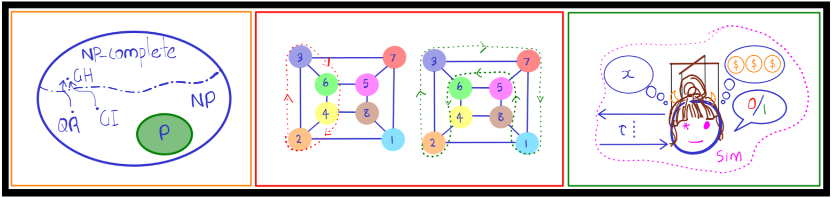

CS783: Theoretical Foundations of Cryptography

Lecture 15 (01/Oct/24)

Instructor: Chethan Kamath

Recall from Last Lecture...

- *Interactive* proof (IP)
  - Compared to traditional "NP" proof
  - IP is powerful: IP for GNI

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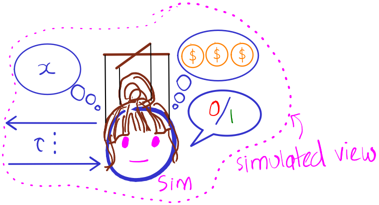

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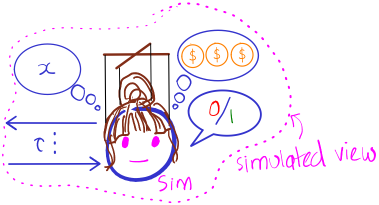

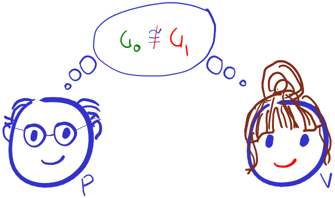

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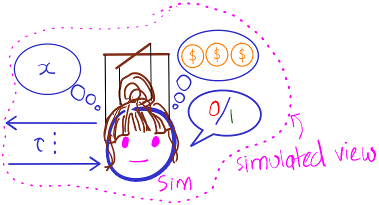

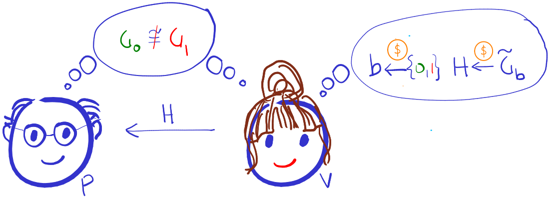

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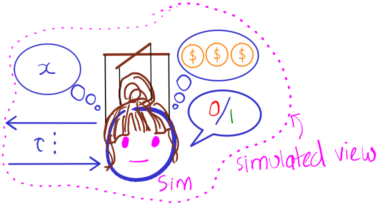

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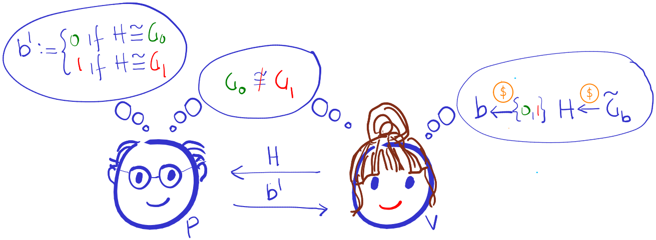

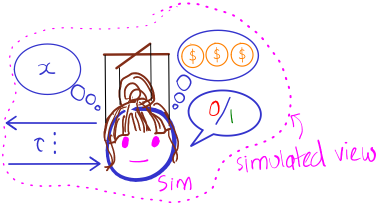

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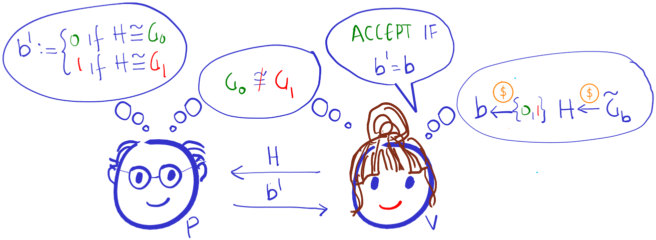

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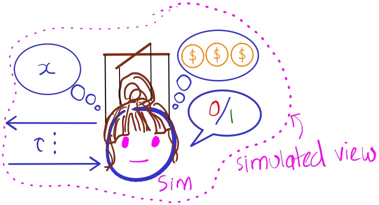

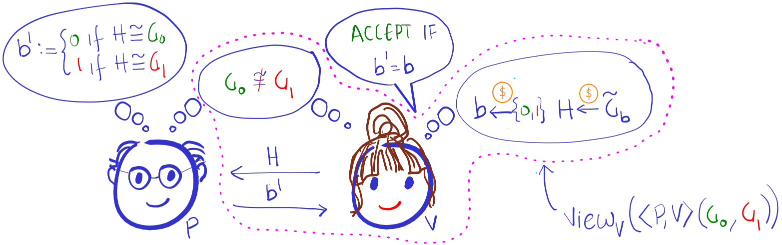

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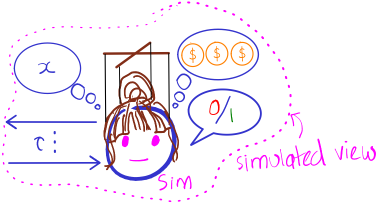

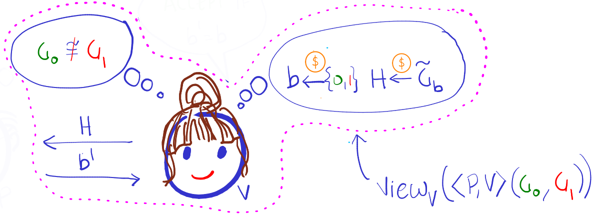

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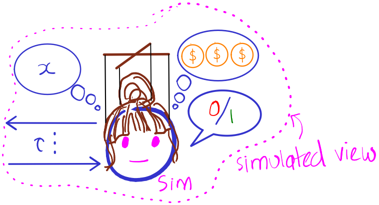

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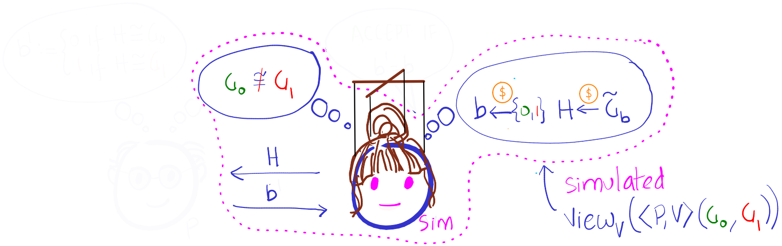

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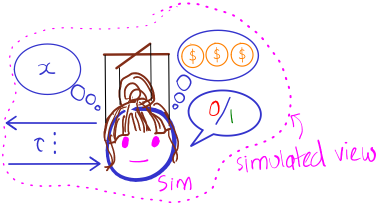

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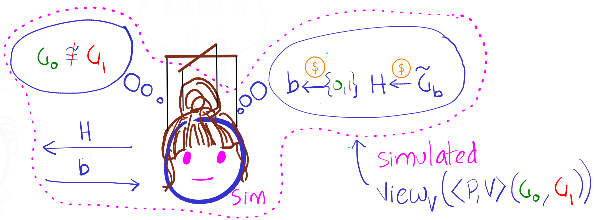
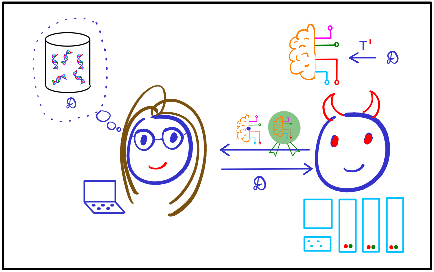


- *Honest-verifier* ZKP for **GI** (Exercise 4: **QR**)

(ZK)IPs are Useful!

