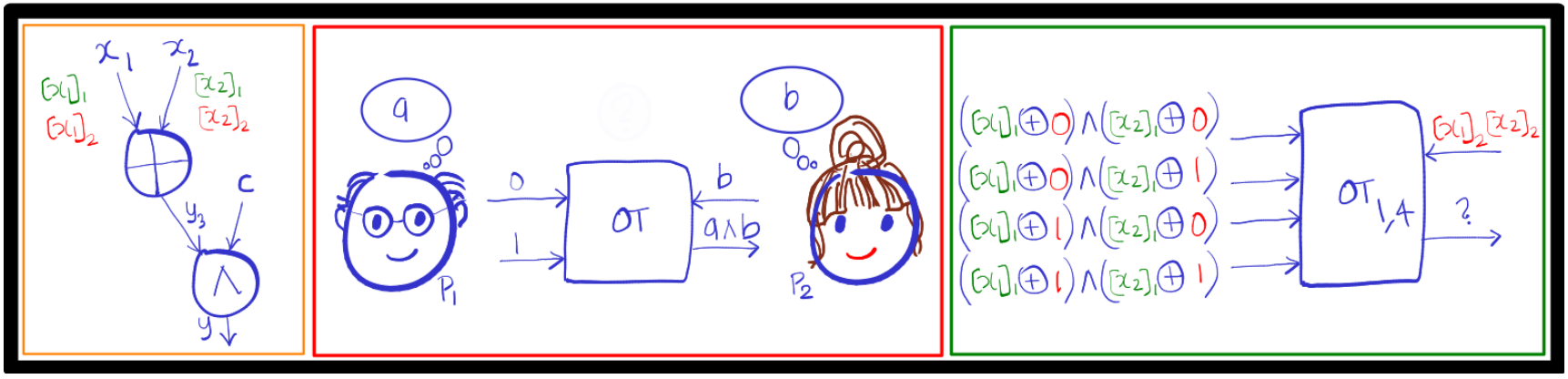


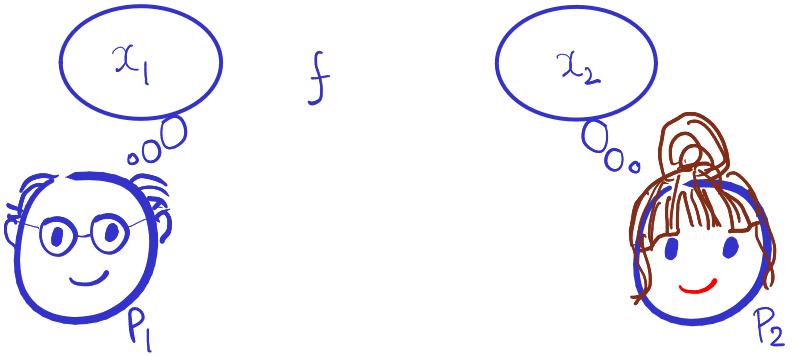
CS783: Theoretical Foundations of Cryptography

Lecture 18 (11/Oct/24)

Instructor: Chethan Kamath

Recall from Last Lecture

- Task 6: Private computation of functions



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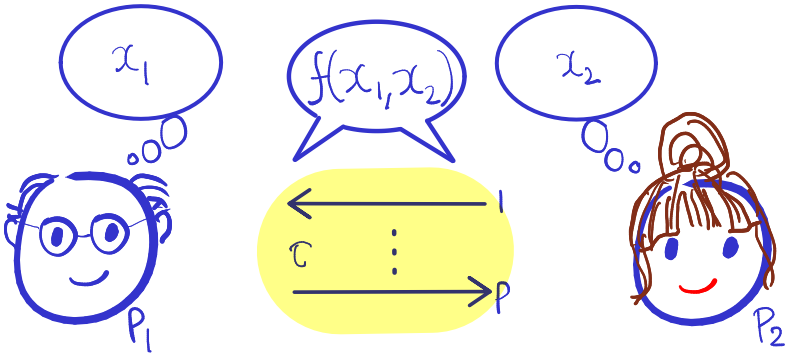

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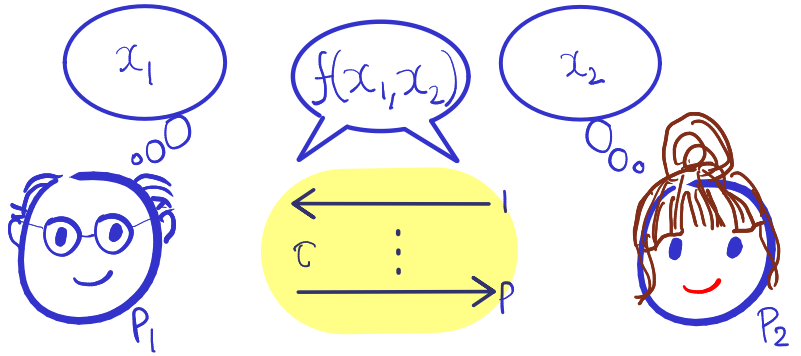
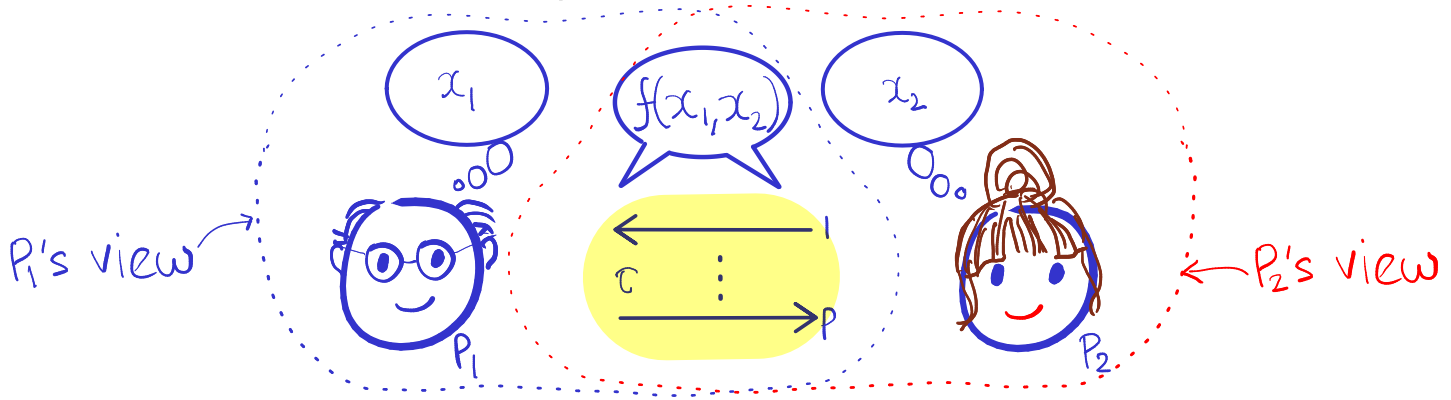


- Defined syntax and security for the two-party case (2PC)
  - Special case of 2PC: ZK proof
  - Extends the simulation paradigm

Recall from Last Lecture

- Task 6: Private computation of functions



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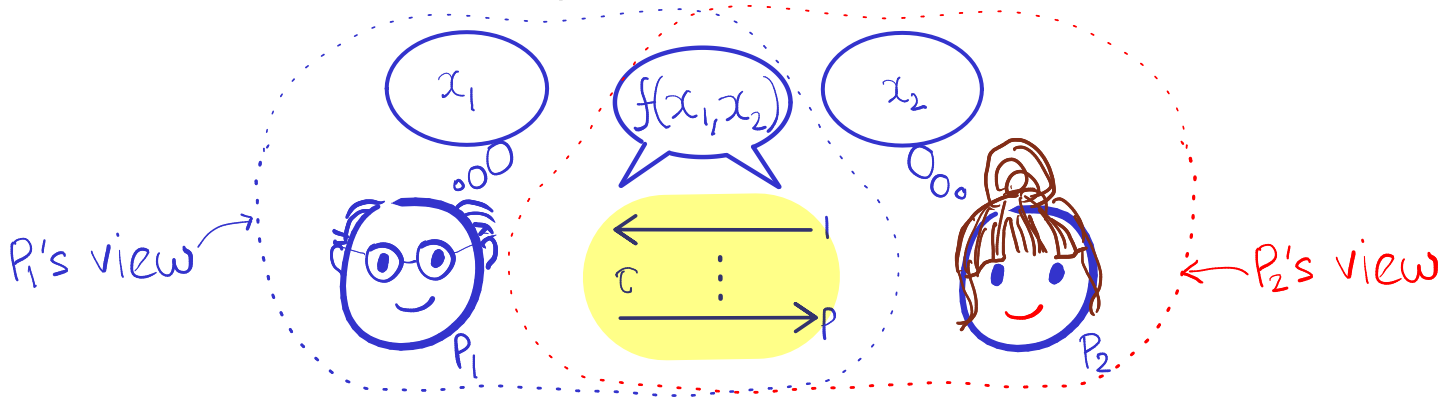

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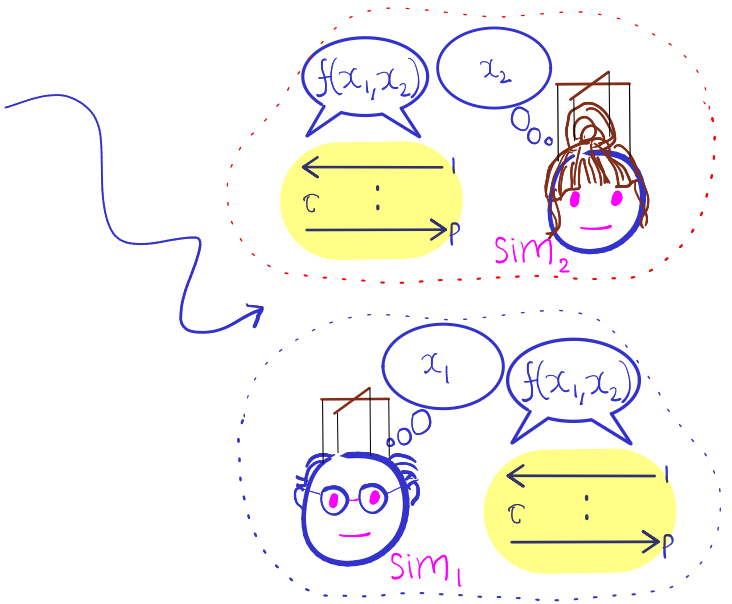

