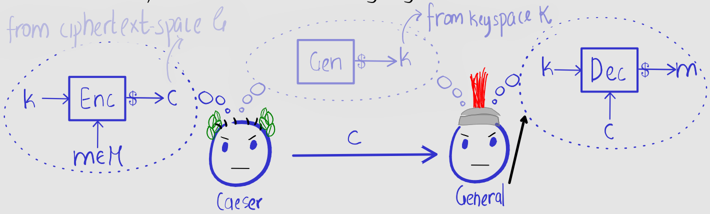

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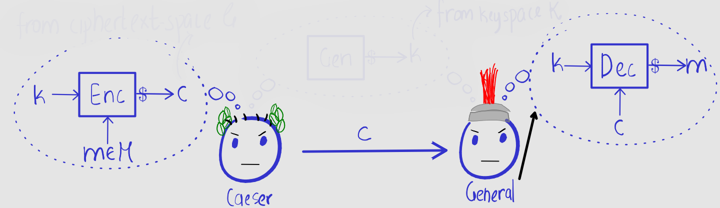


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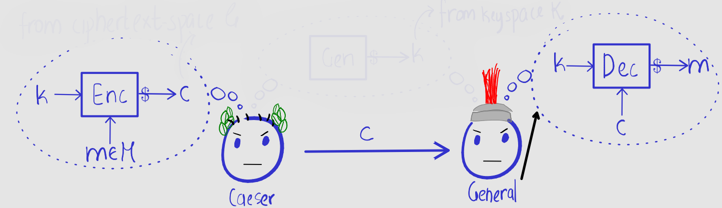


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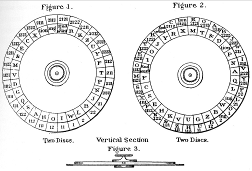
Why can we assume that $\mathsf{Dec}$ is *deterministic* w.l.o.g.?

# Plan for this Lecture

# Shift Cipher (Caeser Cipher)




## Construction 1 (for message space $\{\mathtt{a}, \cdots, \mathtt{z}\}^{\ell}$)

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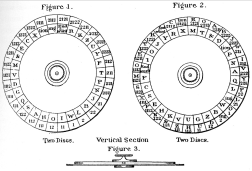


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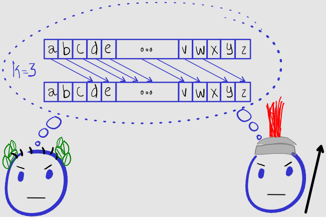


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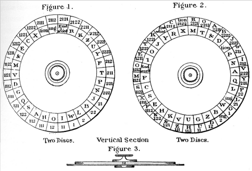


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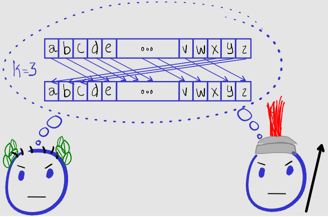


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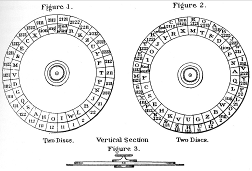


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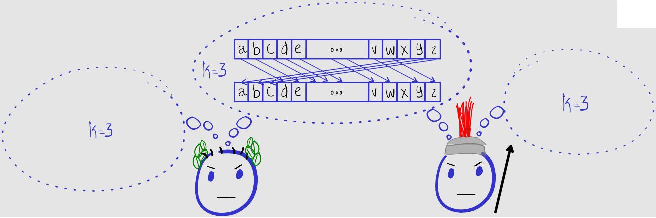


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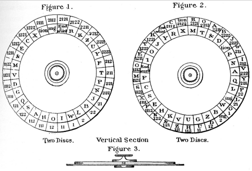


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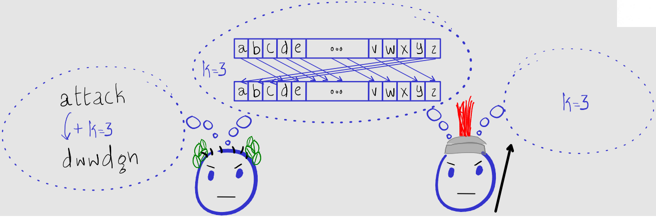


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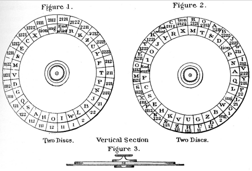


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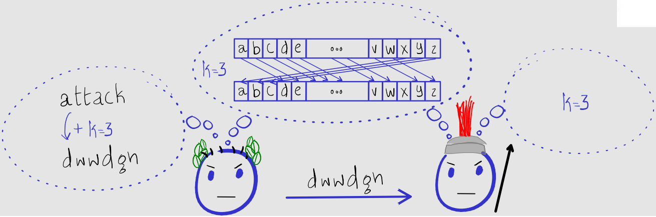


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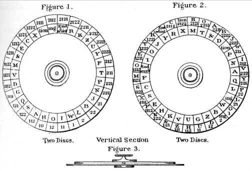


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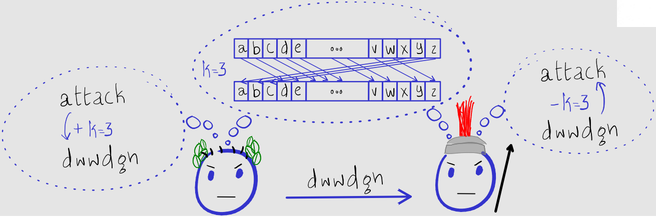