- Applications of IP: Verifiable outsourcing



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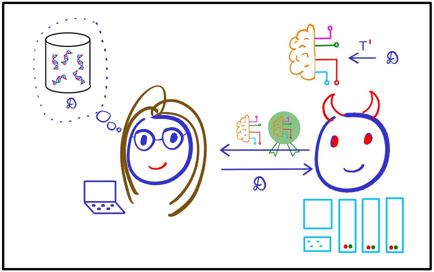

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  - Digital signatures: next lecture

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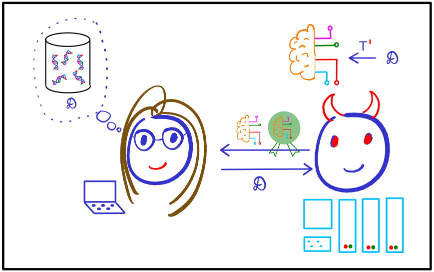

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  - NIST is currently standardising ZKP (projects/pec/zkproof)

Plan for Today's Lecture...

- *Malicious-verifier* ZKP for GI

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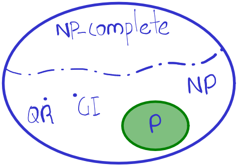

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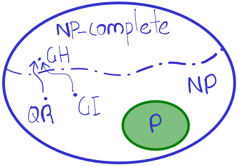
- *Malicious-verifier* ZKP for **GI**
- ZKP for all of **NP**
  - Blum's protocol for Graph Hamiltonicity (**GH**)
    - Given a graph G, decide whether it has a Hamiltonian cycle



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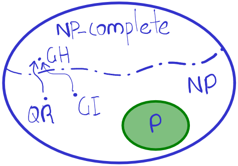

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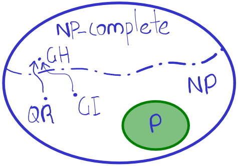

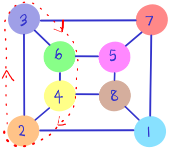

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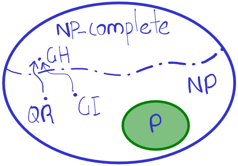

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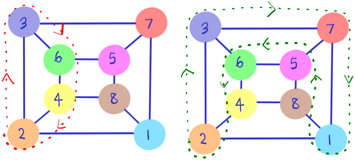

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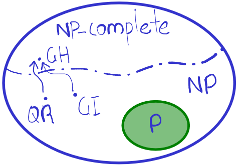

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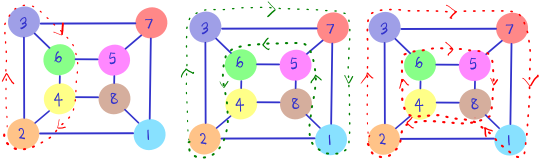

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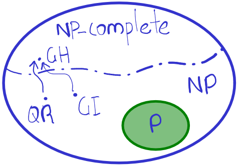

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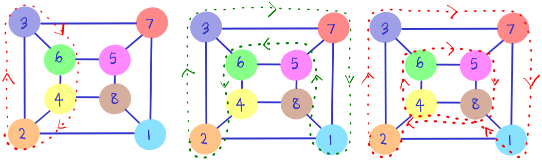

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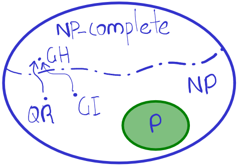

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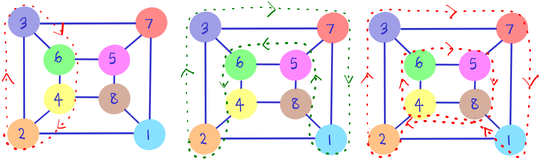

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- Commitment scheme

  - Digital analogues of lockers
  - OWP $\rightarrow$ (non-interactive) commitment scheme

Plan for Today's Lecture

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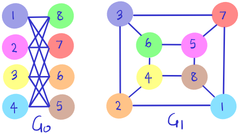

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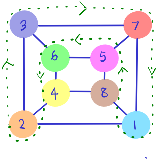

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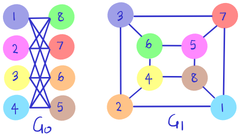

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3 Commitment Scheme

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Observation: transitivity of isomorphism

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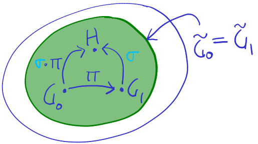

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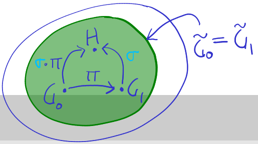

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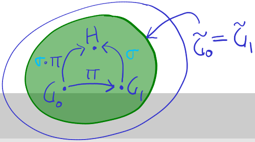

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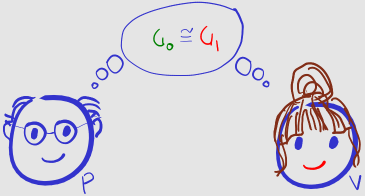

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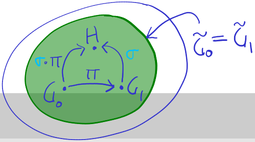

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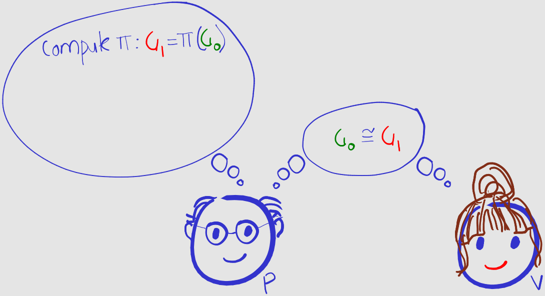

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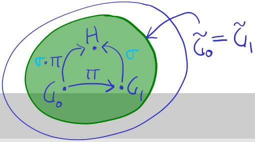

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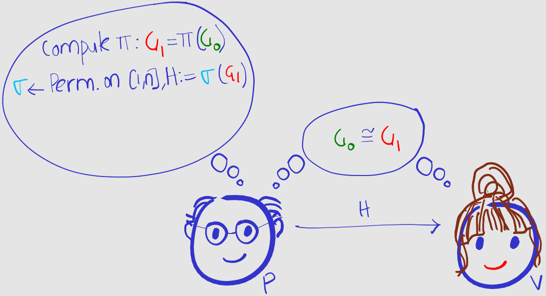

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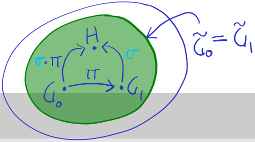

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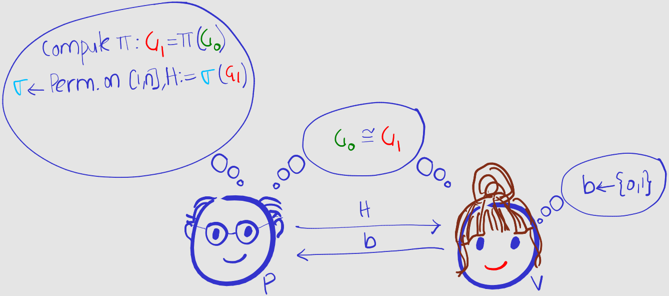

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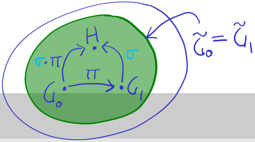

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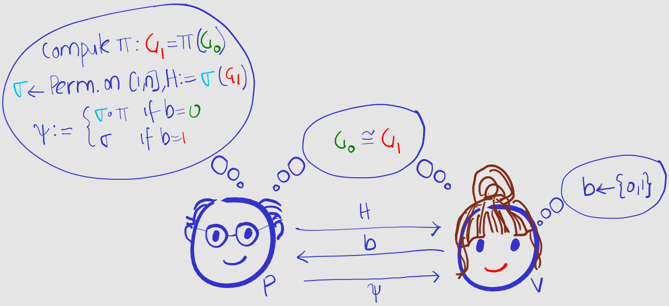