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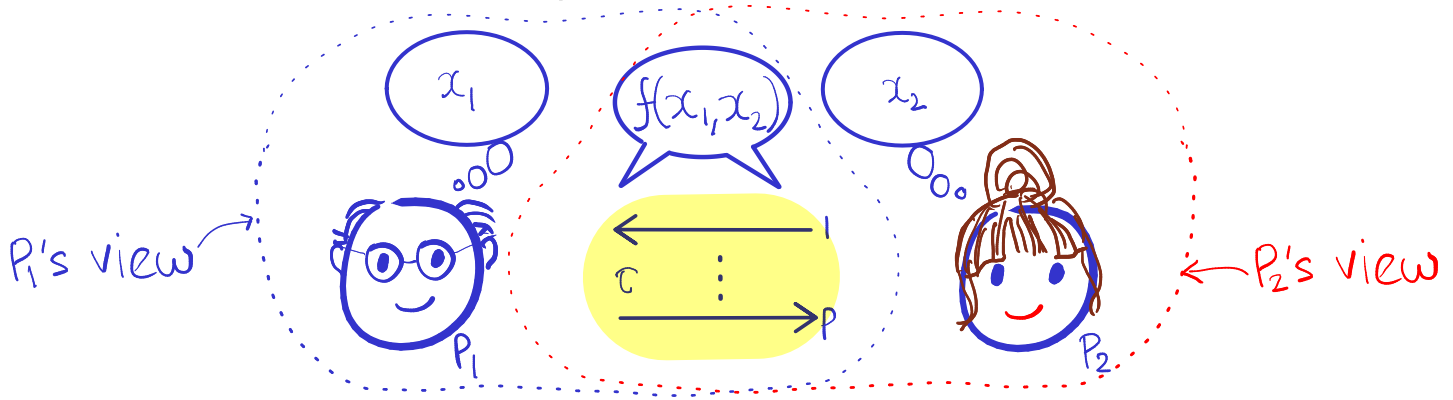

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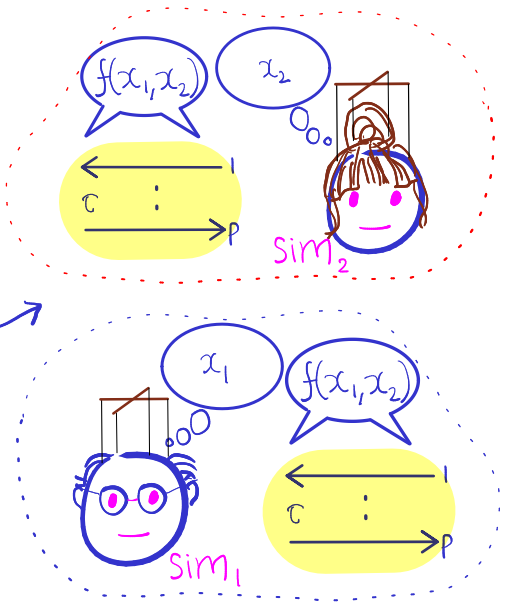

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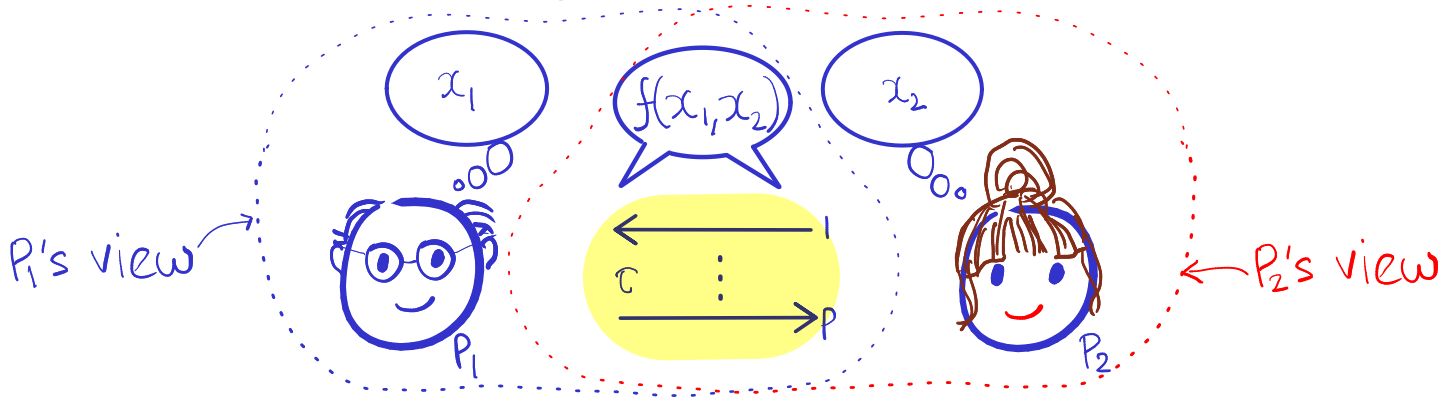

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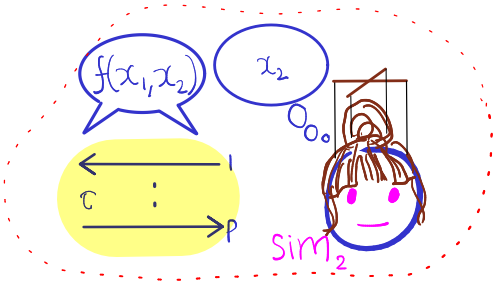
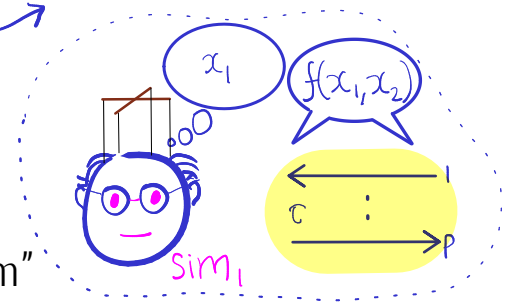
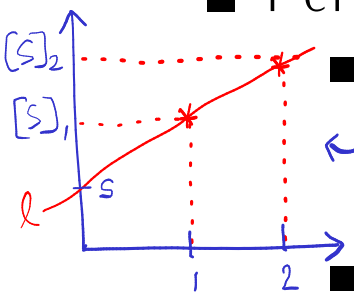




- Perfectly-private MPC for *linear* functions
  - Key tool: threshold secret sharing (TSS)
    - Construction: Shamir's TSS
    - Linearity: "sum of shares → shares of sum"
  - Key idea: "compute over shares"



Plan for Today's Lecture...

- What about perfectly-private MPC for *general* functions?

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- What about perfectly-private MPC for *general* functions?
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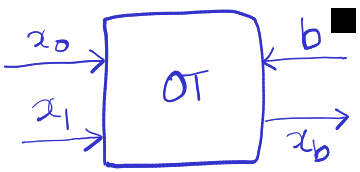
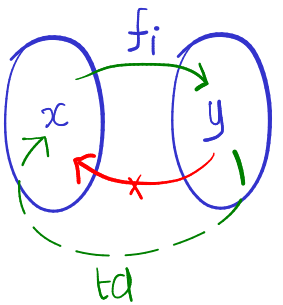
- What do we do? Relax to *computational* privacy
  - New cryptographic primitive: oblivious transfer (OT)
    - Trapdoor permutation (TDP) $\rightarrow$ OT





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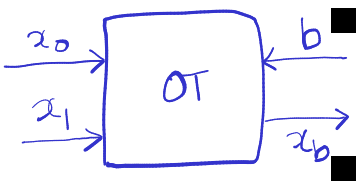

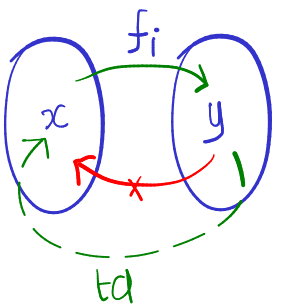

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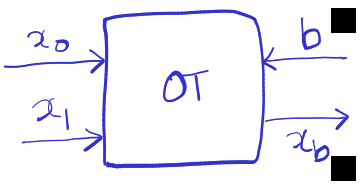

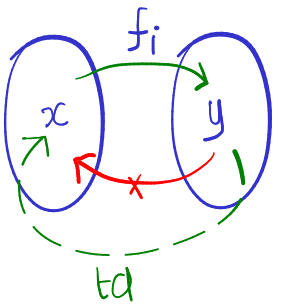

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- Note: perfectly-private MPC for general functions *is* possible *with honest majority*, i.e., if $t < n/2$ (BGW protocol)
  - E.g., three-input $\wedge$ can be computed with perfect privacy if only one of the parties corrupt

Plan for Today's Lecture...

General *template*:

1. Identify the task
2. Come up with precise threat model M (a.k.a security model)
   - Adversary/Attack: What are the adversary's capabilities?
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3. Construct a scheme Π
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General *template*:

1. Identify the task → private computation of (arbitrary) functions for two parties
2. Come up with precise threat model M (a.k.a security model) → semi-honest model
   - Adversary/Attack: What are the adversary's capabilities? → "static corruption"
   - Security Goal: What does it mean to be secure? → "computational privacy"
3. Construct a scheme Π → GMW protocol
4. Formally prove that Π in secure in model M
   ↳ OT ⇒ simulation

Plan for Today's Lecture

1. Computing $\wedge$ with Perfect Privacy is Impossible
2. New Cryptographic Primitive: Oblivious Transfer (OT)
3. Goldreich-Micali-Wigderson (GMW) Protocol

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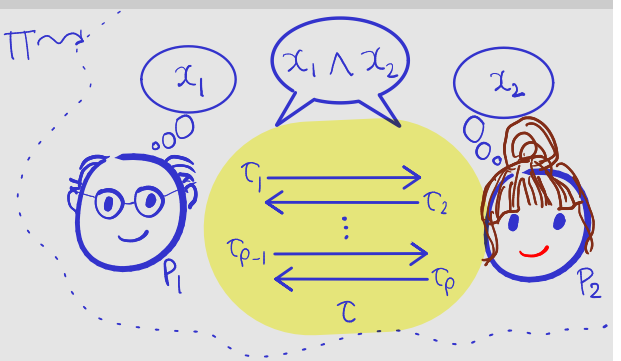

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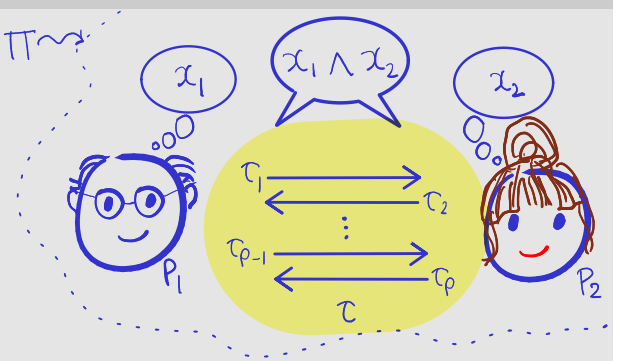

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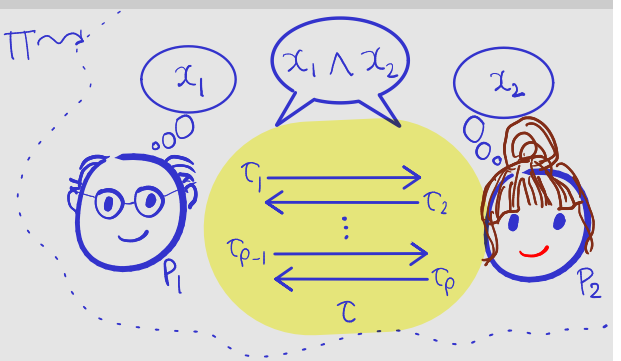

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$$P_{x_1,x_2}(\tau) = \prod_{r=1}^{p} \Pr\left[\tau_r \mid \tau_{r-1}, \ldots \tau_1, x_1, x_2\right]$$

