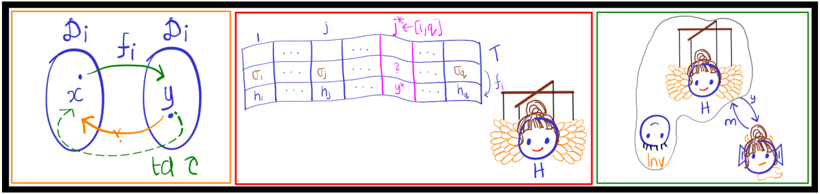

CS783: Theoretical Foundations of Cryptography

Lecture 13 (13/Sep/24)

Instructor: Chethan Kamath

Recall from Last Lecture

- Sub-task 5.a: domain-extension of digital signature/MAC
- Reduces to constructing collision-resistant hash functions

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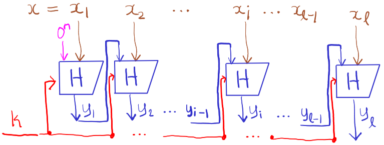
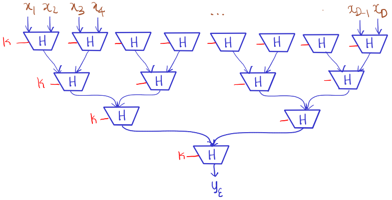
  - Domain extension for compression functions
    - Merkle-Damgård transform
    - Merkle trees





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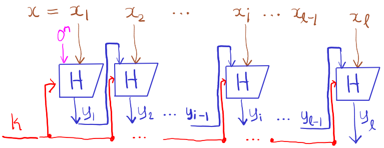

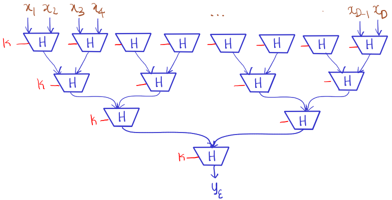

  - Some constructions:
    - Practical/unkeyed: SHA2, MD5
    - Theoretical/keyed: DLog- and SIS-based

Plan for Today's Lecture...

- Motivation: construct efficient signatures

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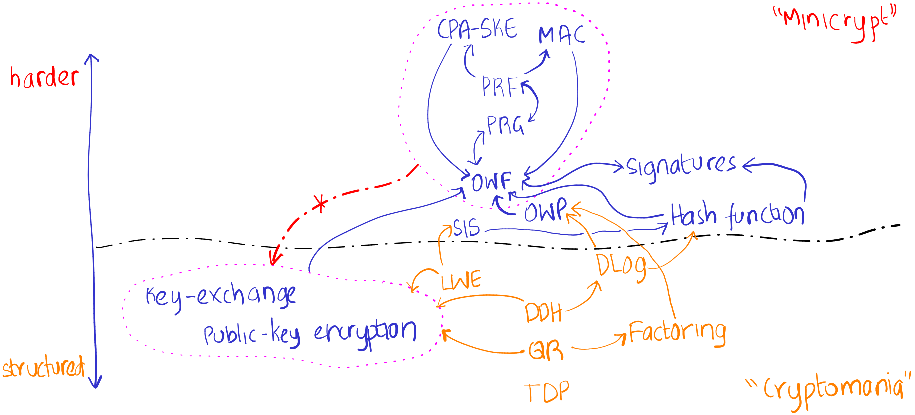

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  - Efficient digital signatures from TDP

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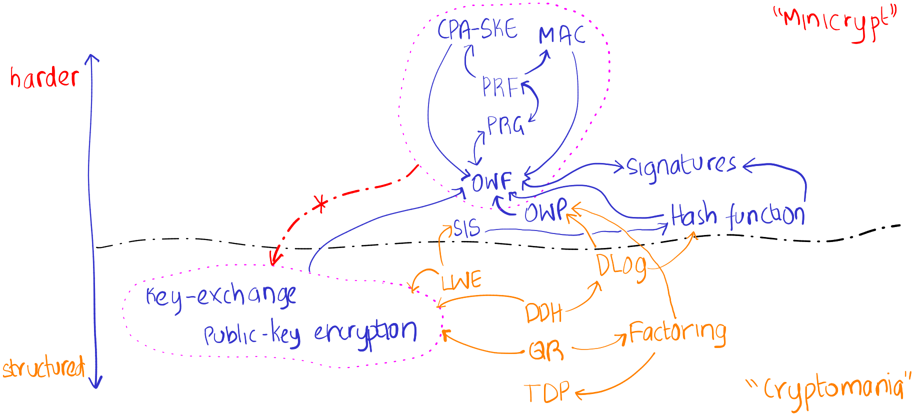