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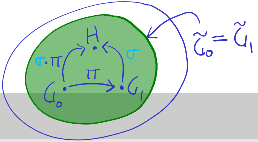
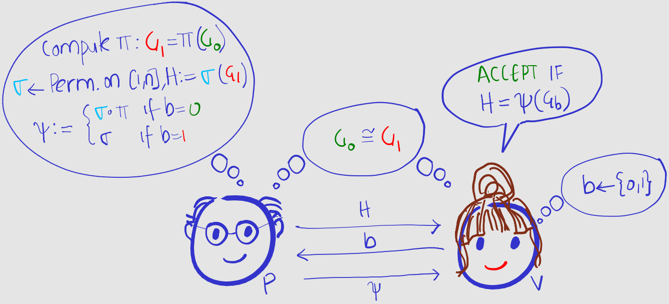


Protocol 1 ($\Pi_{\mathsf{GI}} = (\mathsf{P}, \mathsf{V})$: IP for GI)

1. P *"commits" by sending a random* H *s.t.* $G_1 \cong H$
2. *For* $b \leftarrow \{0,1\}$, V *challenges* P *to "reveal"* $G_b \cong H$
3. V *accepts if the revealed permutation is valid*



Recall Π_{GI}: Honest-Verifier ZK for GI...

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- Completeness: $G_0 \cong G_1 \Rightarrow$ P can reveal on either challenge $\Rightarrow$ V always accepts $\Rightarrow \epsilon_c = 0$
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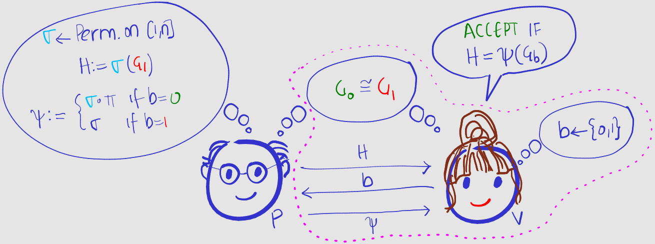

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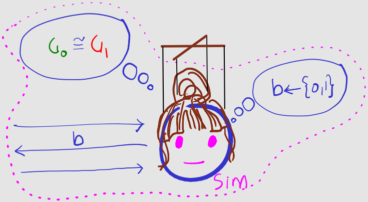

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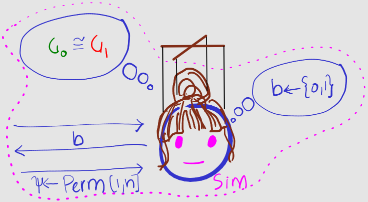

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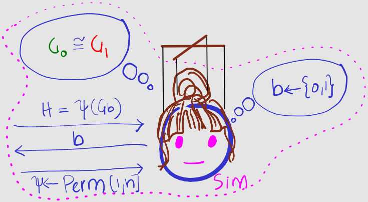

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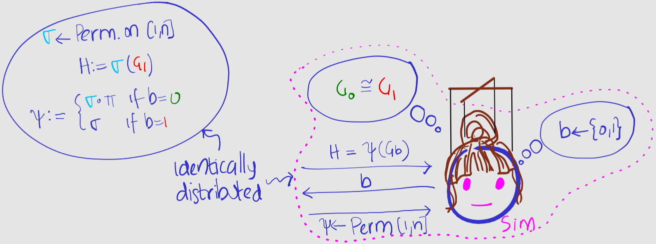

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Defintion 1 ((*Malicious-Verifier*) Perfect ZK)

An IP Π is perfect ZK for $\mathcal{L}$ if for every V^* there exists a PPT simulator $\mathsf{Sim}^{\mathsf{V}^*}$ such that for all distinguishers D and all $x \in \mathcal{L}$, the following is zero

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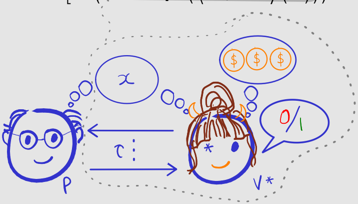

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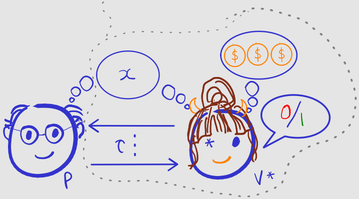

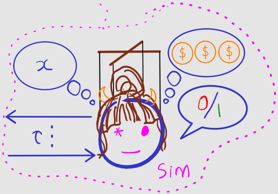

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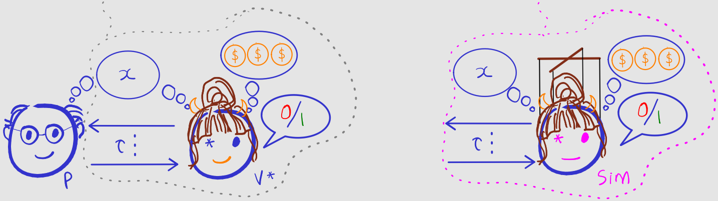

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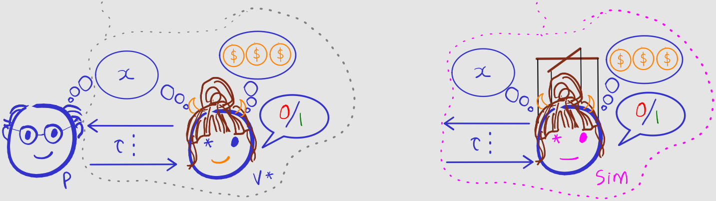

What happens if we use honest-verifier simulator Sim now?

- The distribution of b generated by V^* may not be uniform
- It could depend arbitrarily on P's message H

Π_{GI} Works Also For Malicious Verifiers!...

Just need a different Sim

Theorem 2

Π_{GI} is a malicious-verifier perfect ZKP for $\mathcal{L}_{\mathsf{GI}}$

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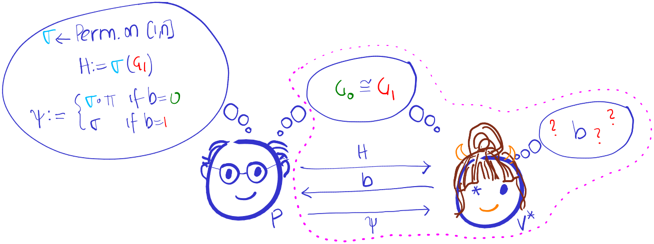

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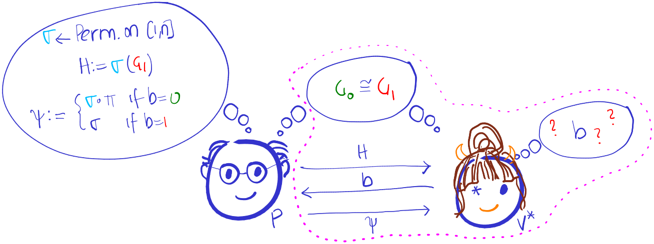

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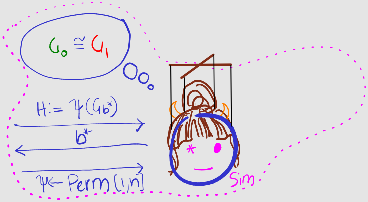
Proof (of ZK) Idea: **Sim** invokes V^*!