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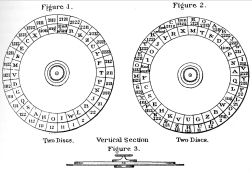


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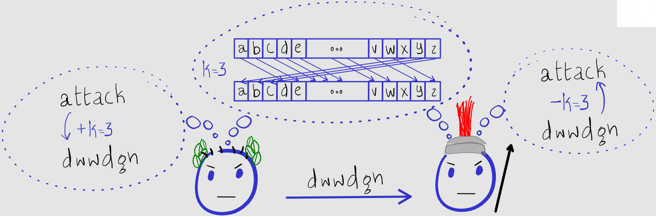


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- Key generation, **Gen**: output $k \leftarrow \{0, \cdots, 25\}$

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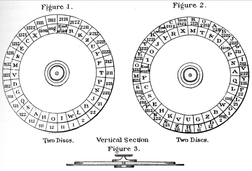


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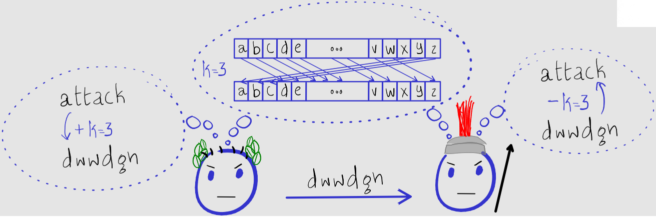


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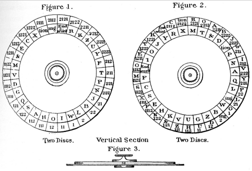


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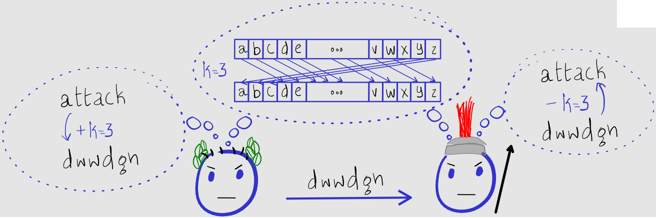


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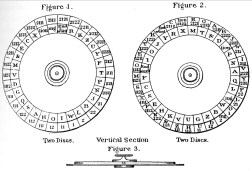


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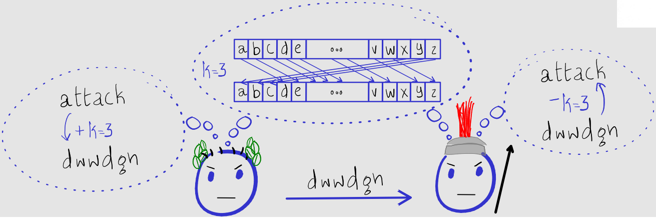


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? Why does correctness of decryption hold?

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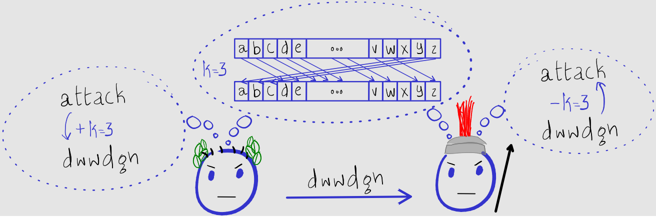


## Exercise 1

1. *What is the key-space? What is the ciphertext-space?*
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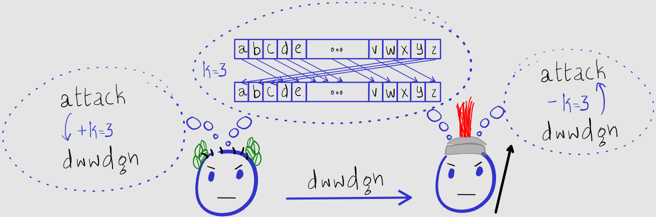


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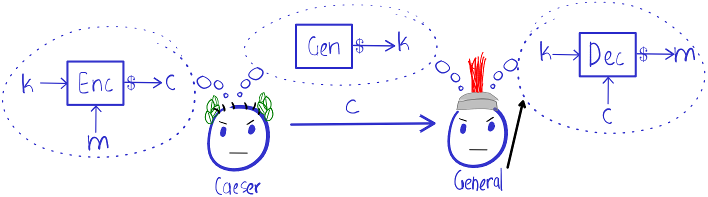
   *Assume that Caeser only sends either* `attack` *or* `defend`*.*
3. *What is the probability that the ciphertext is* `kddkmu`*, (resp.* `kddkmw`*)?*
4. *If ciphertext is* `kddkmu`*, is it possible that message is* `defend`*?*

# First Let's Try to Model our Eavesdropper Eve



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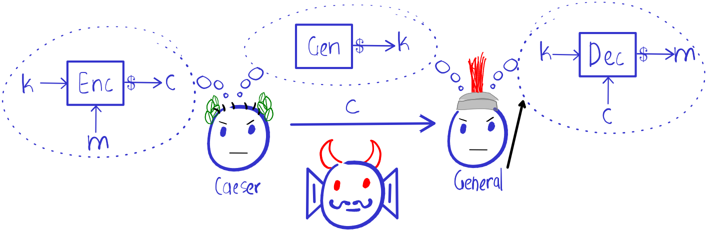