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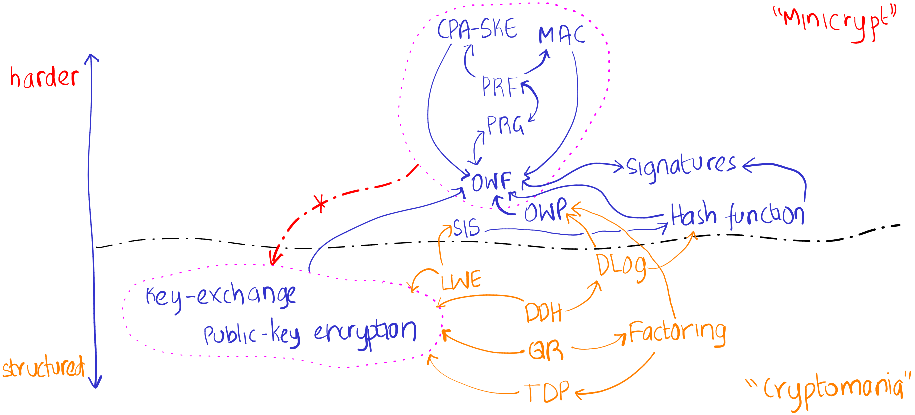
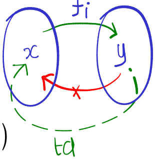


- New primitive: trap-door (one-way) permutation (TDP)
  - Efficient digital signatures from TDP
  - PKE from TDP

Plan for Today's Lecture...

1. Trap-Door (One-Way) Permutation (TDP)



2. Efficient Digital Signatures from TDP (in Random-Oracle Model)

3. Public-Key Encryption from TDP

Plan for Today's Lecture

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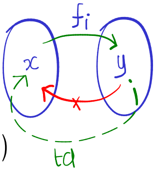


2 Efficient Digital Signatures from TDP (in Random-Oracle Model)

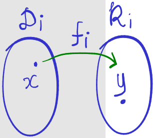
3 Public-Key Encryption from TDP

Recall from Lecture 6: *Collection* of OWFs

Defintion 1 (One-way function (OWF) collection)

A collection of functions $\mathsf{f} := \{\mathsf{f}_i : \mathcal{D}_i \to \mathcal{R}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ *is one-way if*

1. *There is an efficient index-sampling algorithm* Index
2. *Each* f_i *in collection is efficiently computable*



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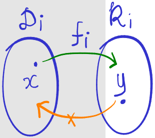

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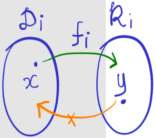

- Recall examples:
  1. Squaring modulo composite $N = pq$: $\mathsf{f}_N(x) := x^2 \bmod N$
  2. Exp. with generator g modulo prime p: $\mathsf{f}_{p,g}(x) := g^x \bmod p$

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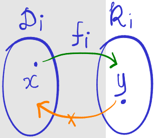

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? Describe $\mathcal{I}$, $\mathcal{D}_i$ and $\mathcal{R}_i$ above. How is $i \in \mathcal{I}$ sampled?

OWP Collection *with Trap-Door*

Defintion 2 (Trapdoor (one-way) permutation (TDP) collection)

A collection of permutations $\mathsf{f} = \{\mathsf{f}_i : \mathcal{D}_i \to \mathcal{D}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ *is trapdoor one-way if*

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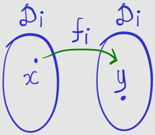

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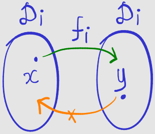

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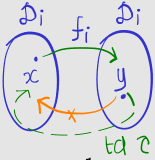

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4. f_i^{-1} *can be efficiently computed given trapdoor* τ *for* i

Candidate TDPs

- RSA TDP $\left\{f_{N,e} : \mathbb{Z}_N^\times \to \mathbb{Z}_N^\times\right\}_{N,e}$, defined as

$$f_{N,e}(x) := x^e \bmod N$$

N=pq for primes pq

- $f_{N,e}$ is permutation when $GCD(e,(p-1)(q-1)) = 1$
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Exercise 1

How can we compute $f_N^{-1}(y)$ given (p,q)?

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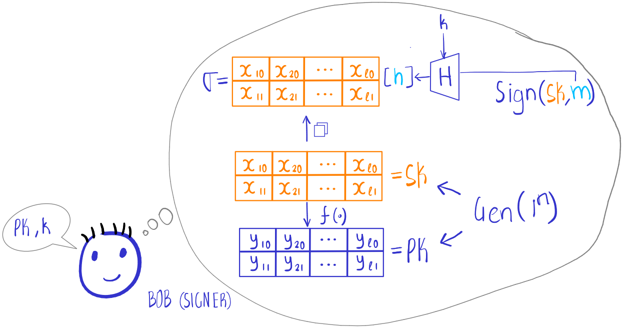

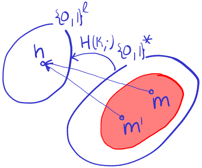

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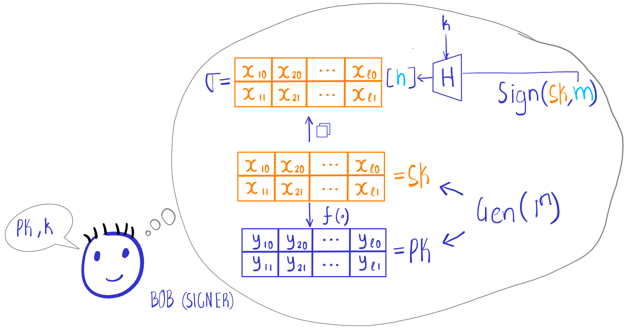

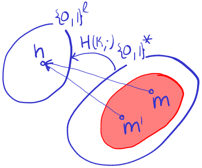

Theorem 1 (Theorem 3, Lecture 12 (rephrased))