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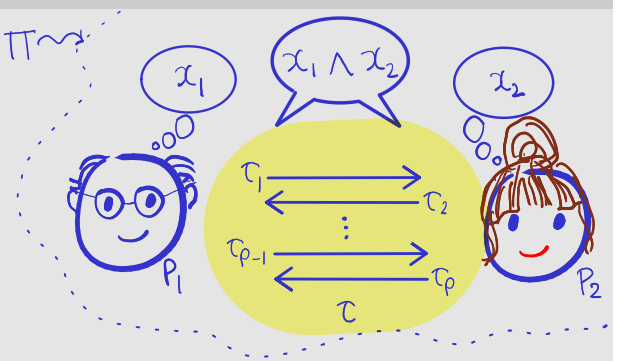

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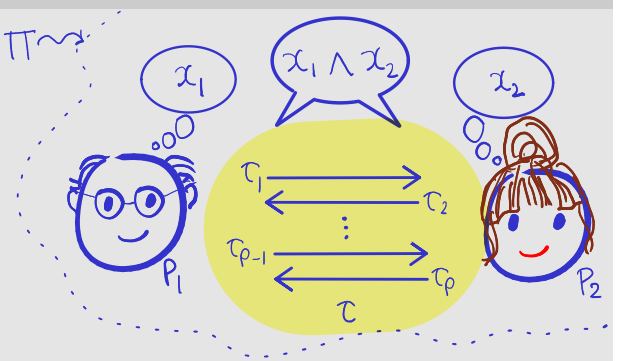

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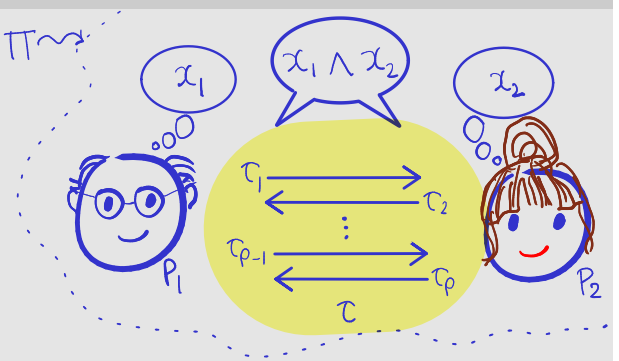

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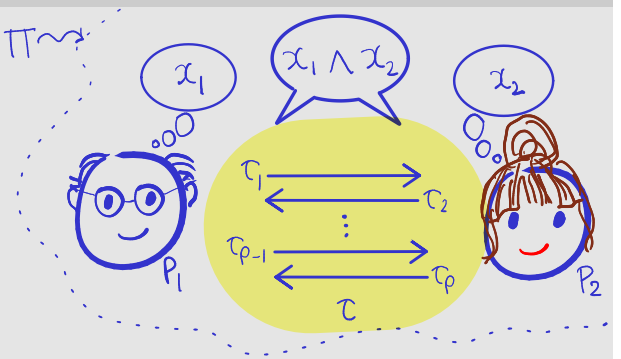

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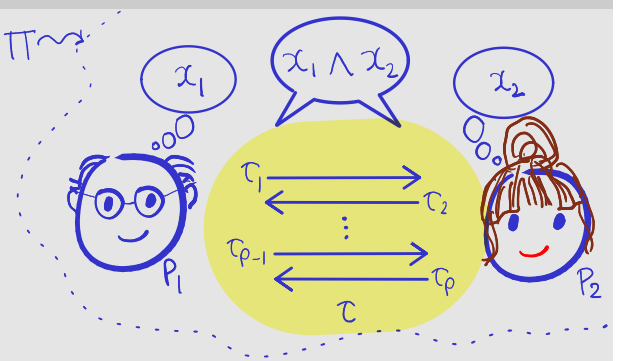

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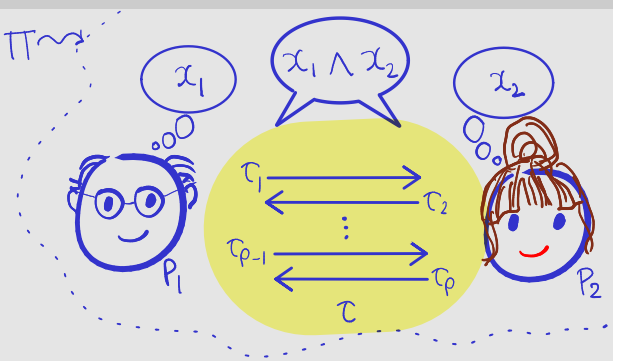

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$$\alpha(\tau,1) = \alpha(\tau,0) \;\&\; \beta(\tau,0) = \beta(\tau,1)$$

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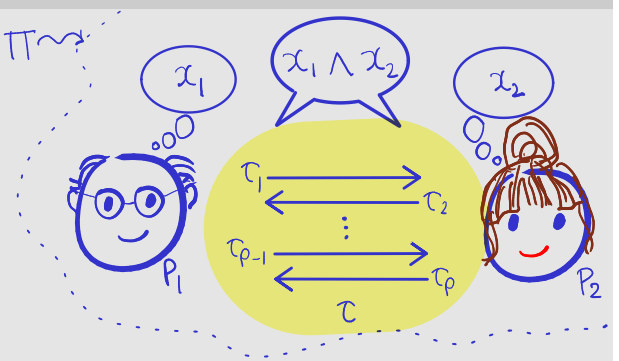

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- Need to deal with $\wedge$ over $\mathbb{F}_2$/multiplication over $\mathbb{F}_p$

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Proof (Idea: $\exists$ perfectly private $\Pi \implies \Pi$ incorrect).

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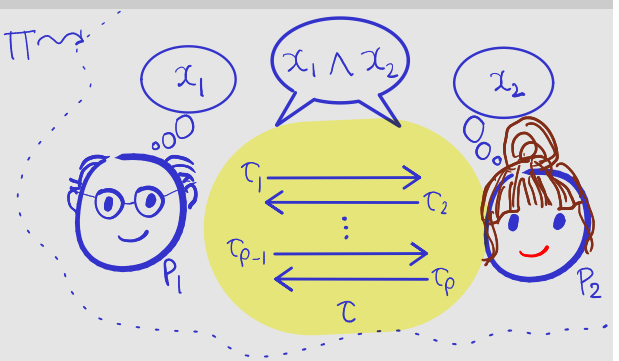

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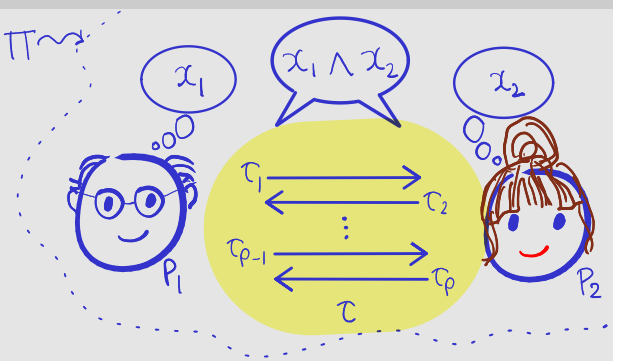

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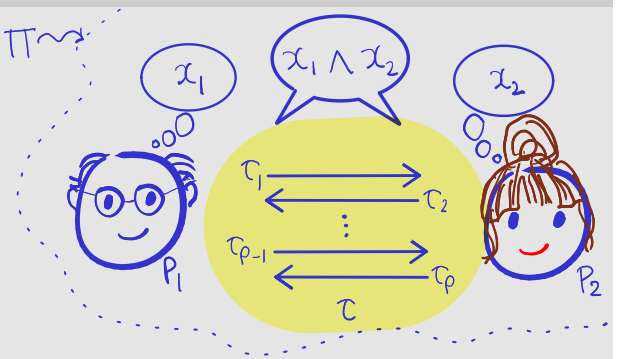

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Plan for Today's Lecture

1. Computing $\wedge$ with Perfect Privacy is Impossible
2. New Cryptographic Primitive: Oblivious Transfer (OT)
3. Goldreich-Micali-Wigderson (GMW) Protocol

Oblivious Transfer (OT)

- Intuitively: 2PC protocol for "choice function/multiplexer"

$f_b(x_0, x_1) := x_b = (1-b)x_0 + b x_1$

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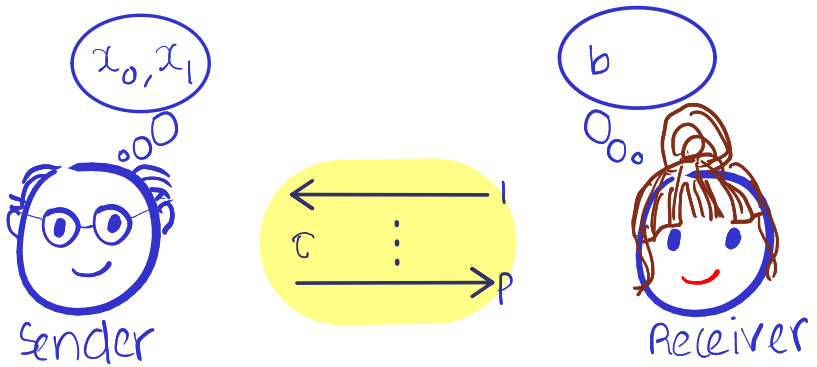

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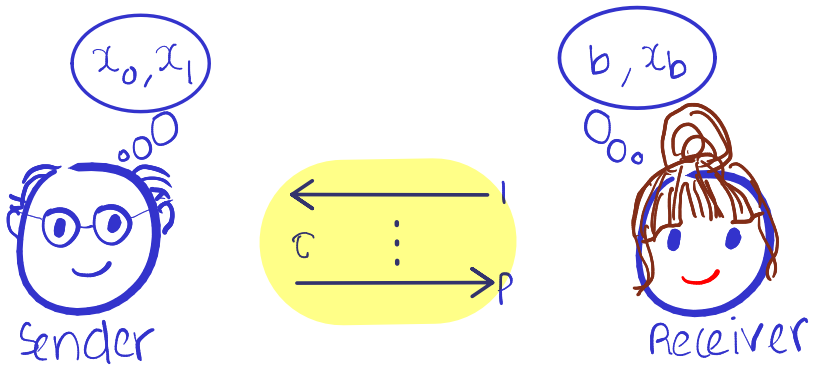

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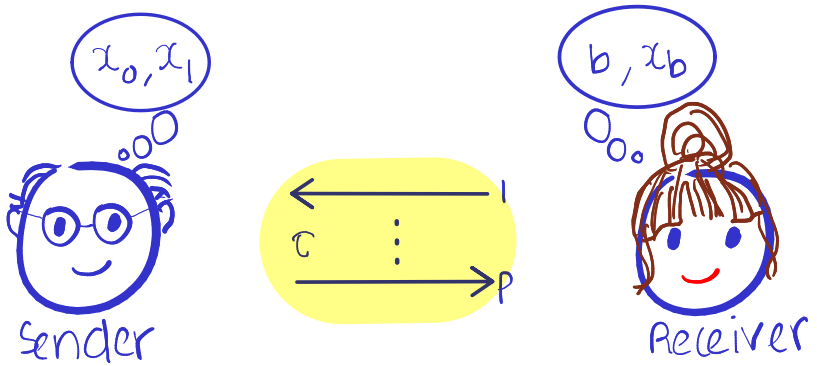