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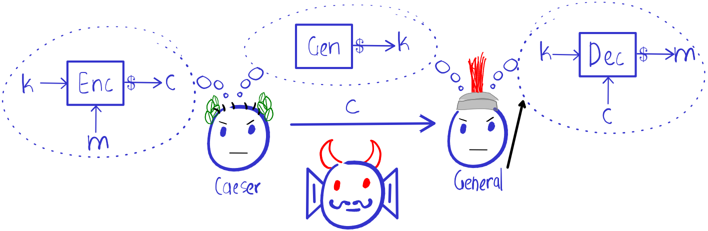


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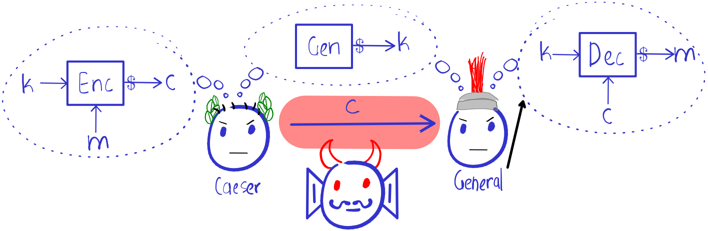


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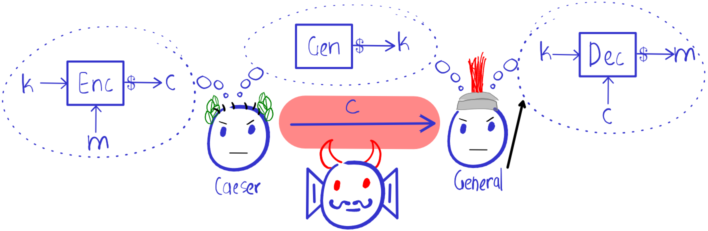
? What does Eve have access to?

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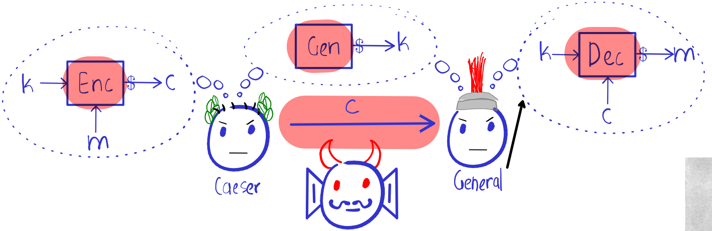


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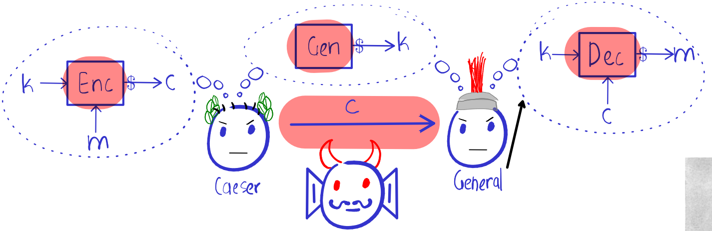


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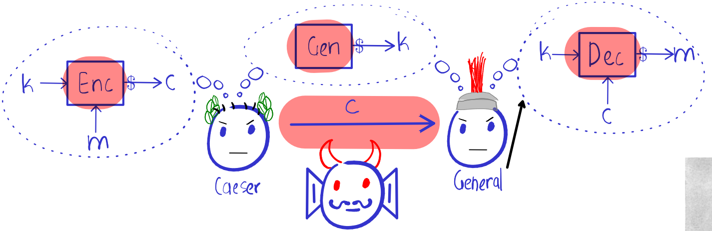


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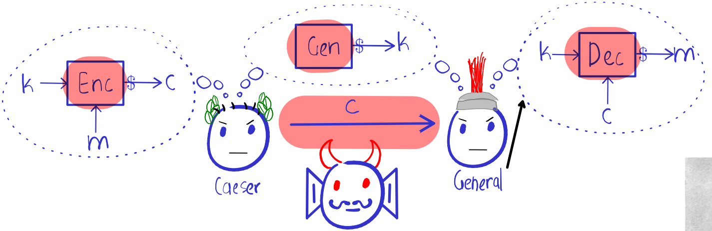


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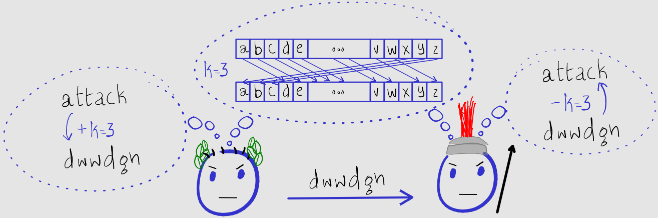
# Shift Cipher (Caeser Cipher)...