If Lamport's scheme is OTS and H *is CRHF then "hash-then-sign" scheme is a one-time EU-CMA for arbitrarily-long messages.*

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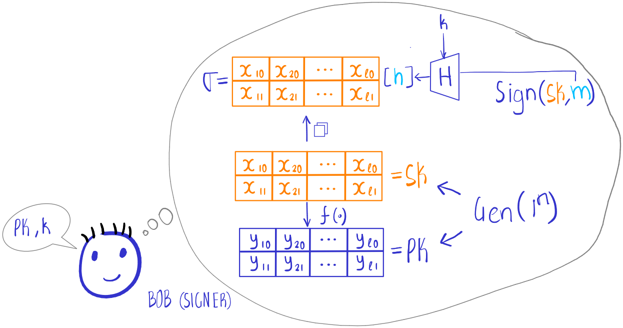

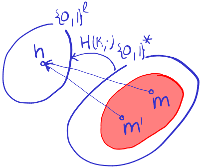

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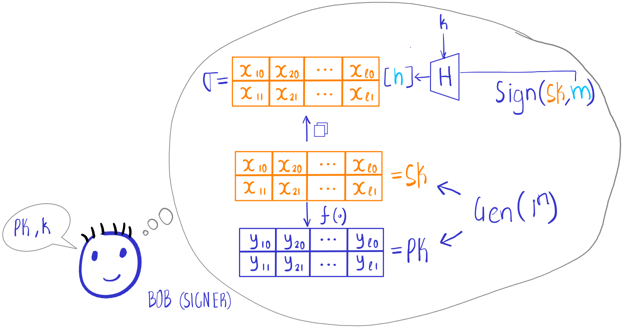

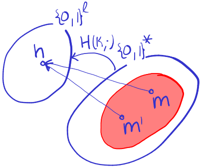

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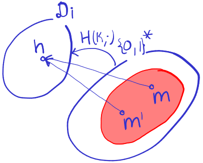
- How can a TDP be useful here? To replace Lamport's OTS

TDP-based Signature: "Hash-then-Invert"

- 1) Compute "hash" $h = \mathsf{H}(\mathsf{k}, m)$ 2) invert h using trapdoor
  - "Full domain" hash function $\mathsf{H} : \mathcal{K} \times \{0,1\}^* \to \mathcal{D}_i$



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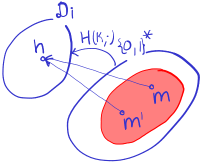

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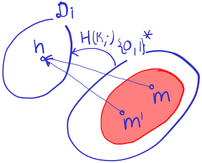

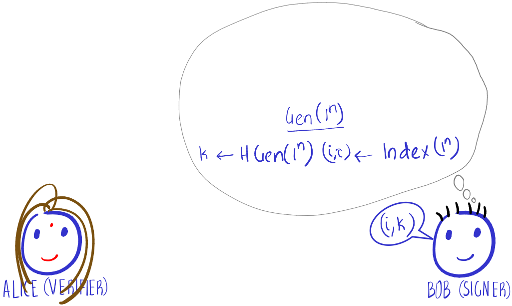

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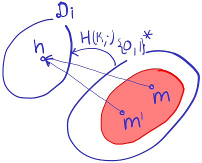

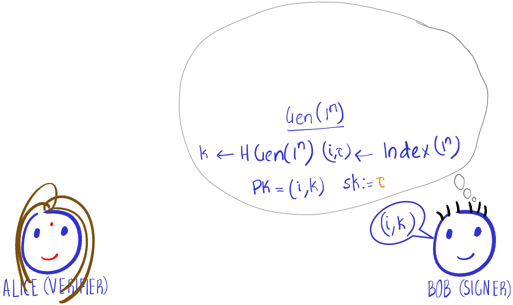

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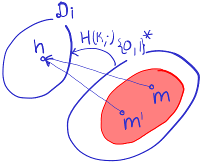

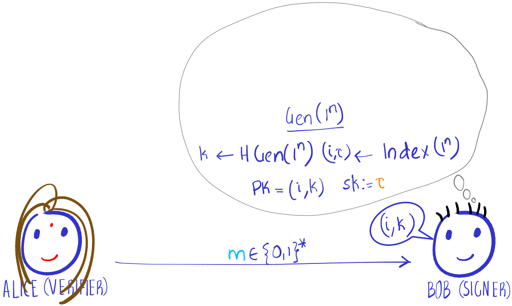

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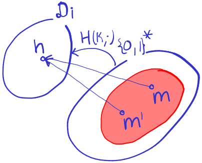

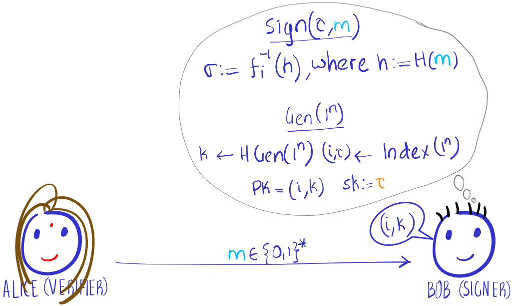

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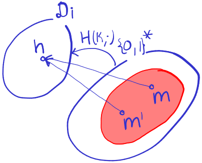