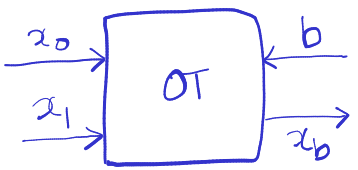

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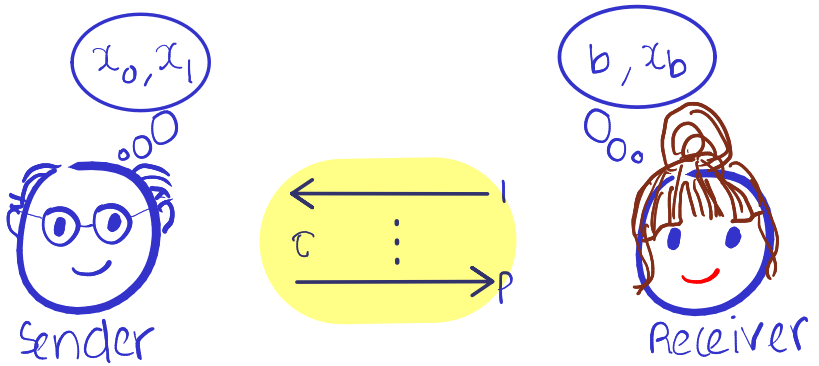

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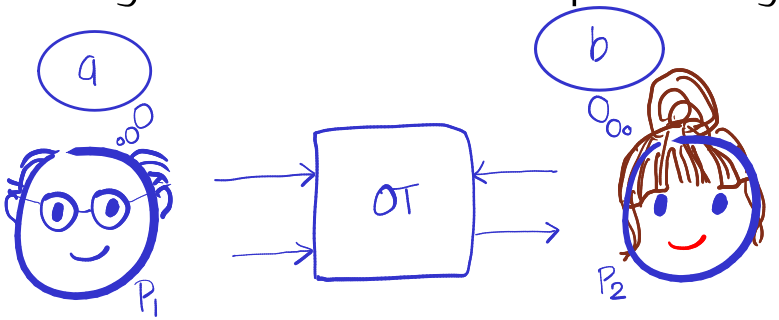

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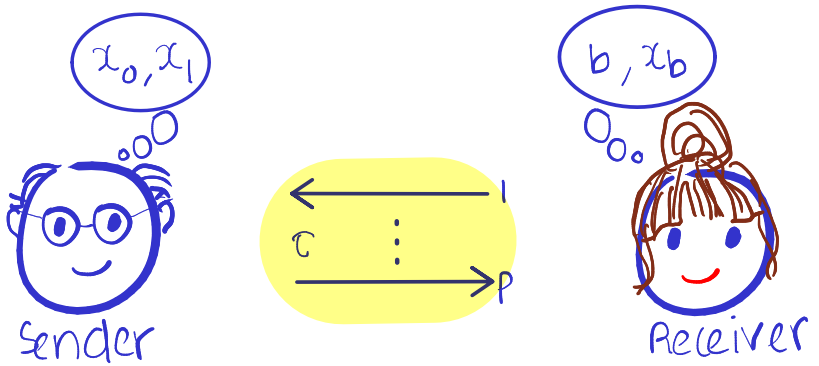

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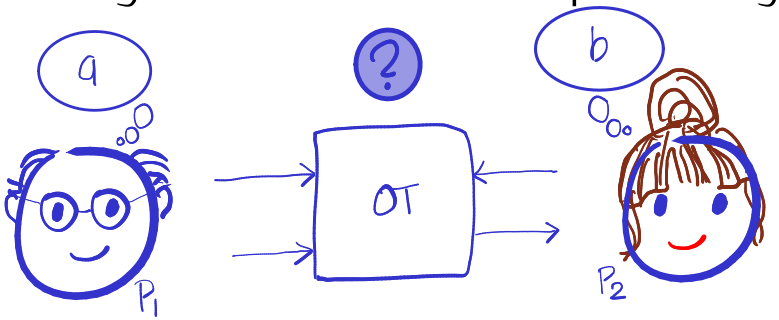

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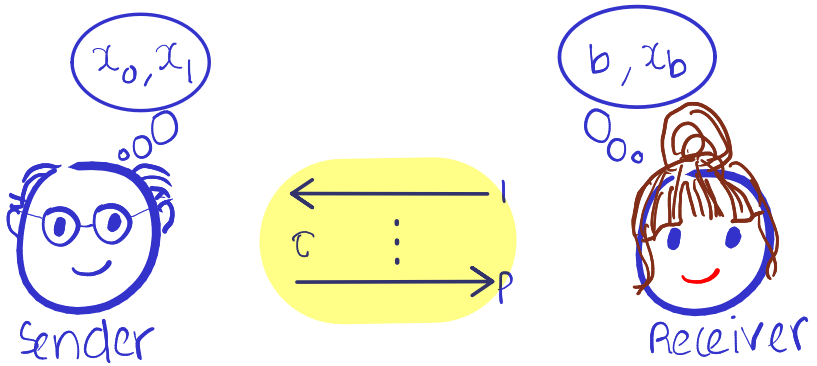

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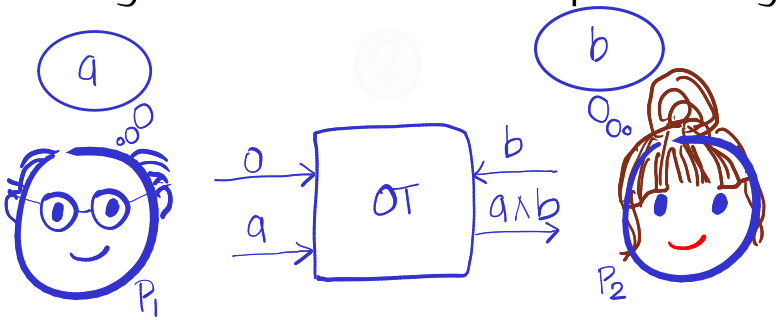

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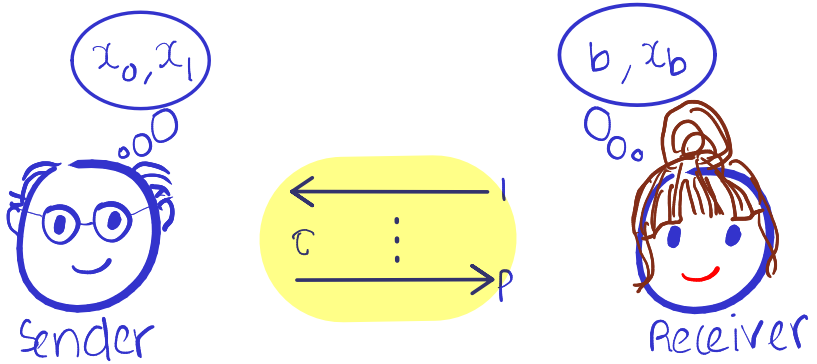

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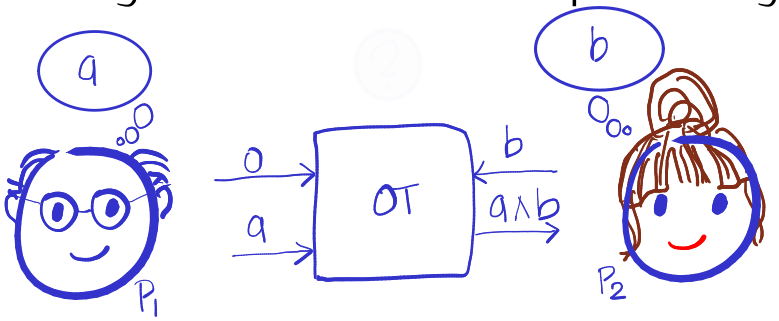


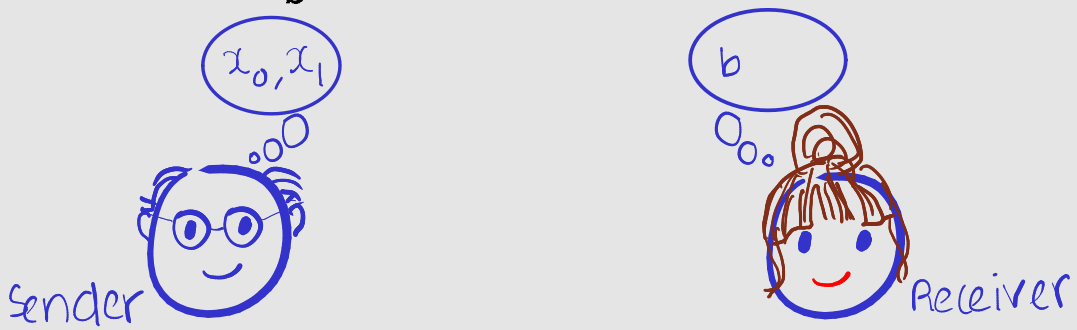
Exercise 1 (Hint: Need to invoke OT multiple times)

Implement comparison operator $\leq$ for n-bit integers using OT

Syntax and Security of OT

Defintion 1 (Oblivious Transfer)

An interactive protocol $\Pi = (\mathsf{S}, \mathsf{R})$ *between a sender* S *with input bits* $x_0, x_1 \in \{0,1\}$ *and receiver* R *with choice bit* $b \in \{0,1\}$*, at the end of which* R *learns* x_b



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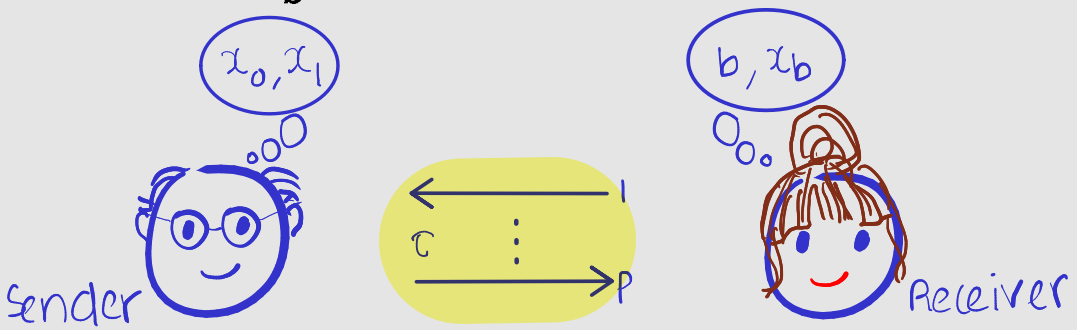

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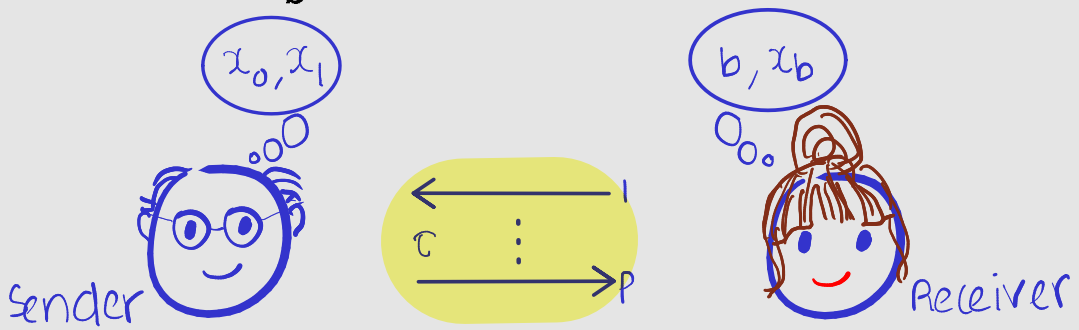