- New simulator $\mathsf{Sim}^{\mathsf{V}^*}$: repeat till required
  1. Sample random $H \overset{\psi}{\cong} G_{b^*}$ for $b^* \leftarrow \{0,1\}$
  2. Invoke V^* on H to obtain challenge b (with fresh random coins)
  3. If $b^* = b$ output $((G_0, G_1), (H, b, \psi))$ $\square$

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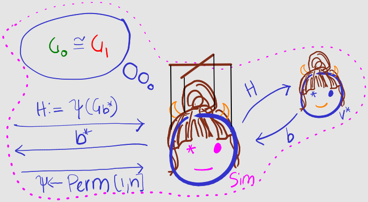

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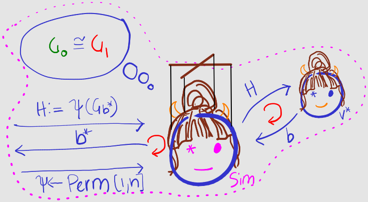

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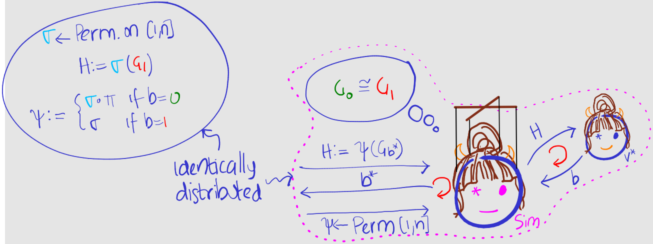

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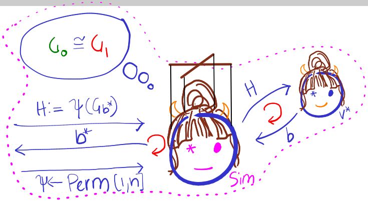

Why is b independent of b^*?

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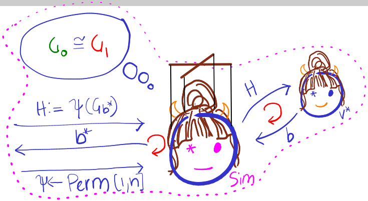

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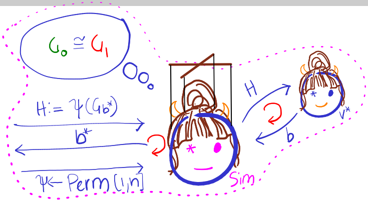

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  - In expectation: polynomial time
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Exercise 1

Can you come up with a strict PPT simulator?

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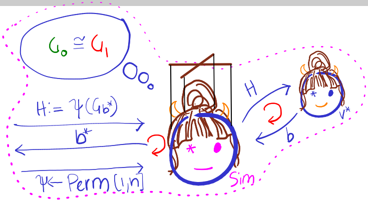

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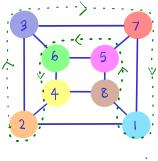
Can you come up with a strict PPT simulator?

Exercise 2

1. *Design malicious-verifier perfect ZKP for $\mathcal{L}_{\mathsf{QR}}$*
2. *Think about malicious-verifier perfect ZKP for $\mathcal{L}_{\mathsf{GNI}}$*
   - *Hint: you need to somehow use Π_{GI} as sub-routine*

Plan for Today's Lecture

1 Malicious-Verifier ZKP for Graph Isomorphism

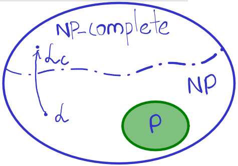
2 (Computational) ZKP for NP 

3 Commitment Scheme

ZKP for Any Problem in NP

Claim 1

ZKP for an NP*-complete language* $\mathcal{L}_c$ *implies ZKP for any* $\mathcal{L} \in$ NP



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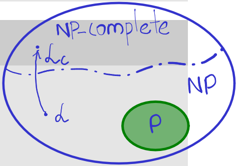

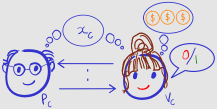

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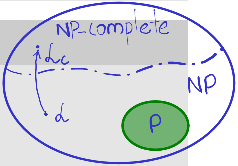

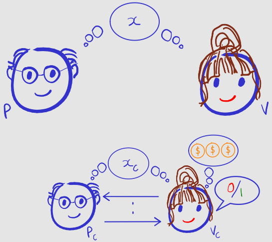

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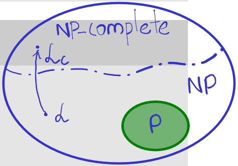

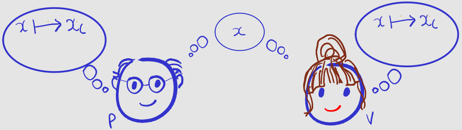

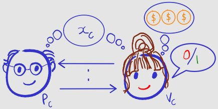

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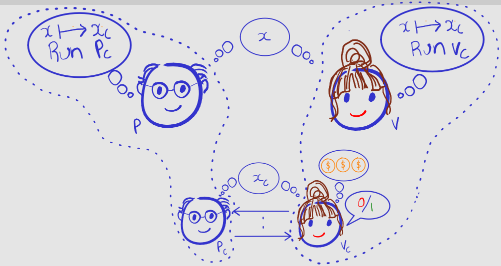

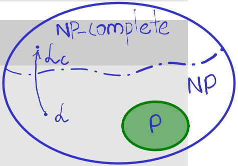

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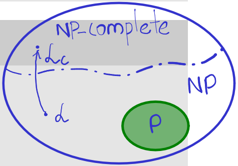

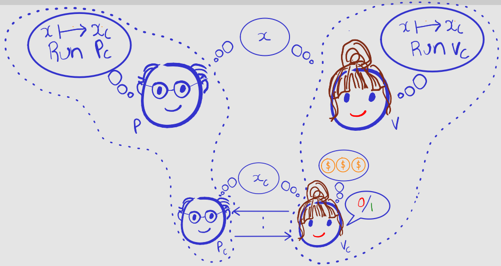

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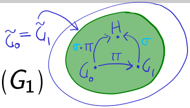
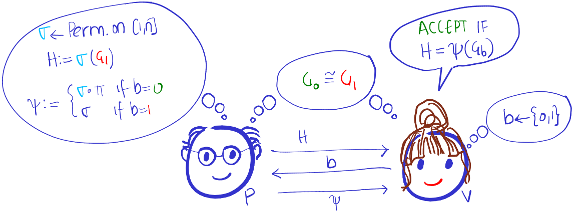
Exercise 3

Show that if Π_c *is a ZKP for* $\mathcal{L}_c$ *then* Π *is a ZKP for* $\mathcal{L}$

Let's Construct ZKP for Graph Hamiltonicity...

- Let's recall/rephrase Π_{GI}:
  - Honest P "commits" to G_0 *and* G_1 by sending $H = \sigma(G_1)$





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- Let's recall/rephrase Π_{GI}:
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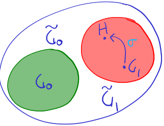
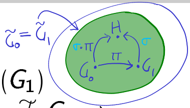
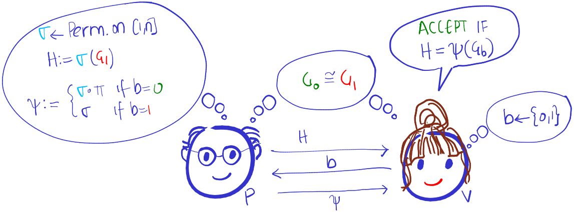
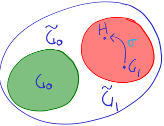


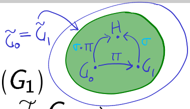




Let's Construct ZKP for Graph Hamiltonicity...