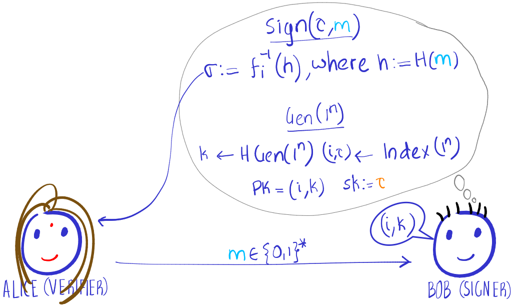

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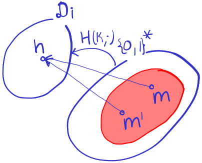

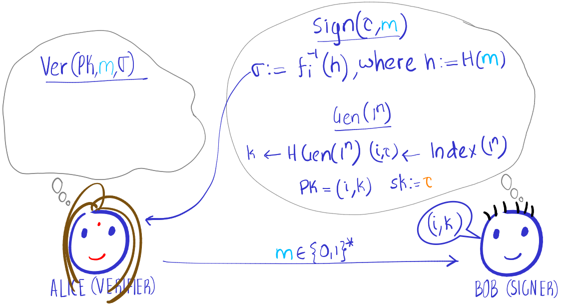

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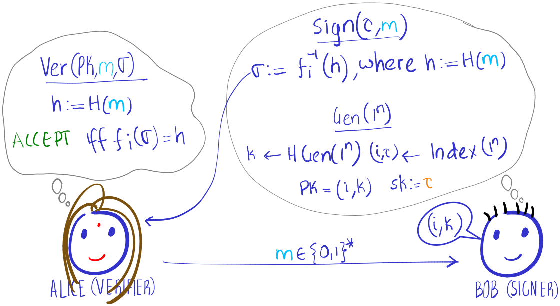

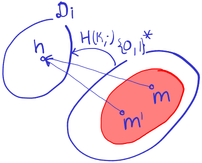

TDP-based Signature: "Hash-then-Invert"

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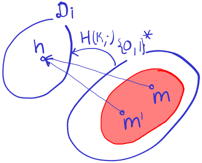

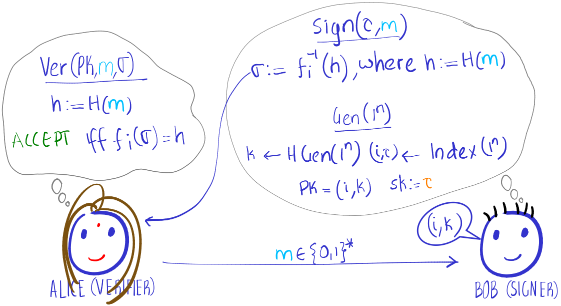

- Efficiency, when using RSA TDP (i.e., RSA-FDH)

$$\mathsf{f}_{N,e}(x) := x^e \bmod N$$

  - Public key: (N, e) and description of H
  - Signatures: one element of $\mathbb{Z}_N^\times$
  - Signing/verification: one exponentiation + hash evaluation

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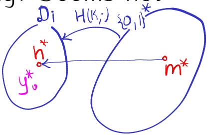

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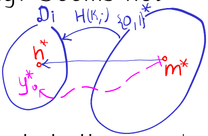

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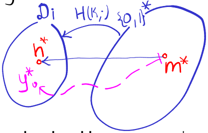

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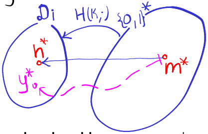

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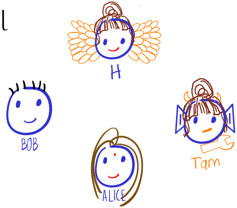

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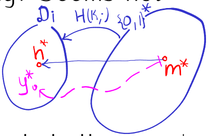

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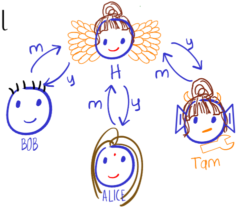

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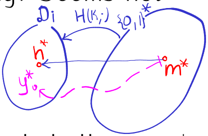

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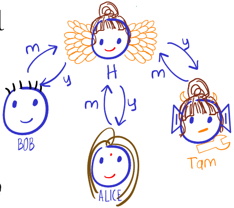

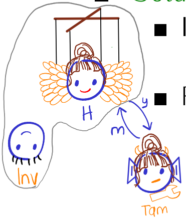

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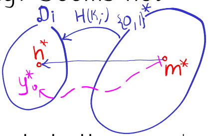

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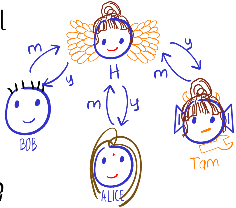

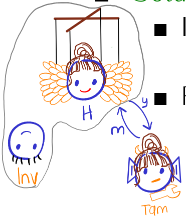

- Warning:
  - Only heuristic security guarantee
  - Adversary could exploit specific implementation of H

Let's Prove Security of "Hash-then-Invert"...

Theorem 2

If f is a TDP and $\boldsymbol{H}$ is a random oracle then "hash-then-invert" is EU-CMA for arbitrarily-long messages.