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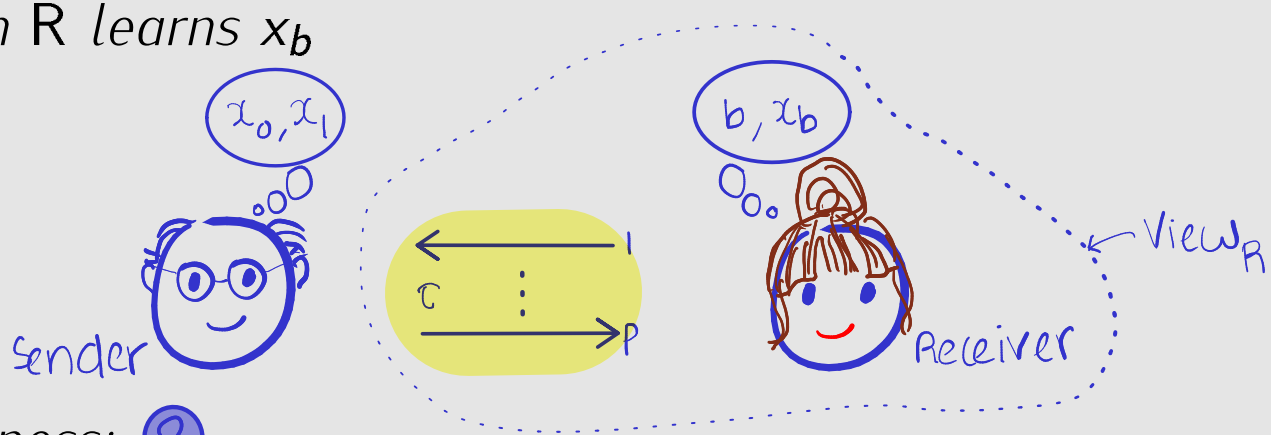

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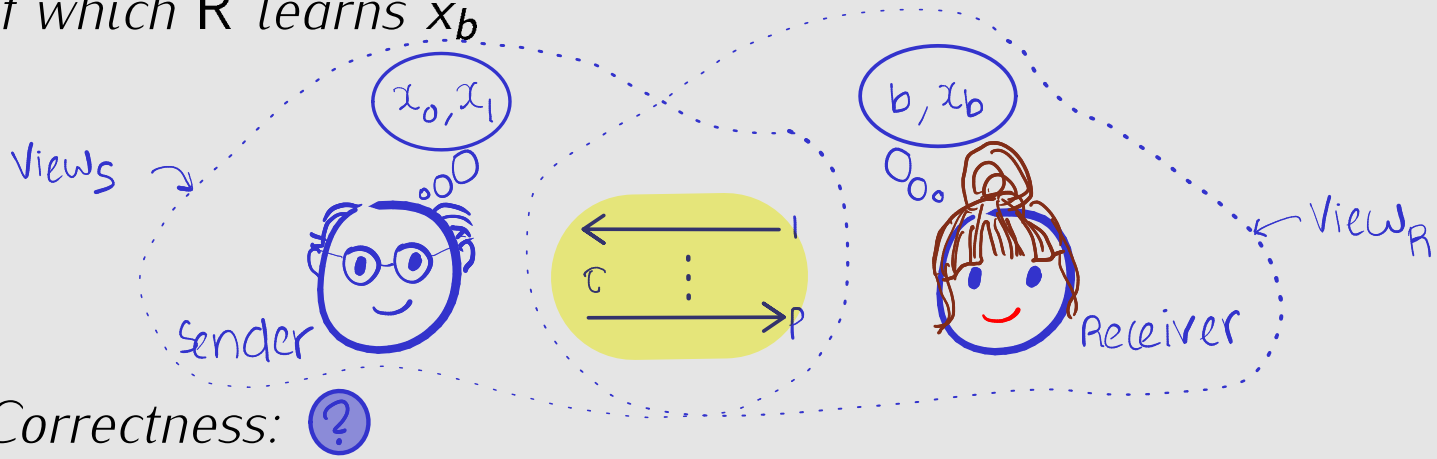

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Syntax and Security of OT

1-out-of-2

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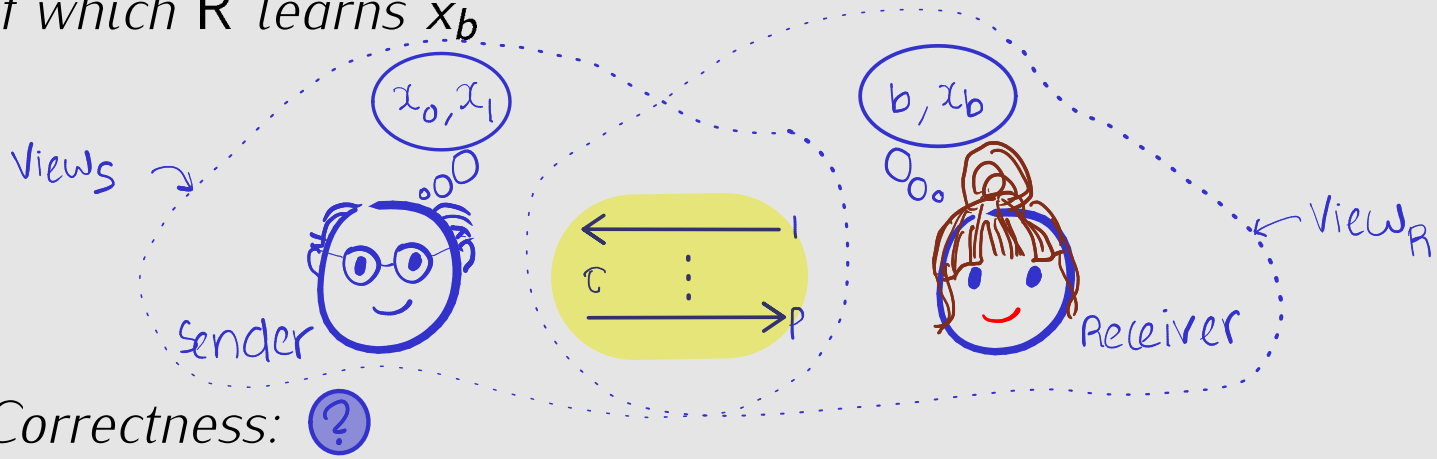

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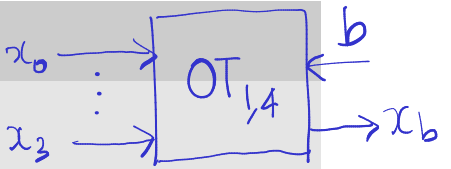
Exercise 2

$S: x_0, x_1, x_2, x_3 \in \{0,1\}^\lambda \quad R: b \in \{0, \ldots, 3\}$

Construct a 1*-out-of-*4 *OT from a* 1*-out-of-*2 *OT.*



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- One-way permutation that is easy to invert given a "trapdoor"

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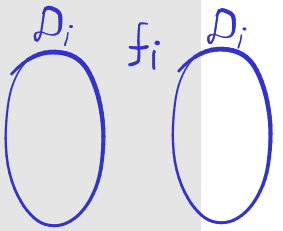

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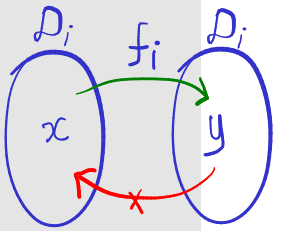
$$p(n) := \Pr_{\substack{(i,\tau) \leftarrow \mathsf{Index}(1^n) \\ x \leftarrow \mathcal{D}_i}} [\mathsf{Inv}(\mathsf{f}_i(x)) \in \mathsf{f}_i^{-1}(\mathsf{f}_i(x))]$$



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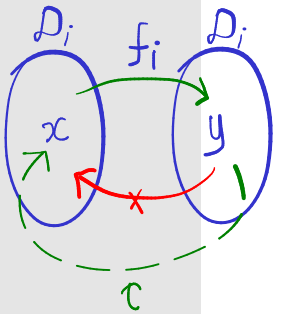

4. f_i^{-1} *can be efficiently computed given trapdoor* τ *for* i

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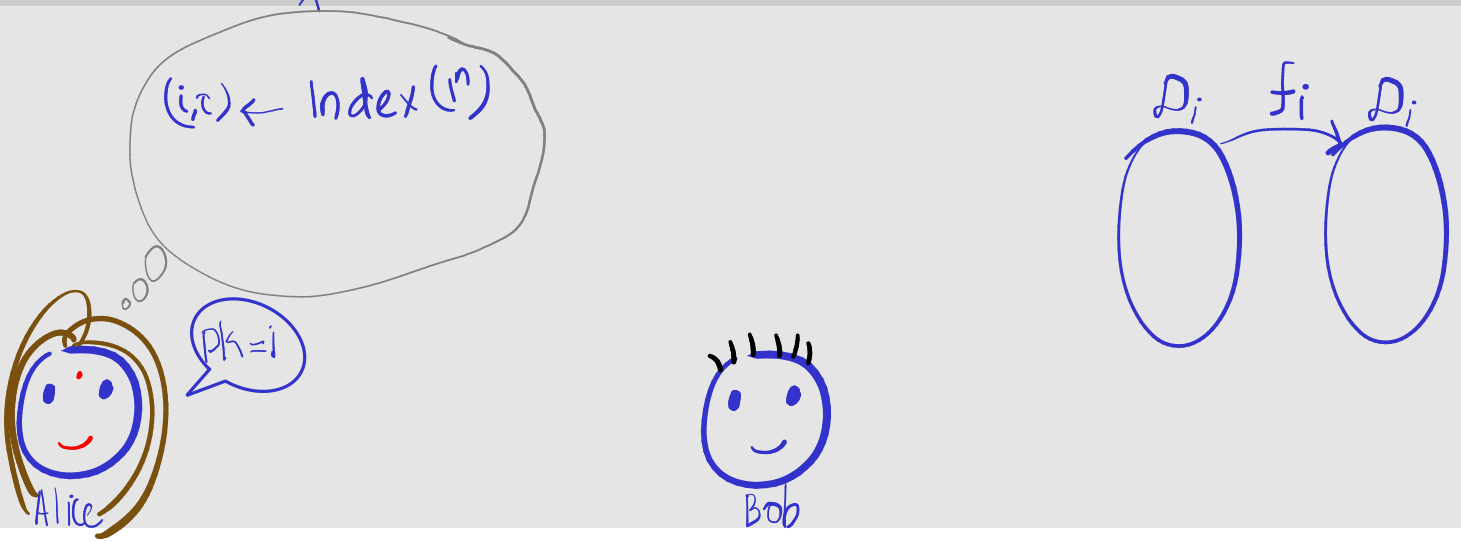