## Construction 3

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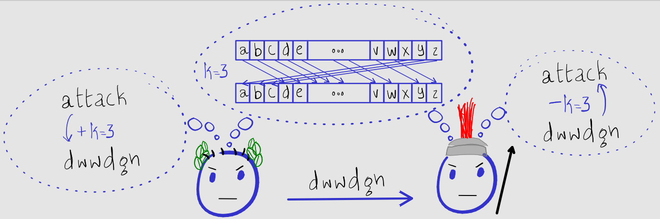


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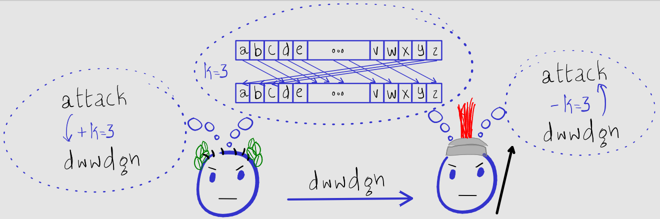


- What can Eve learn?
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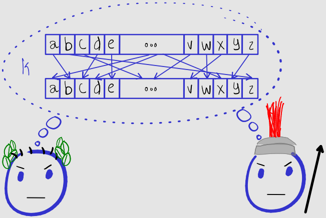
- What have *we* learnt?
  - *Large-enough* key-space is necessary to thwart *brute force*

### Exercise 2

*What happens if the length of the message* $\ell = 1$*?*

**Construction 4 (Message space $\{\mathtt{a}, \cdots, \mathtt{z}\}^{\ell}$)**

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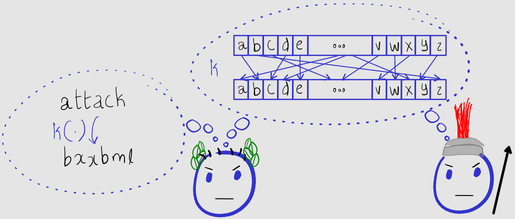


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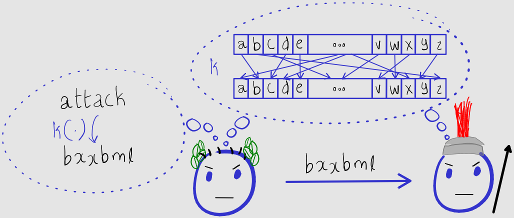


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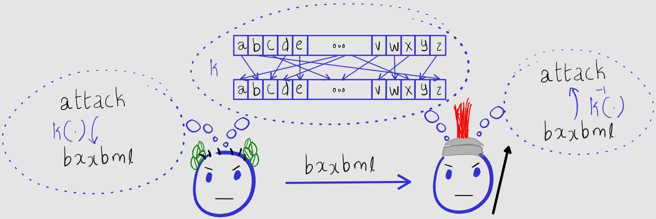


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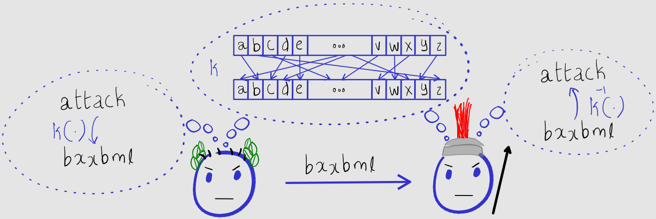


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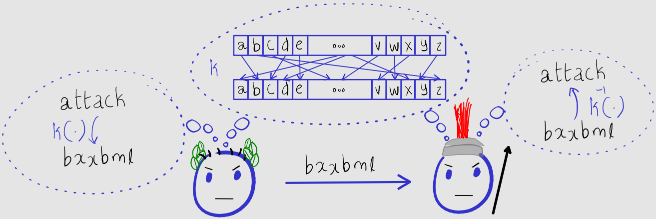


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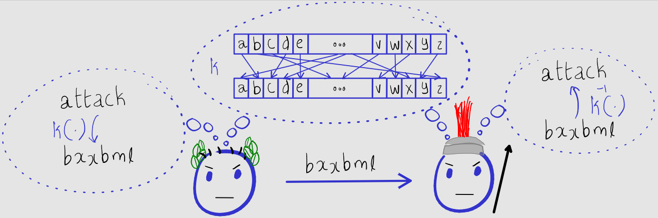
## Exercise 3

- *Write down the pseudocode for substitution cipher.*
- *Why does correctness of decryption hold?*

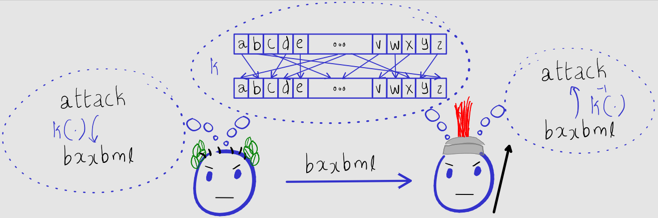
## Construction 5




What can Eve learn?