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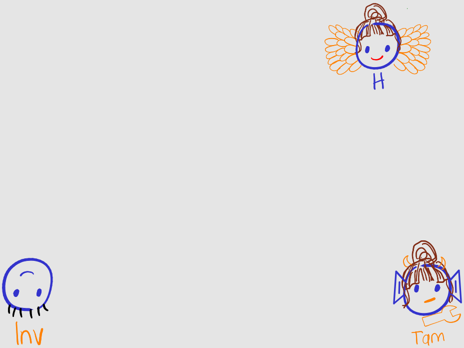

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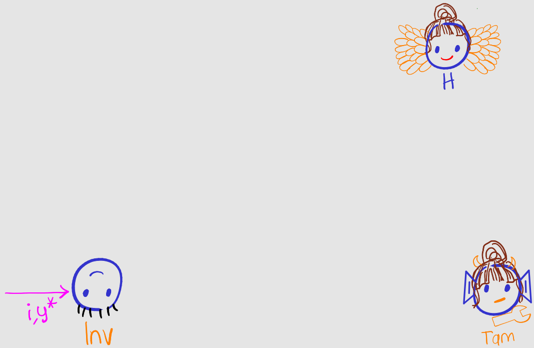

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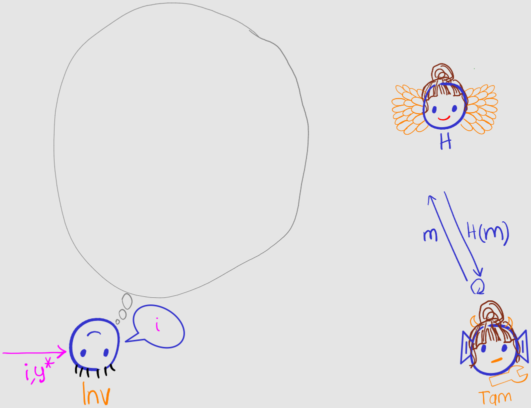

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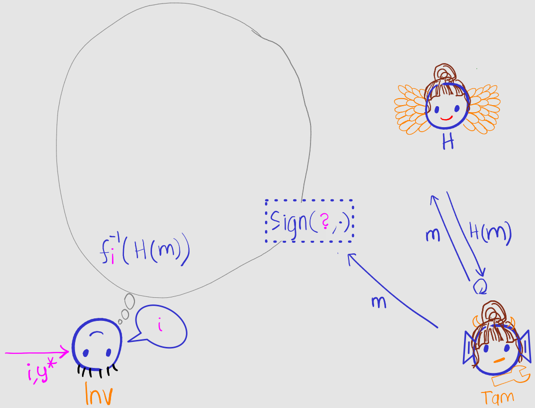

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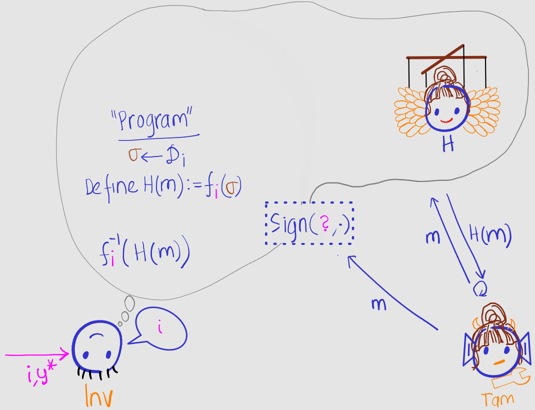

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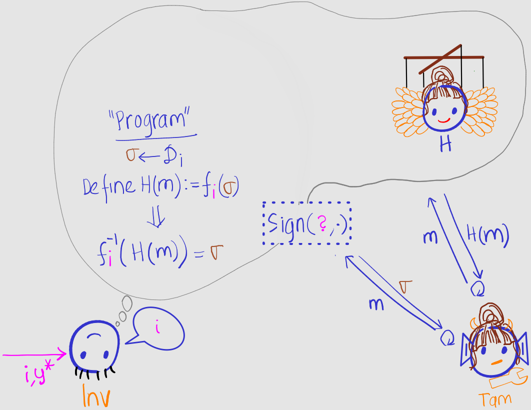

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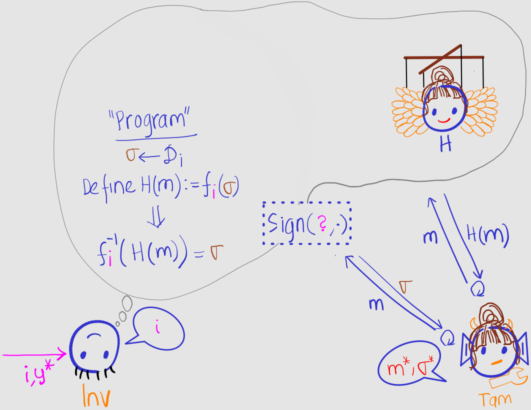

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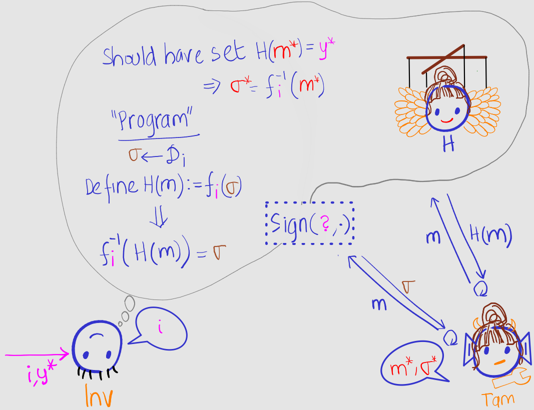