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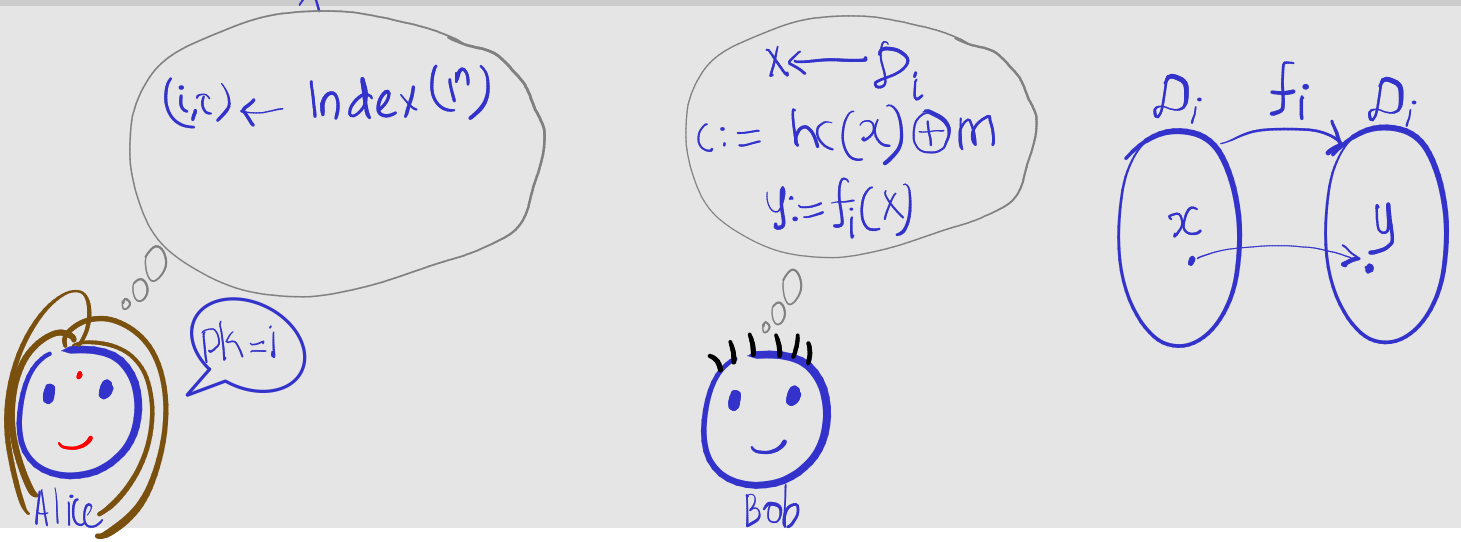

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- Example: RSA TDP $\mathsf{f}_{N,e} : \mathbb{Z}_N^\times \to \mathbb{Z}_N^\times$ defined as

$$\mathsf{f}_{N,e}(x) := x^e \bmod N$$

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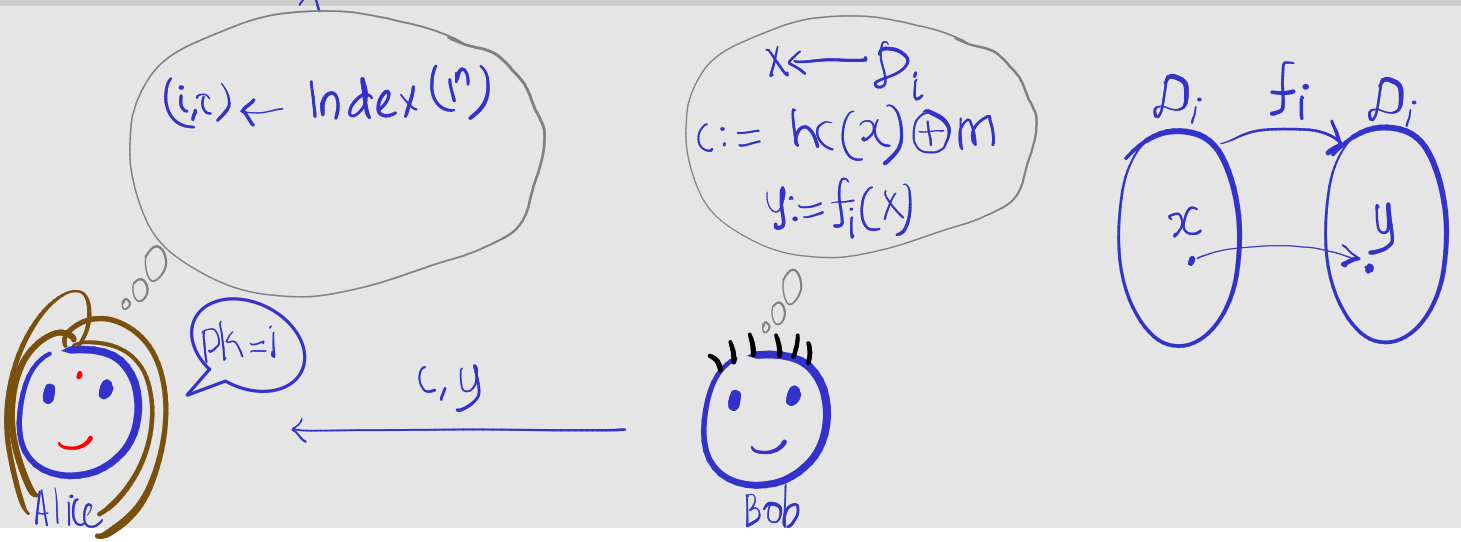

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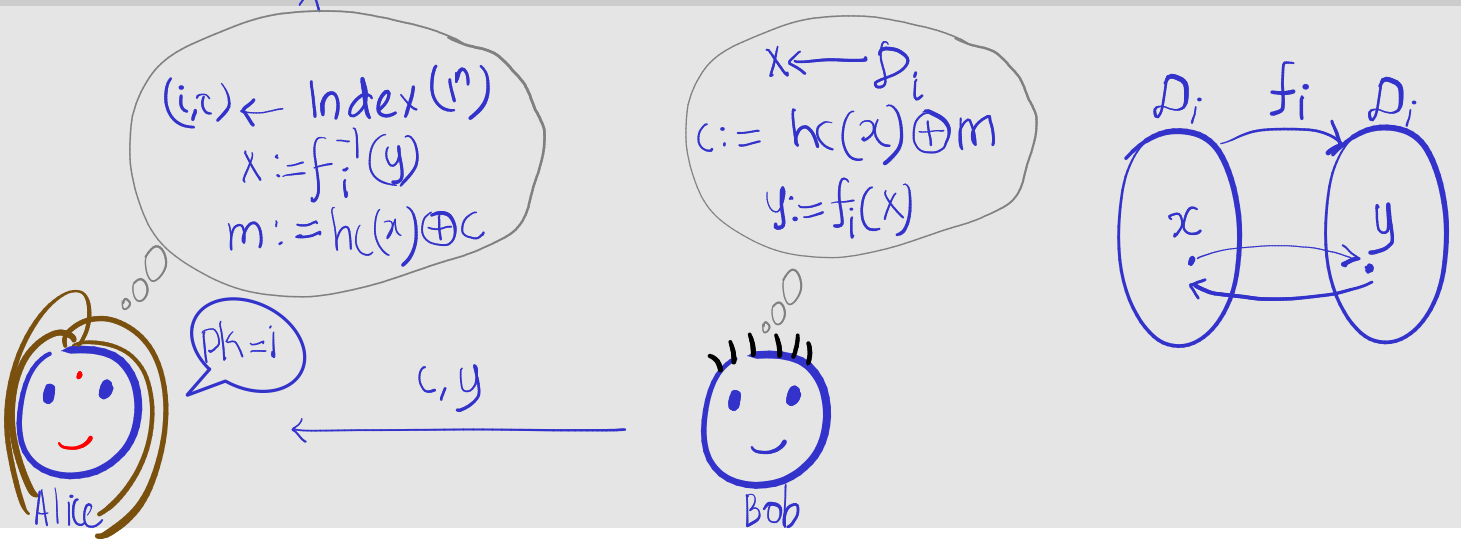

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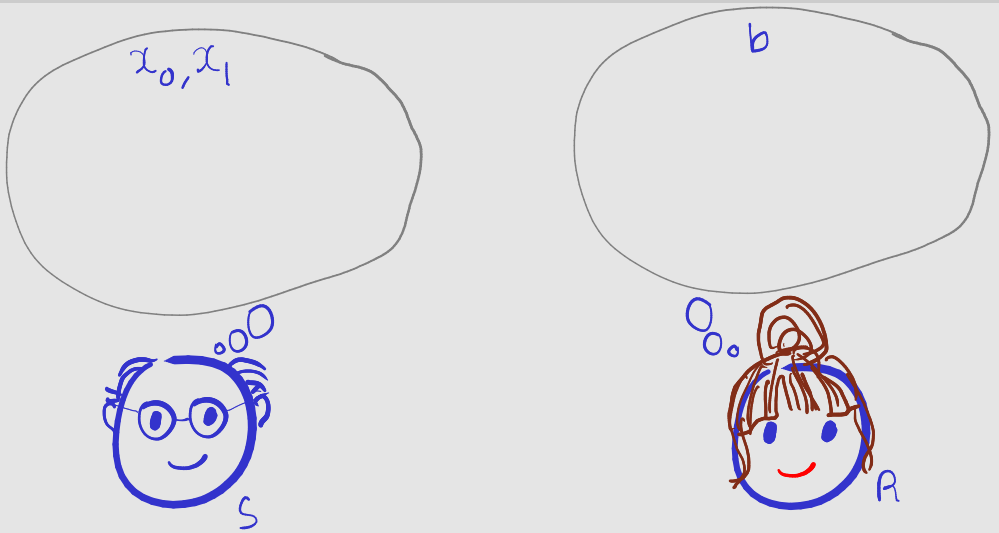

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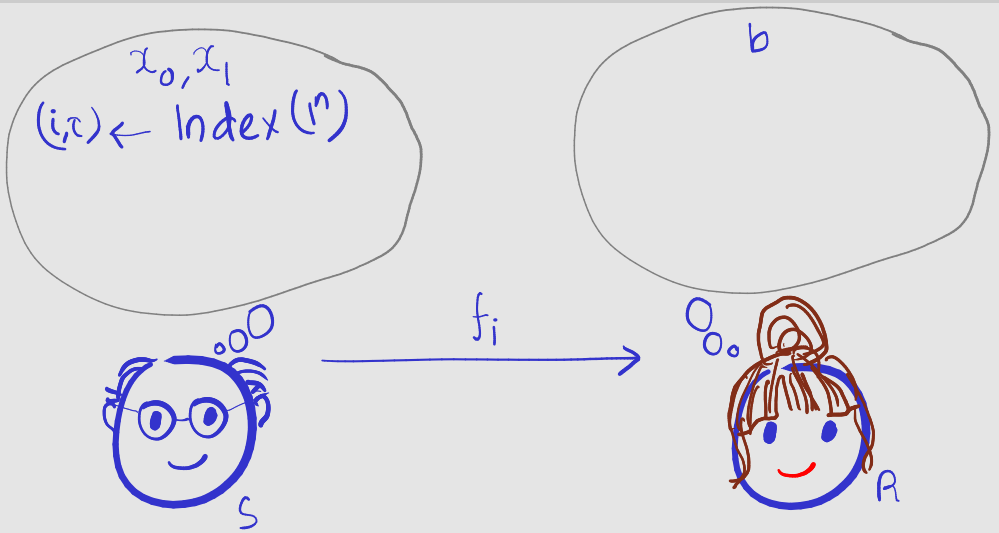