# Substitution Cipher...

## Construction 5

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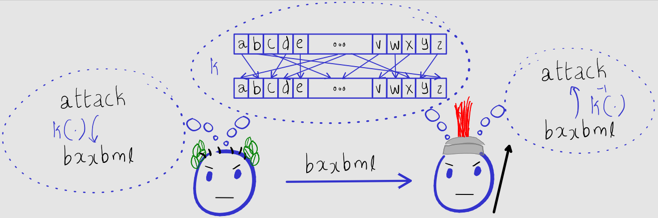


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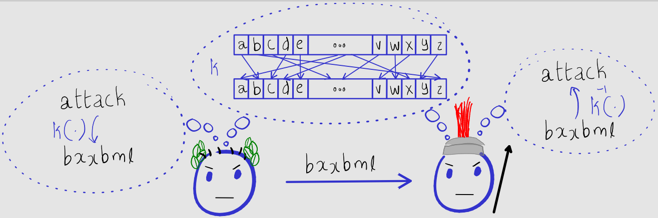


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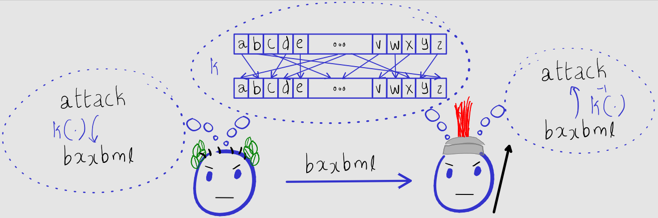


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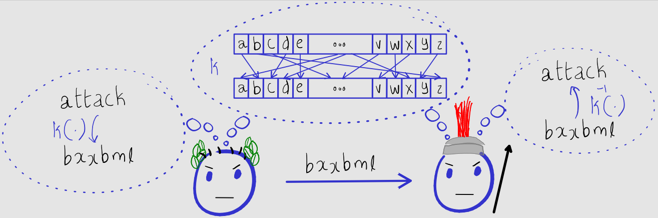


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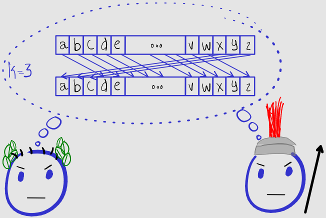

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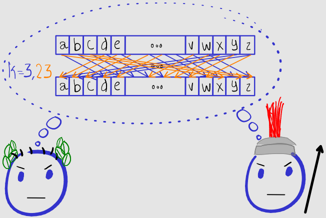

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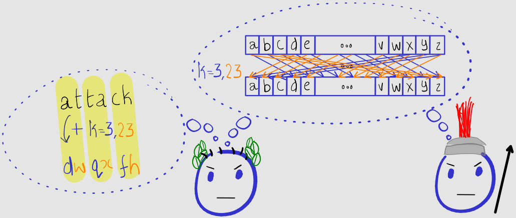

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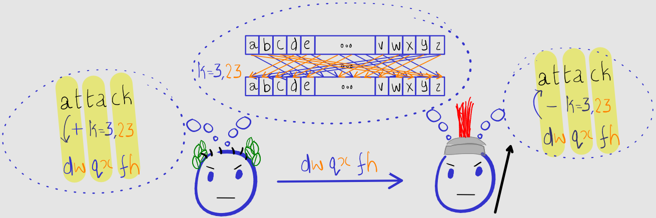

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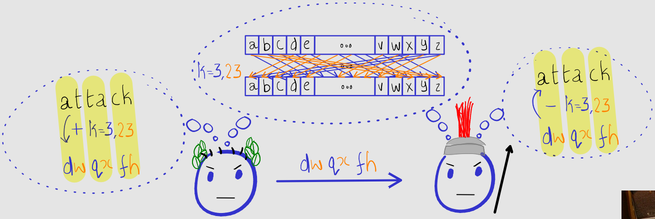
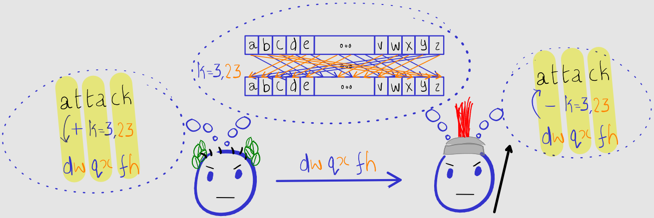



## Exercise 4

1. *Write down the pseudocode for polyalphabetic shift cipher.*
2. *Work out the details of polyalphabetic substitution cipher.*

# *Polyalphabetic* Ciphers...

**Construction 7** (*Polyalphabetic* shift cipher (Vignère cipher))



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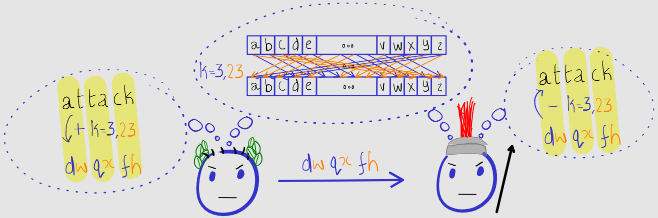


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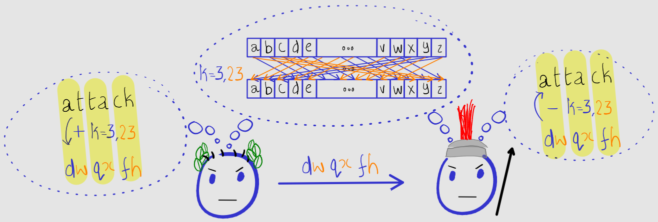