- Let's recall/rephrase Π_{GI}:





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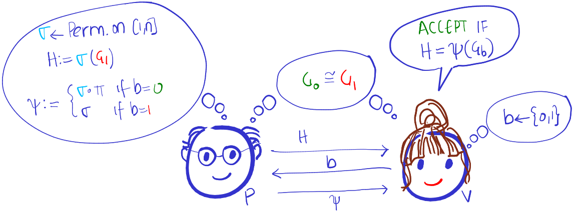

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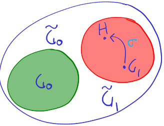

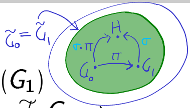

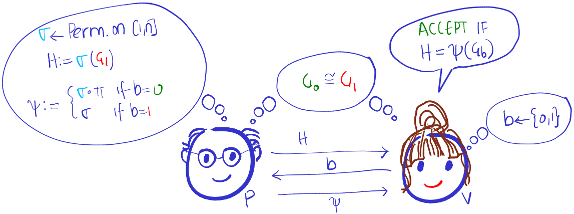

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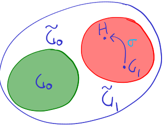
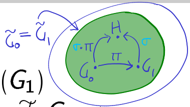
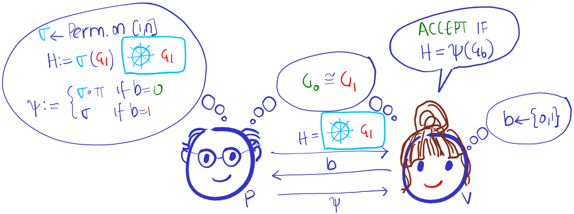


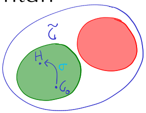




- Physical analogy: H acts as a secure "locker"
  1. *Hides* its contents from the verifier V
  2. *Binds* P^* by forcing it to store either G_0 or G_1 *before* seeing challenge b

Let's Construct ZKP for Graph Hamiltonicity... Using Lockers 

Observation: G Hamiltonian and $G \overset{\sigma}{\cong} H$ then H Hamiltonian



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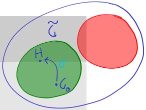

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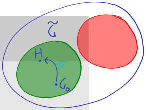

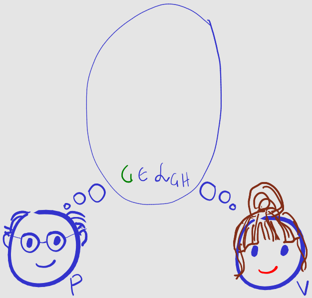

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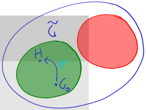

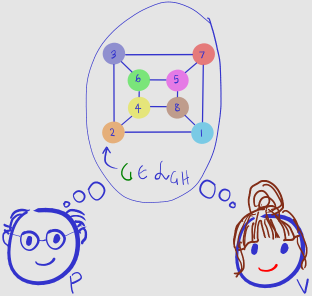

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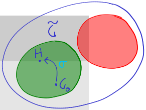

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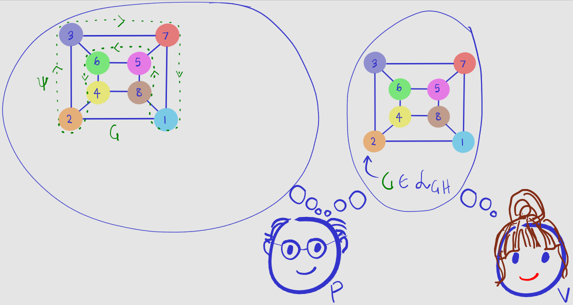

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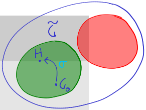

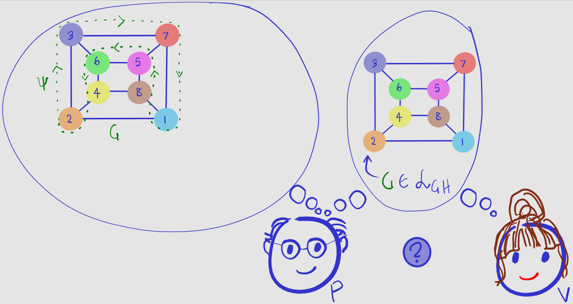

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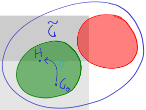

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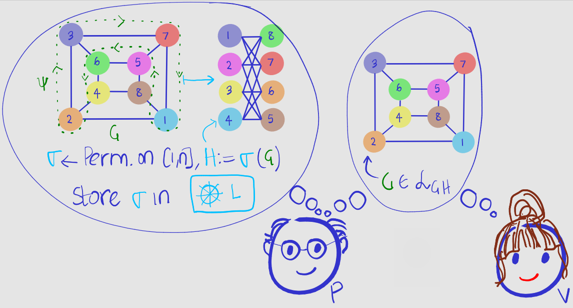

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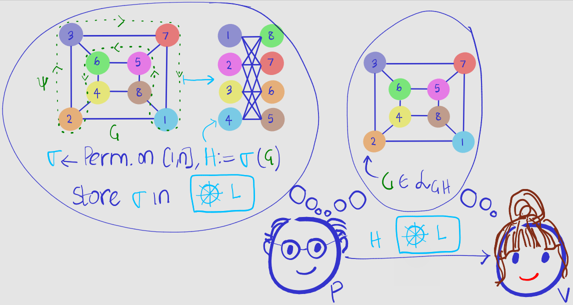

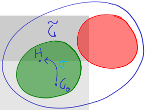

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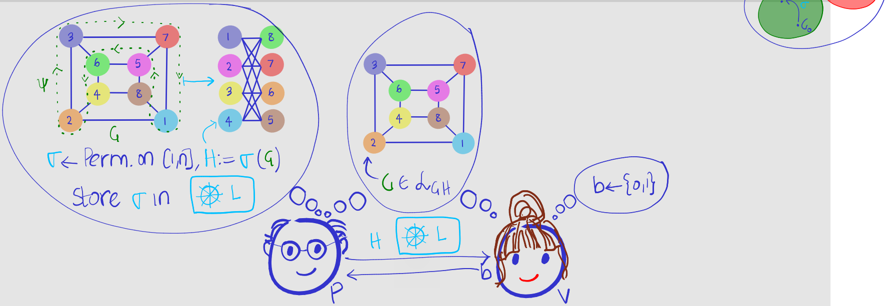

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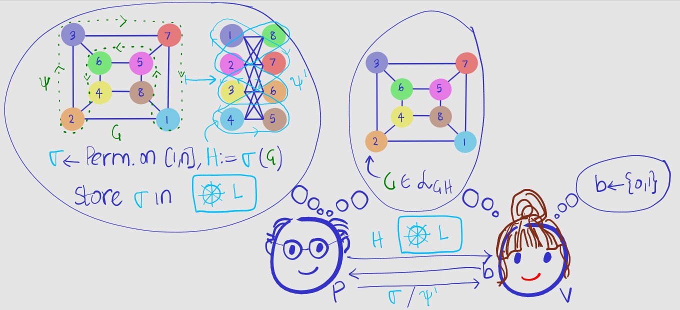

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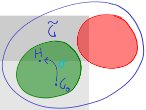

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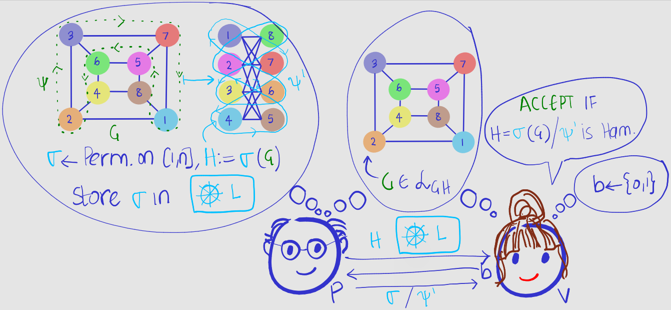

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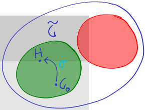