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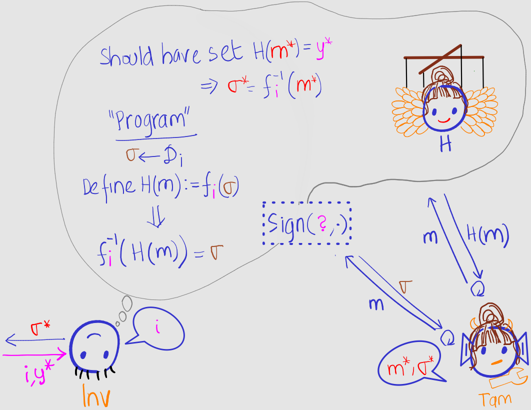

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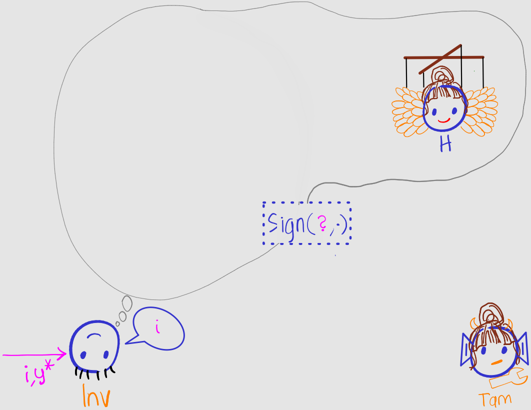

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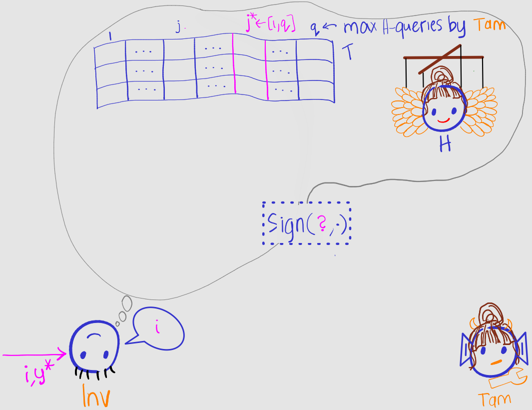

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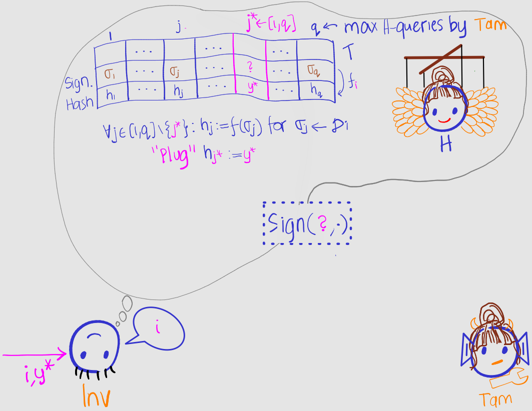

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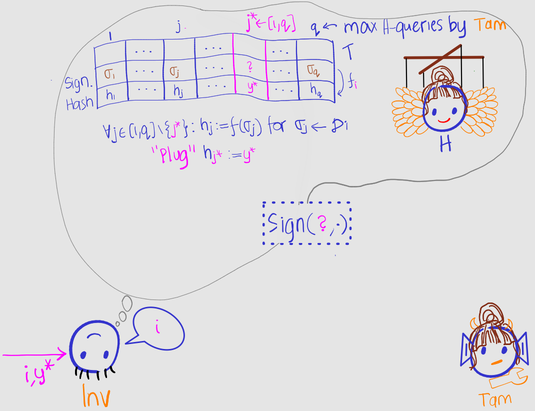

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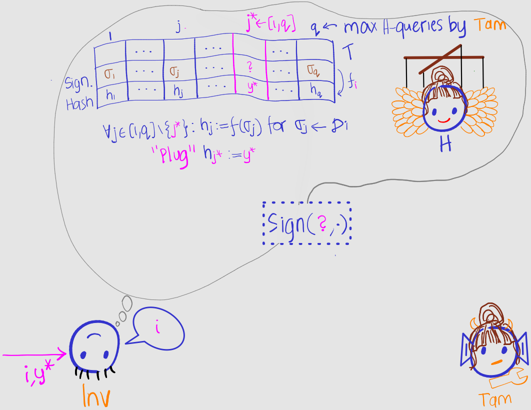

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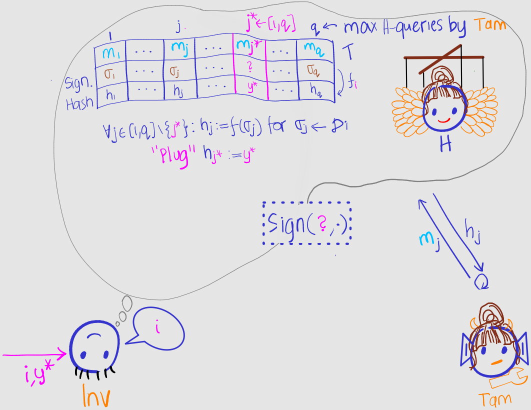