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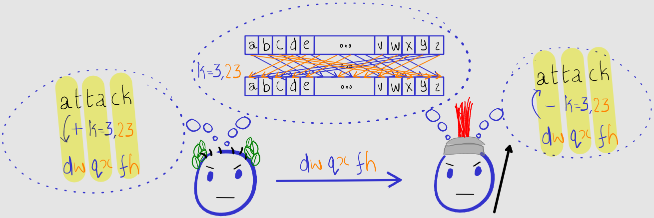


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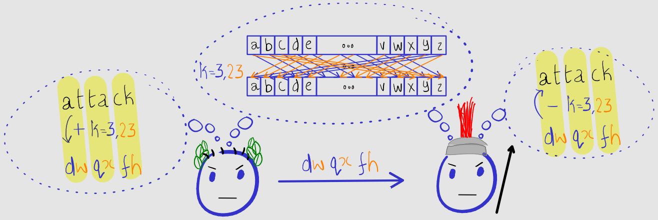


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- What have we learnt?
  - Must hide *all* statistical patterns of the plaintext
  - Equivalently: Eve must *learn no information* about the plaintext

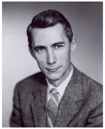
# Plan for this Lecture

# How to Model 'No Information Learnt'?

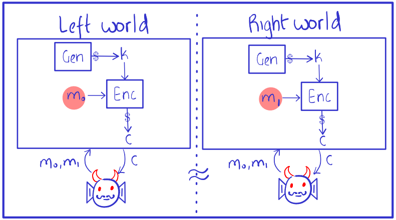
- We will look at two ways:

"Information theoretic"

$$\Pr_{k \leftarrow \mathsf{Gen}}[M = m^* | C = c^*] = \Pr[M = m^*]$$



"Imitation game"



# Modelling 'No Information Learnt': Shannon's Take...

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**Definition 2 (Shannon'49)**

Let $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ be an SKE with message space $\mathcal{M}$.
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- Definition essentially says $M$ and $C$ are *independent* random variables
- Definition *does not* refer to Eve at all!

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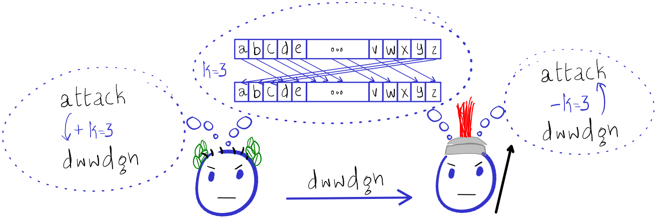

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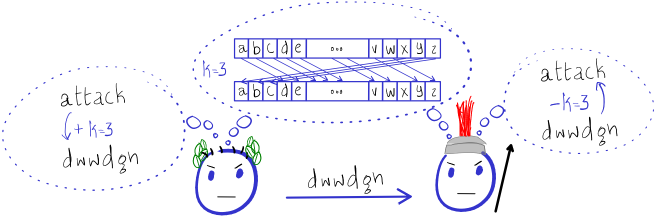

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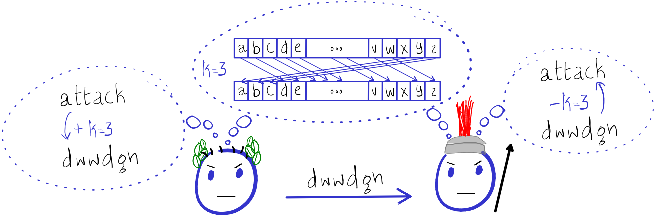

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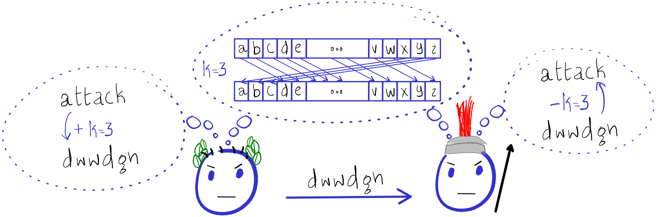

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$\Pr_{k \leftarrow \mathsf{Gen}}[M = m^* | C = c^*]$: 0!  $\Pr[M = m^*]$: 1/2

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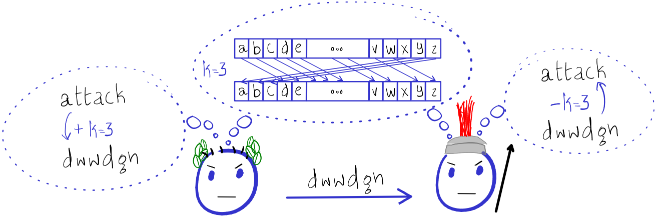

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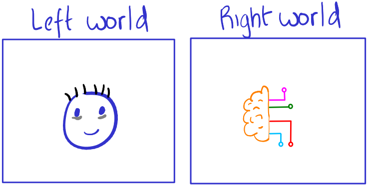

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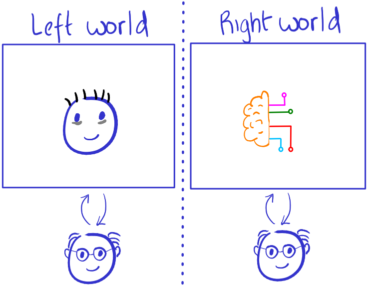