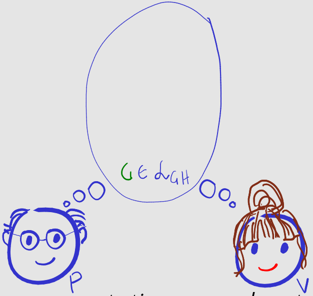

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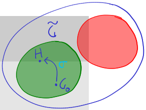

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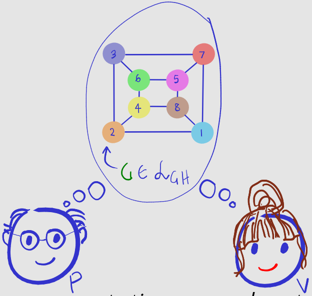

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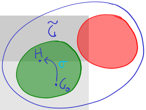

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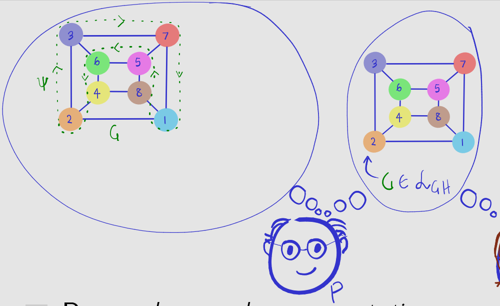

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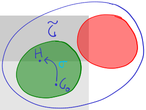

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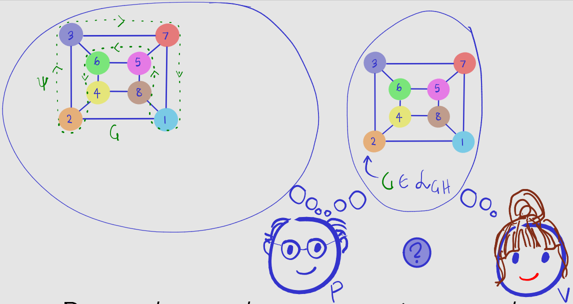

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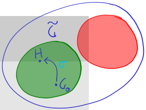

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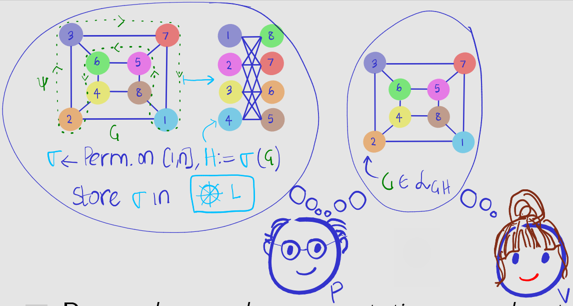

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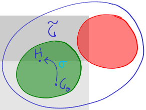

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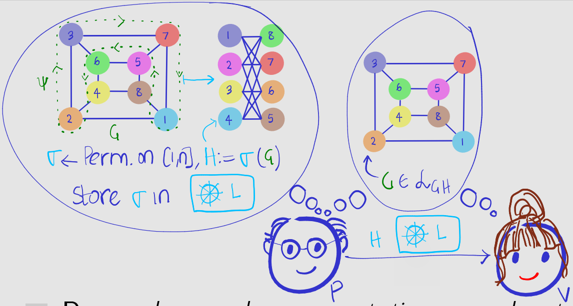

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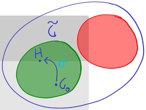

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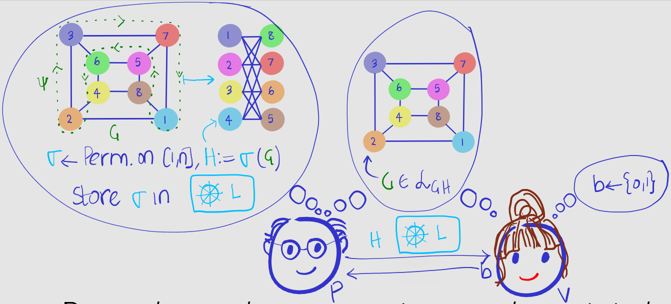

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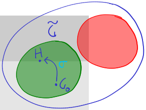

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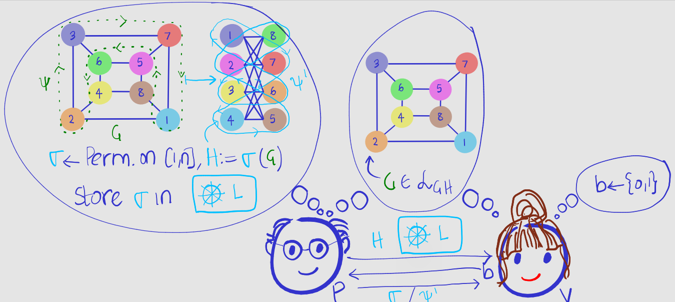

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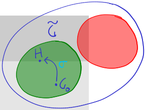

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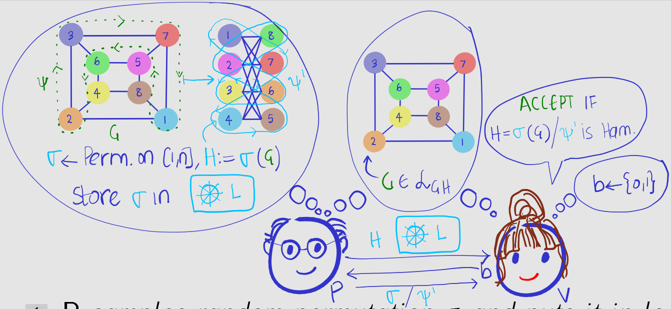

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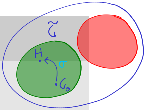

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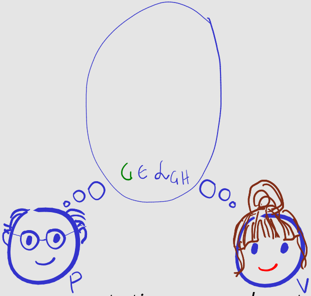

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- Problem: not clear if zero knowledge. How to simulate?

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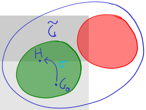

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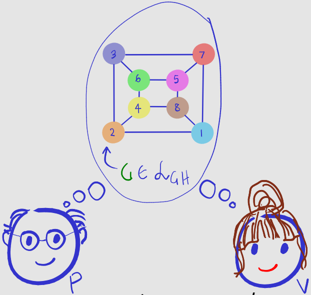

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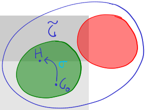

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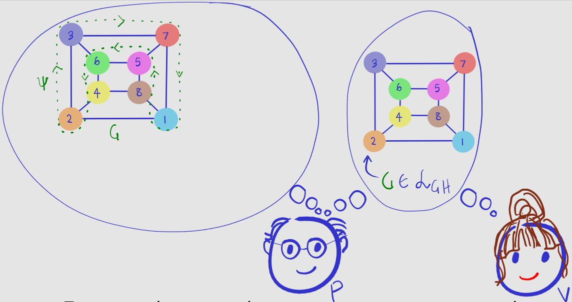

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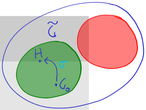

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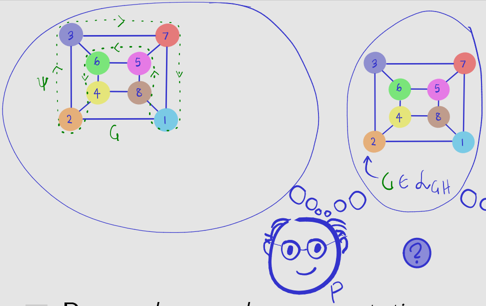

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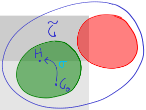