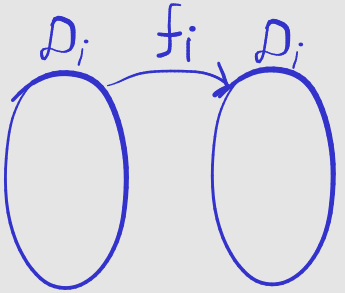

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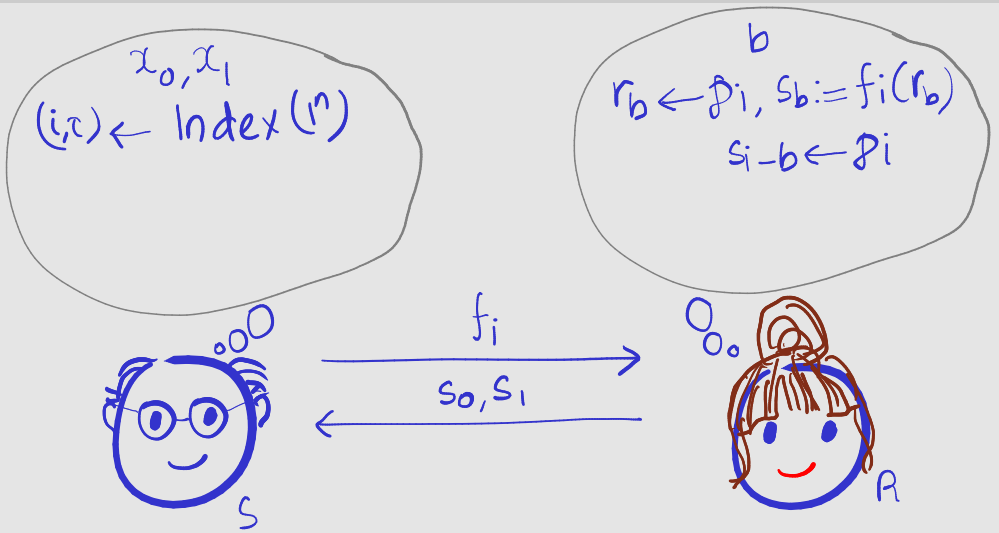

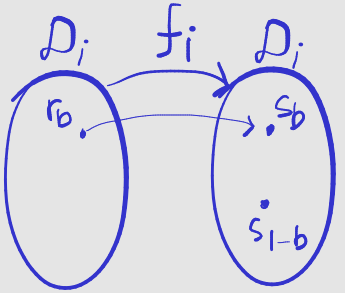

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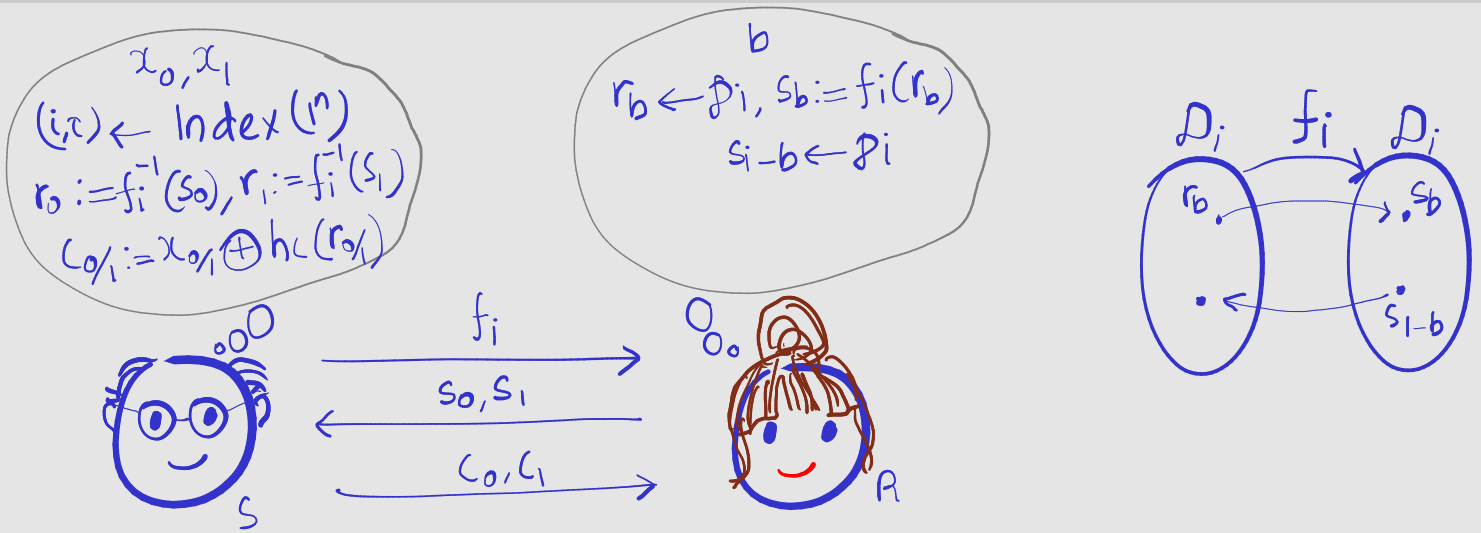

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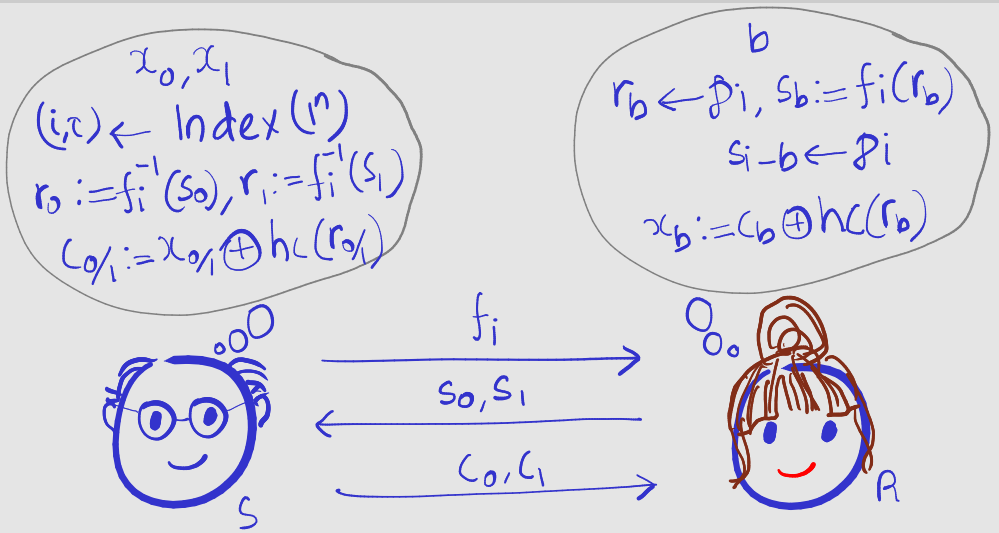

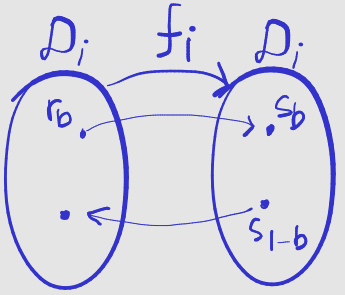

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Theorem 1

If f_i is a secure TDP then Π is a secure OT.

Proof (of sender privacy). i) construct Sim_R ii) show real view ≈ simulation

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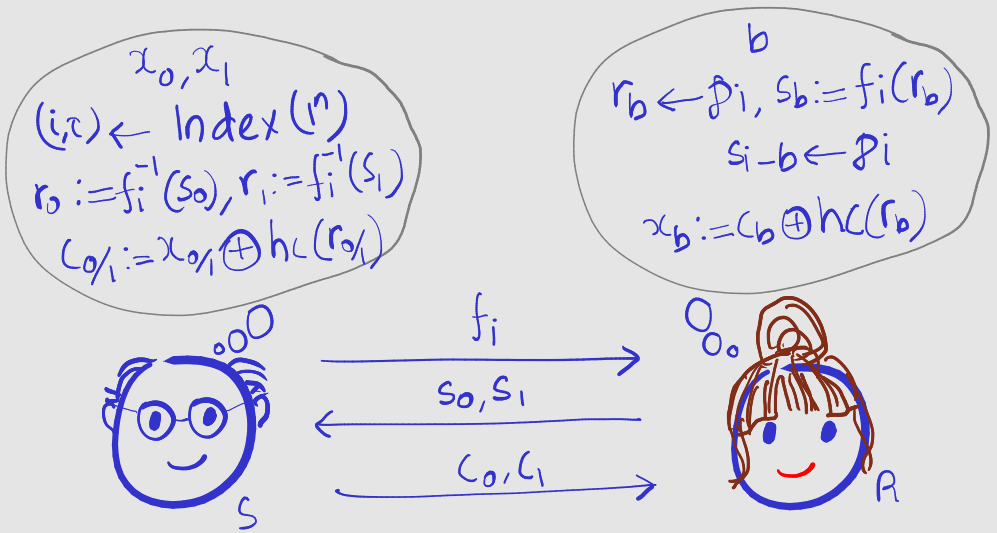

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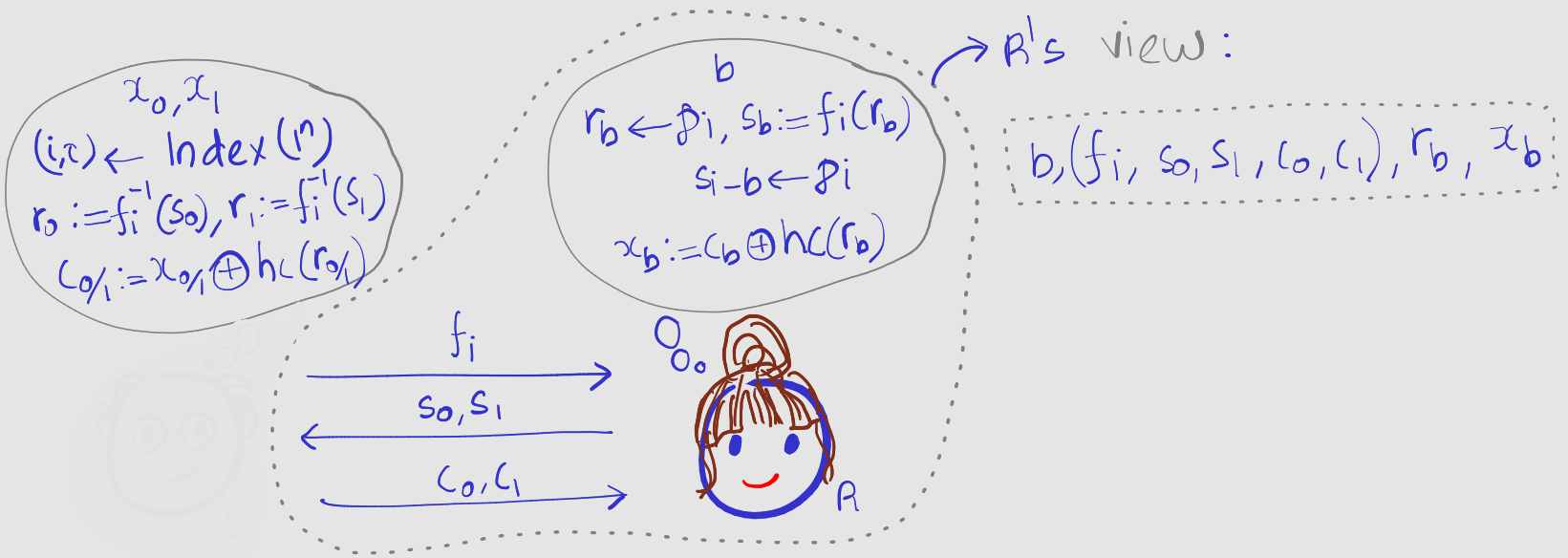