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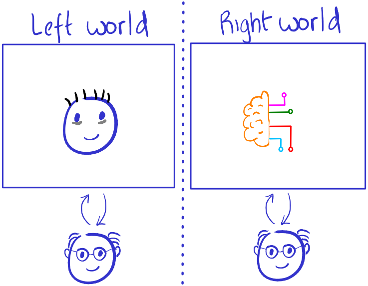
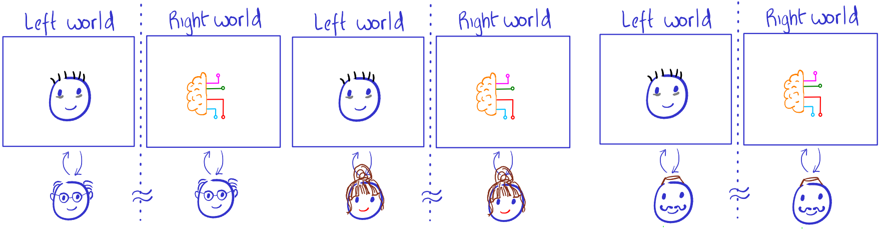



- Turing, on artificial intelligence: *"Are there imaginable digital computers which would do well in the imitation game?"*

# Modelling 'No Information Learnt': Imitation Game...

- Turing's Imitation Game (Turing Test)




- Turing, on artificial intelligence: *"Are there imaginable digital computers which would do well in the imitation game?"*
- To paraphrase: sign of artificial (human) intelligence if no human can tell the two worlds apart.

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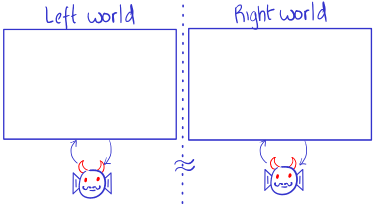
What are our two worlds?



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- ‘Left” world: always encrypt $m_0$
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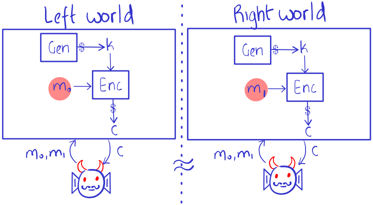

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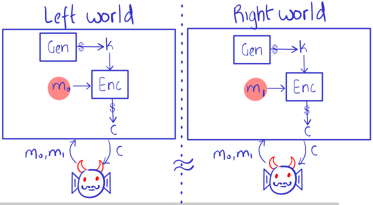


**Definition 4**

An SKE $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is perfectly-secure if *for any* eavesdropper *Eve* and messages $(m_0, m_1) \in \mathcal{M}$:

$$\Pr_{\substack{k \leftarrow \mathsf{Gen} \\ c \leftarrow \mathsf{Enc}(k, m_0)}}[\mathsf{Eve}(c) \text{ outputs ‘left'}] = \Pr_{\substack{k \leftarrow \mathsf{Gen} \\ c \leftarrow \mathsf{Enc}(k, m_1)}}[\mathsf{Eve}(c) = \text{ outputs ‘left'}]$$

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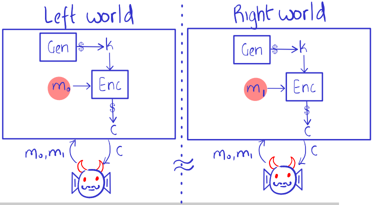


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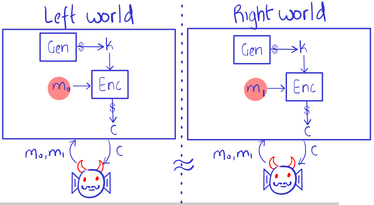


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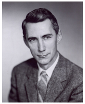
### Exercise 5

*Show that shift and substitution ciphers are not perfectly secure w.r.to above definition.*

- We saw two definitions.

‘Information theoretic’

$$\Pr_{k \leftarrow \mathsf{Gen}}[M = m^* | C = c^*] = \Pr[M = m^*]$$



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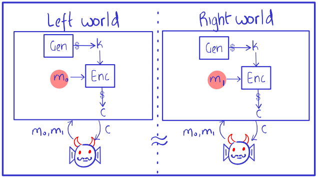

# How to Model ‘No Information Learnt’?...

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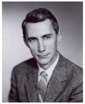

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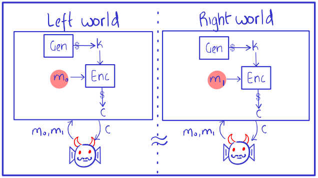


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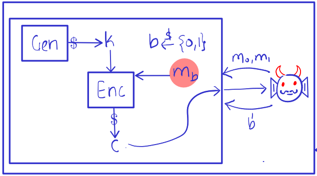


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- Ciphertext indistinguishability: variant of imitation game

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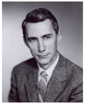

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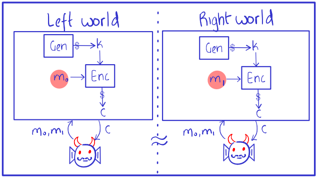


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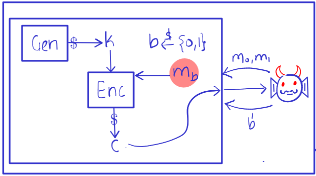