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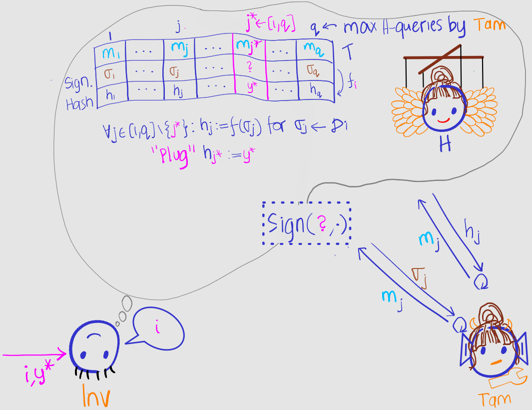

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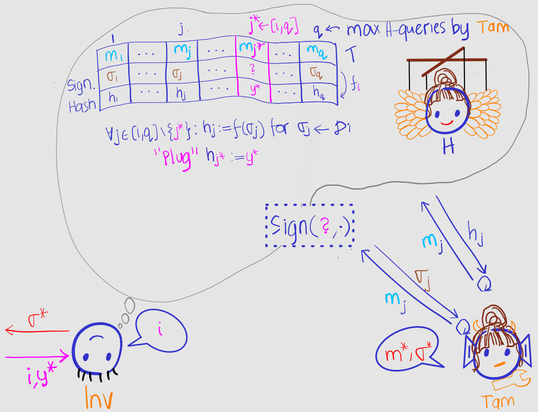

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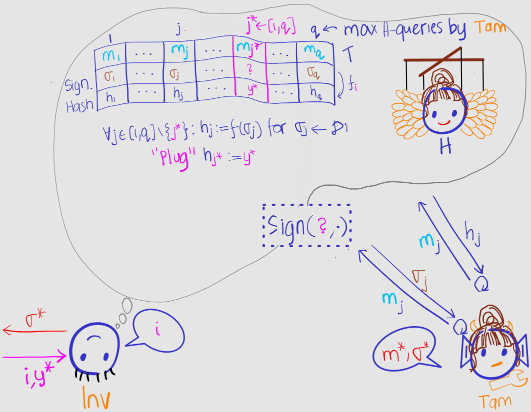

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Analysis: similar to Lamport's $\square$

1 Trap-Door (One-Way) Permutation (TDP)

2 Efficient Digital Signatures from TDP (in Random-Oracle Model)

3 Public-Key Encryption from TDP

Let's Construct CPA-Secure PKE from TDP...

- Recall from Lecture 6:

Defintion 3 (Definition 2, Lecture 6)

A predicate $\mathsf{hc} : \{0,1\}^n \to \{0,1\}$ *is hard-core for a function family* $\mathsf{f}_n : \{0,1\}^n \to \{0,1\}^m$, *if for every PPT predictor* P, *the following is negligible*

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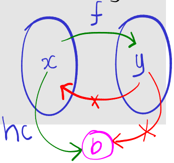

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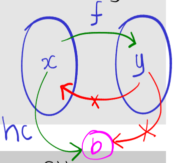

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For a OWP f*, let* $\mathsf{f}'(x,r) := (\mathsf{f}(x), r)$*. Then* $\mathsf{hc}(x,r) := \langle x, r \rangle_2$ *is a hard-core predicate for* f'*.*

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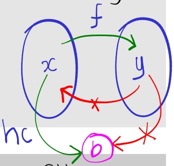

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Exercise 2

Extend Goldreich-Levin theorem for TDP $\mathsf{f} = \{\mathsf{f}_i : \mathcal{D}_i \to \mathcal{D}_i\}_{i \in \mathcal{I}}$

Let's Construct CPA-Secure PKE from TDP...

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Construction 1 (PKE $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec}) \leftarrow$ TDP $\mathsf{f}_i : \mathcal{D}_i \to \mathcal{D}_i$)

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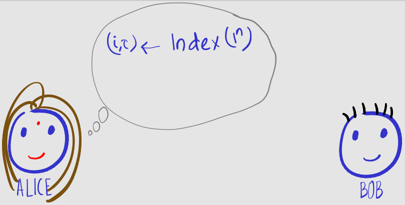

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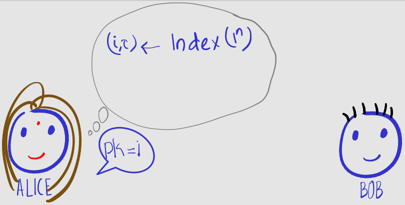

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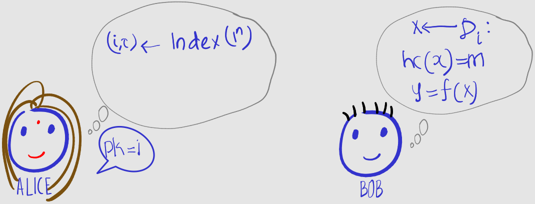

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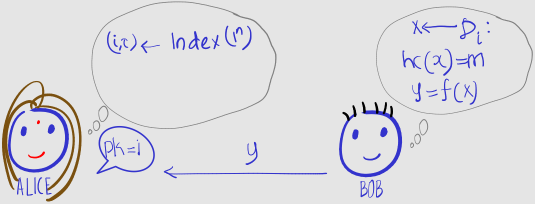