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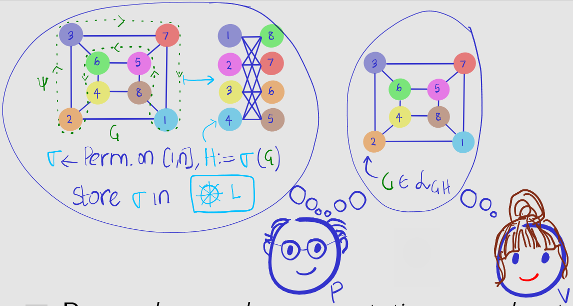

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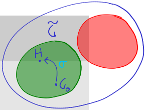

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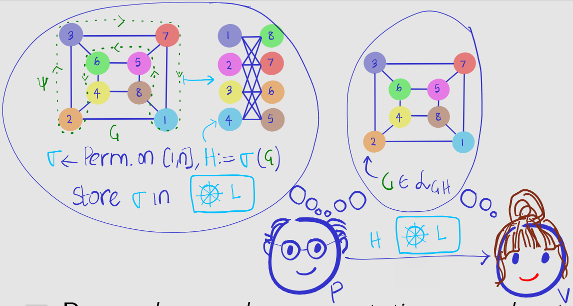

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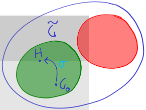

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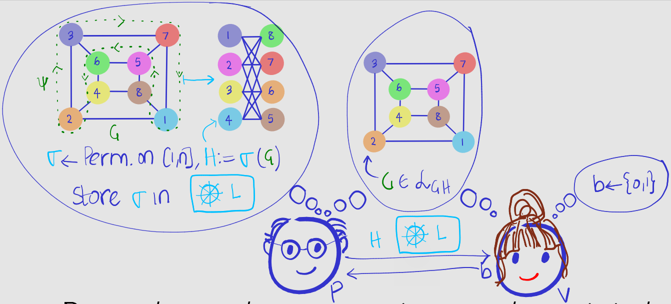

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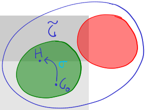

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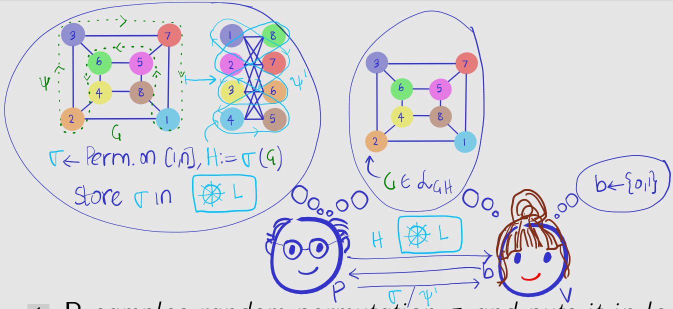

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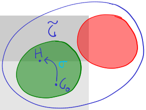

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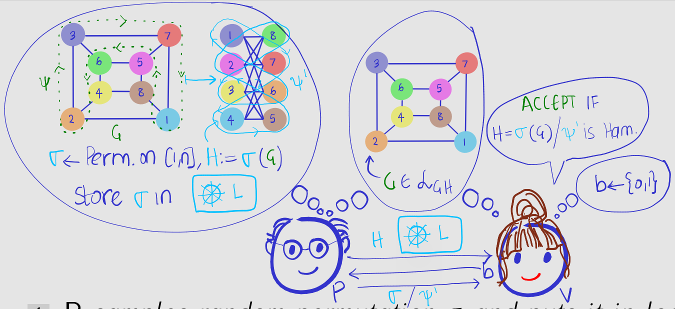

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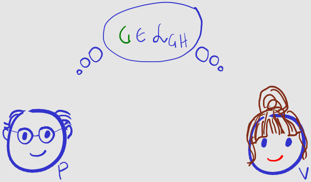

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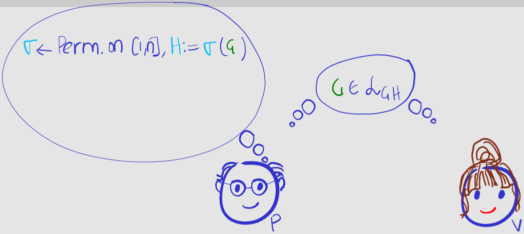

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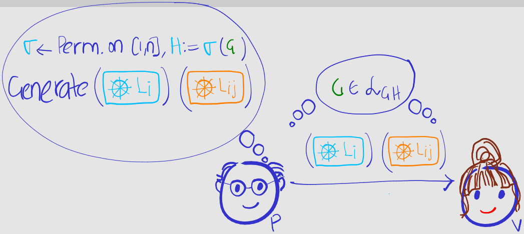

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- *Lockers $\left(L_{i,j}\right)_{(i,j)\in\binom{n}{2}}$ store H's adjacency matrix*

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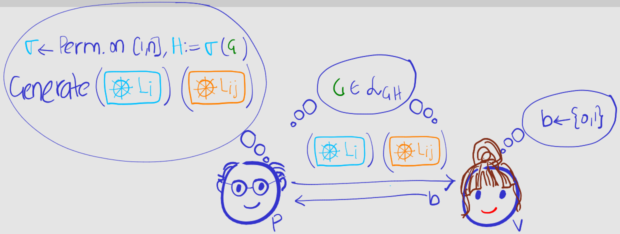

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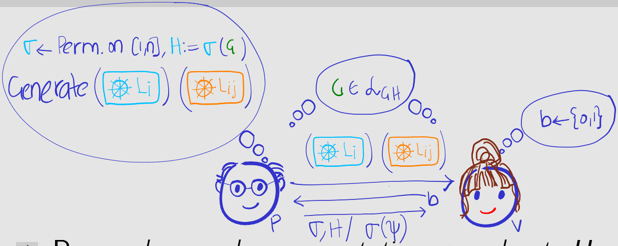

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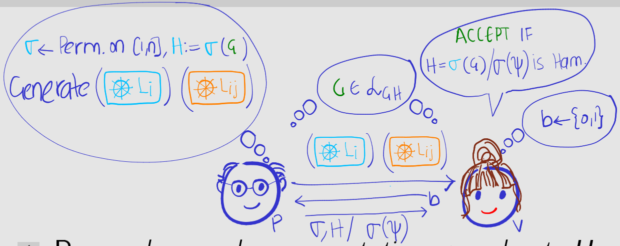

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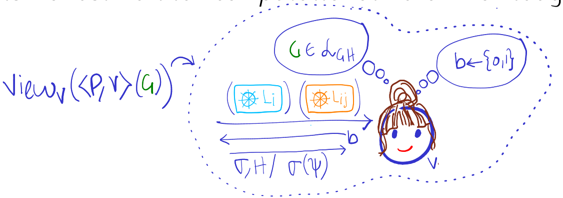
Π'_{GH} is Computational ZKP for Graph Hamiltonicity...

- Soundness: locker binding $\Rightarrow$ Π'_{GH} is sound
- Zero-knowledge: locker "computationally" hides its content $\Rightarrow$ Π'_{GH} is honest-verifier *computational* zero-knowledge for $\mathcal{L}_{\mathsf{GH}}$



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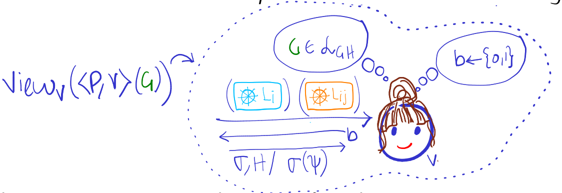

- Simulator: again, sample out of order
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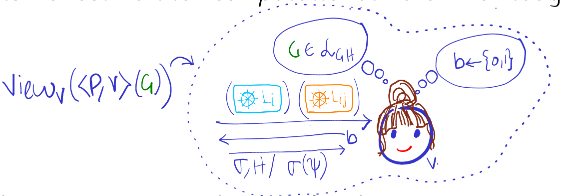

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  3. If $b = 1$
     - Sample random cycle C over $[1, n]$
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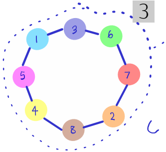


Π'_{GH} is Computational ZKP for Graph Hamiltonicity...