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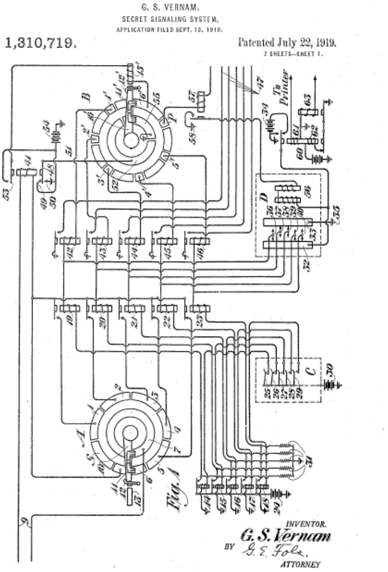
## Exercise 6

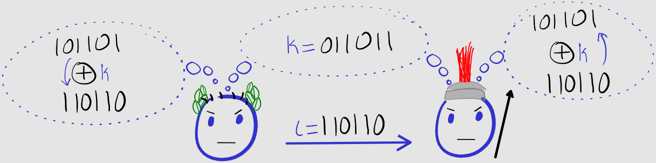
*Show equivalence of all these definitions.*

# One-Time Pad (Vernam' Cipher)...



# One-Time Pad (Vernam' Cipher)...

Construction 8 (Message space $\{0,1\}^{\ell=6}$)



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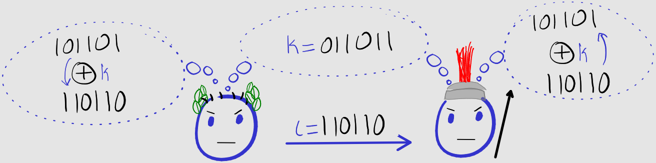


**Pseudocode 2 (Message space $\{0,1\}^{\ell}$)**

- Key generation $\mathsf{Gen}$: output $k \leftarrow \{0,1\}^{\ell}$
- Encryption $\mathsf{Enc}(k,m)$: output $c := k \oplus m$
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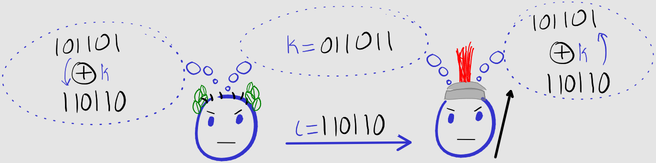


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## Exercise 7

1. *Design OTP for message space* $\{\mathtt{a}, \cdots, \mathtt{z}\}^{\ell}$
2. *How is this different from polyalphabetic shift cipher?*

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Theorem 5 (Shannon'49)

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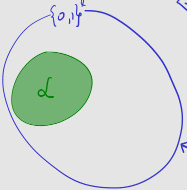


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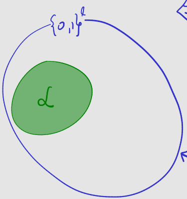


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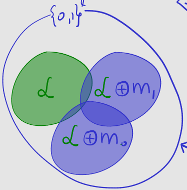


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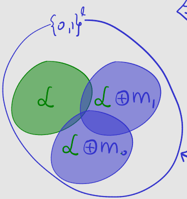


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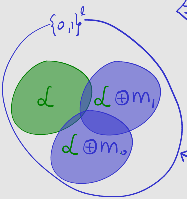


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# OTP IRL

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'Red telephone'

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## Operation Vula: A secret Dutch network against apartheid

Published 9th September 1999

## Moscow–Washington hotline

Article Talk

From Wikipedia, the free encyclopedia

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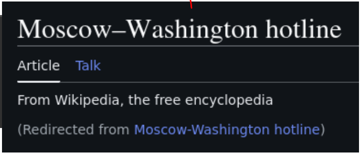

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The Register

Declassified files reveal how pre-WW2 Brits smashed Russian crypto

Moscow's agents used one-time pads, er, two times – ой!

Venona project

Article Talk

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## To Recap

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  - Saw various formulations of perfect secrecy
- Saw first formal proof (OTP is perfectly secure)

# Next Lecture

- Shannon's impossibility: $|\mathcal{K}| \geq |\mathcal{M}|$ for any perfectly-secure SKE
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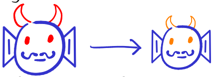
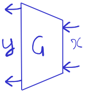
- Shannon's impossibility: $|\mathcal{K}| \geq |\mathcal{M}|$ for any perfectly-secure SKE
- What do you do in face of Shannon's impossibility?
- You compromise.
    - Kerckhoffs' principle
        - *'The system should be, if not theoretically unbreakable, unbreakable in practice.'*

# Next Lecture

- Shannon's impossibility: $|\mathcal{K}| \geq |\mathcal{M}|$ for any perfectly-secure SKE
- What do you do in face of Shannon's impossibility?
- You compromise.
  - Kerckhoffs' principle
    - *'The system should be, if not theoretically unbreakable, unbreakable in practice.'*
  - Restrict to *computationally-bounded* Eve

  - How to model computationally-bounded adversaries?
  - Pseudo-random generators (PRG)



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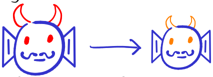
  - Restrict to *computationally-bounded* Eve 
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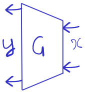


More Questions?

# References

1. [KL14, Chapters 1 and 2] for details about this lecture
2. Shannon's paper on perfect secrecy and proof of perfect secrecy one-time pad: [Sha49]
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