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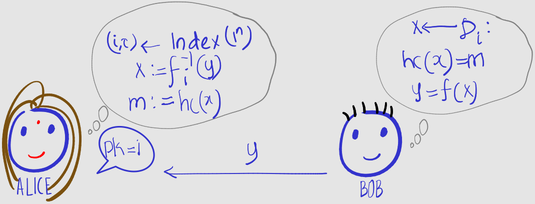

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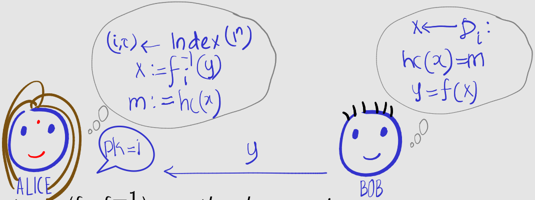

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- *Bob: to encrypt a bit* m*, sample* $x \leftarrow \mathcal{D}_i$ *such that* $\mathsf{hc}(x) = m$ *and send* $y = \mathsf{f}_i'(x)$ *as "hint"*
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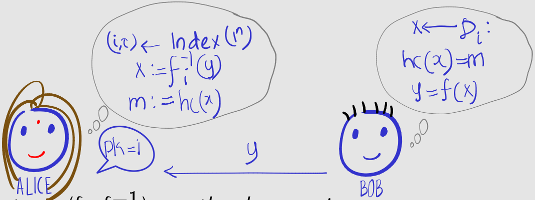

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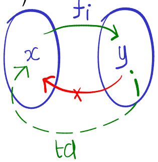
Theorem 4

If f' *is a TDP then* Π *is IND-CPA secure.*

To Recap Today's Lecture

- Introduced a new primitive: trap-door permutation (TDP)
  - Motivation: efficient signature schemes



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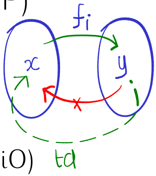

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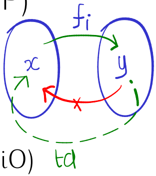

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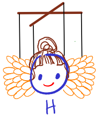

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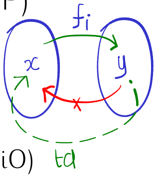

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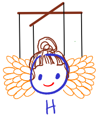
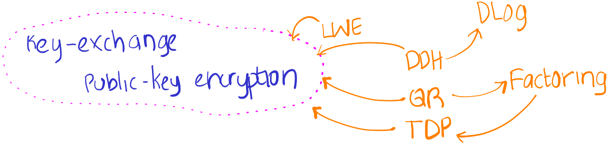


- PKE from TDP:
  - New PKE based on RSA assumption
    - Not same as (textbook) RSA encryption

- We learnt: secure communication in the public-key setting
- Cryptographic primitives encountered: key-exchange, public-key encryption, signature, hash function, TDP
- Hardness assumptions: Factoring, DLog, QR, LWE, RSA



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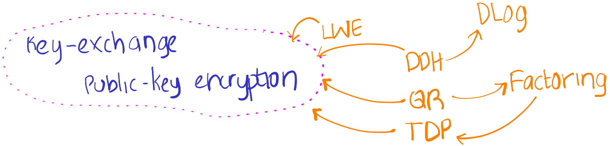
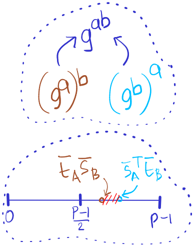


- Key conceptual takeaway: structure vs. hardness



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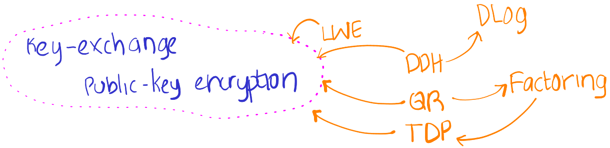

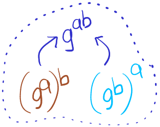

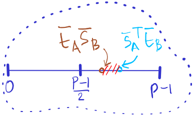

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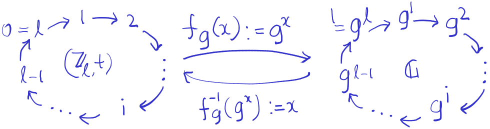
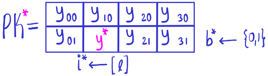




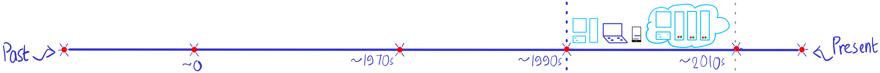
- Key tools: groups, random self-reducibility, plug and pray

MODULE 3
(Secure comp.)

- Zero-knowledge proofs
- Private computation of functions (MPC)
- Private outsourcing: fully-homomorphic encryption
- Verifiable outsourcing: SNARGs

Ubiquity of computing

**



References

1 [KL14, §13.3 and §15.1] for details of this lecture.
2 Trap-door permutations were introduced in [DH76]. Yao [Yao82] who showed how to construct PKE using TDPs.
3 The random oracle model was proposed in [FS87]. But it was in [BR93] that it was shown how it can be fully exploited. E.g., the random-oracle-based of "hash-then-invert" construction is from there.

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