Exercise 4

Describe the simulator for malicious-verifier ZK for Π'_{GH}

Exercise 5

Think of ZKP for other NP*-complete problems like* $n \times n$ *Sudoku and graph three-colouring*

Plan for Today's Lecture

1 Malicious-Verifier ZKP for Graph Isomorphism

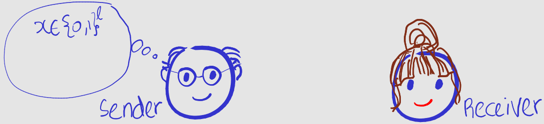
2 (Computational) ZKP for NP

3 Commitment Scheme

Commitment Schemes are Digital Lockers

Defintion 2

A (non-interactive) commitment scheme is a pair of algorithms (S, R) *with the following syntax:*



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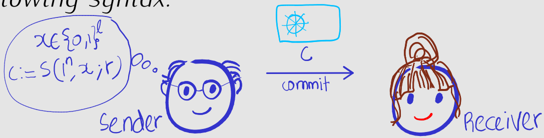

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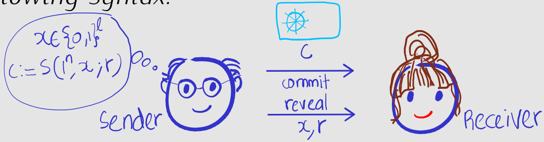

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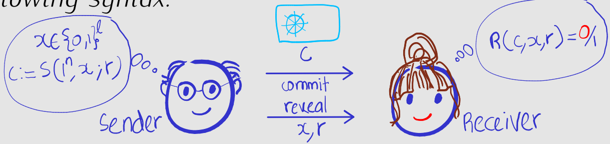

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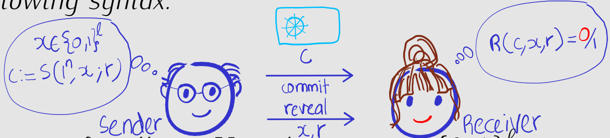

- *Correctness: for all* $n \in \mathbb{N}$ *and inputs* $x \in \{0,1\}^\ell$*:*

$$\Pr_r\left[R(S(1^n, x; r), x, r) = 1\right] = 1$$

- *Computational hiding:* c *reveals no information about* x *to PPT adversaries*

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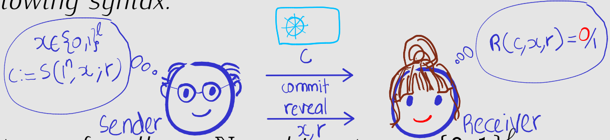

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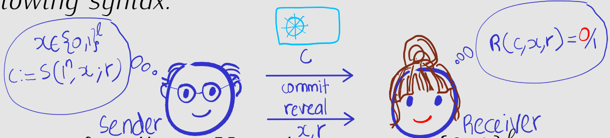

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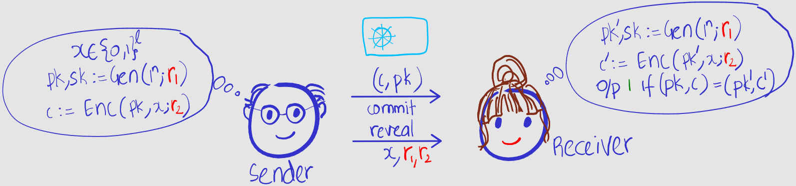
- In general the commit phase can be interactive

Construction 2 (PKE $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec}) \to$ commitment scheme Σ)



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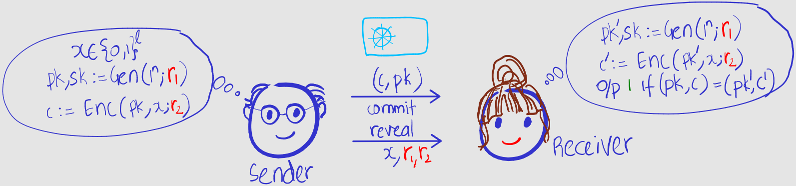

What are the properties we require from Π?

How to Construct Commitment Schemes?

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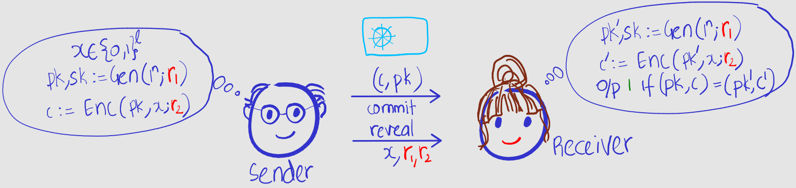

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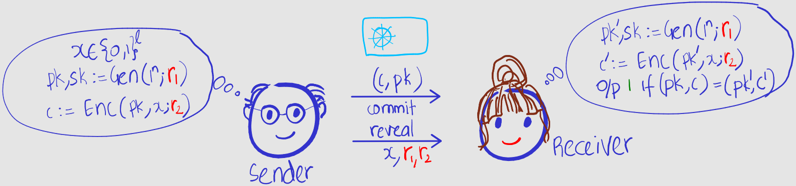

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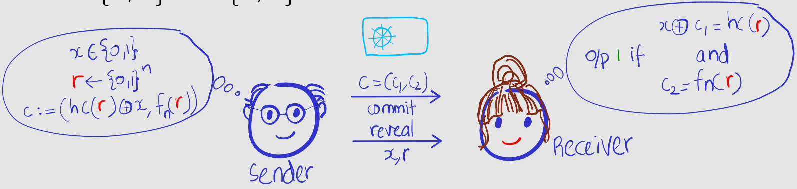
Exercise 6

Which of the PKEs we have seen satisfy the above properties?

Construction 3 (OWP $f_n : \{0,1\}^n \to \{0,1\}^n \to$ *bit*-commitment Σ)

- *Recall: every (leaky)* f_n *has hard-core predicate* $hc : \{0,1\}^n \to \{0,1\}$



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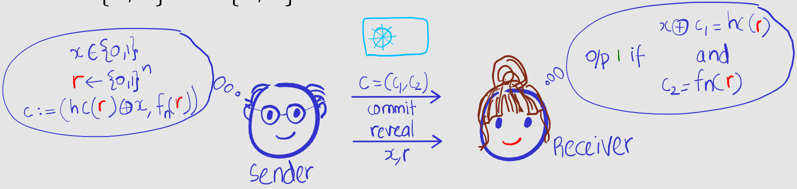

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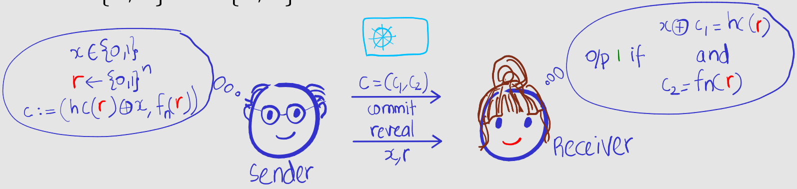

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Next Lecture

- Proofs of knowledge (PoK)
- PoK for the discrete-logarithm problem: Schnorr's protocol
- Fiat-Shamir Transform
  - Digital signatures from discrete-logarithm problem in the random-oracle model

References

1. [Gol01, Chapter 4] for details of today's lecture
2. [GMR89] for definitional and philosophical discussion on ZK
3. The ZKP for graph Hamiltonicity is due to Blum [Blu86]
4. The constructions of commitment scheme from OWP and PRG is from [GMW91] and [Nao90]