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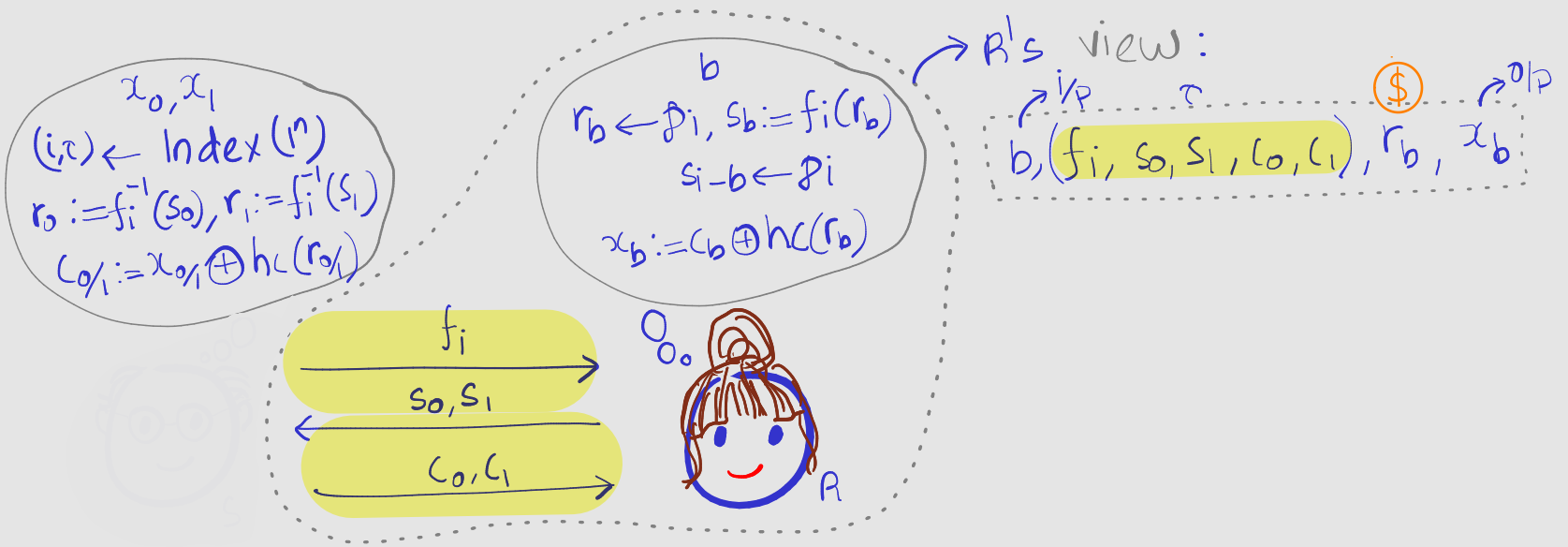

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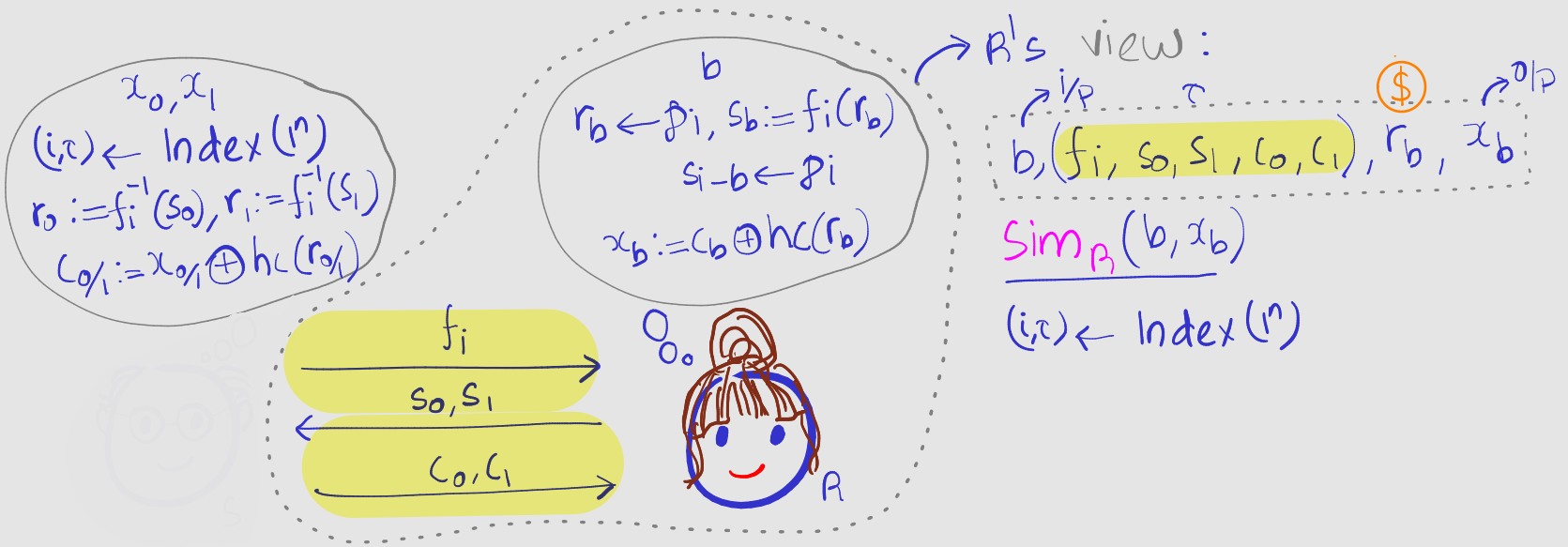

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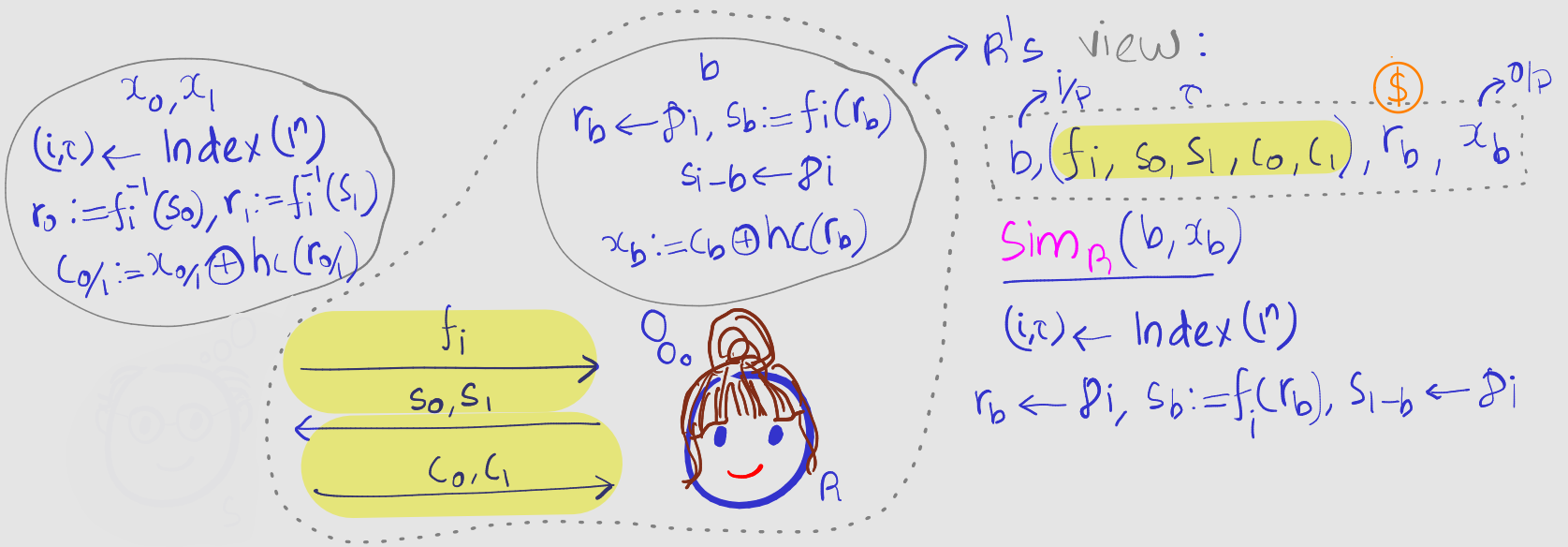

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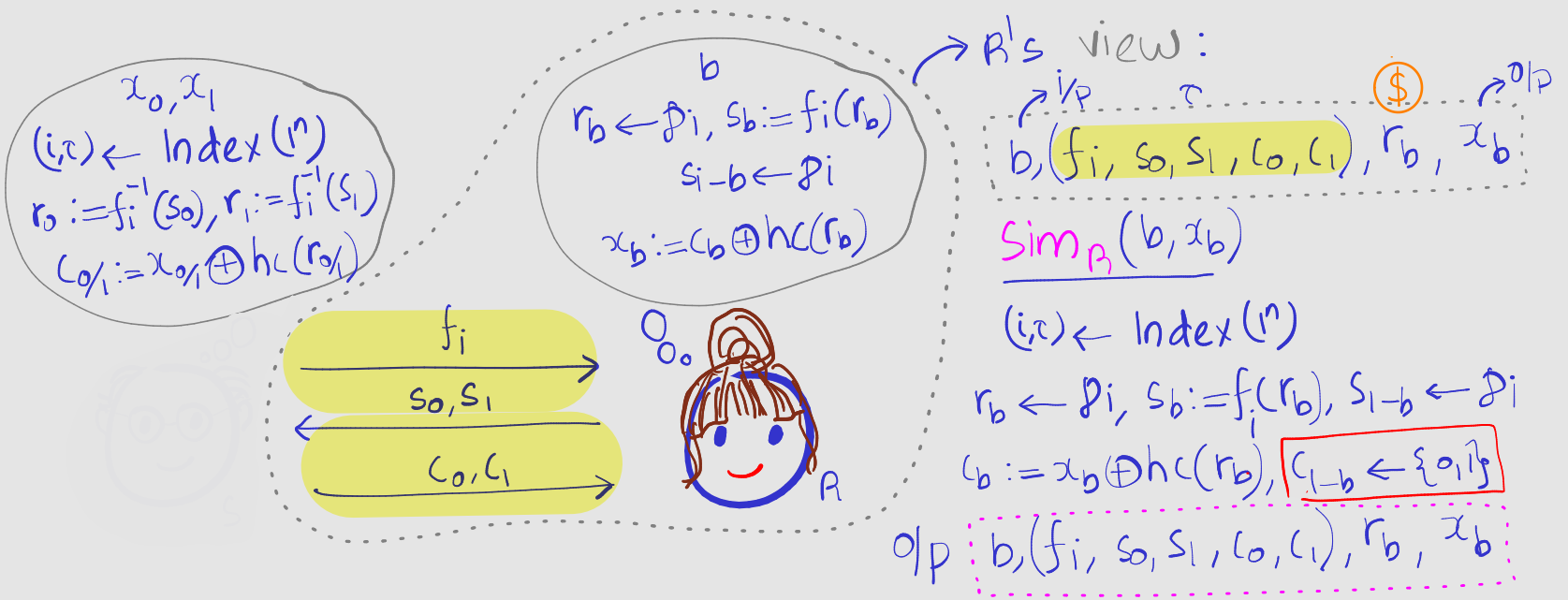

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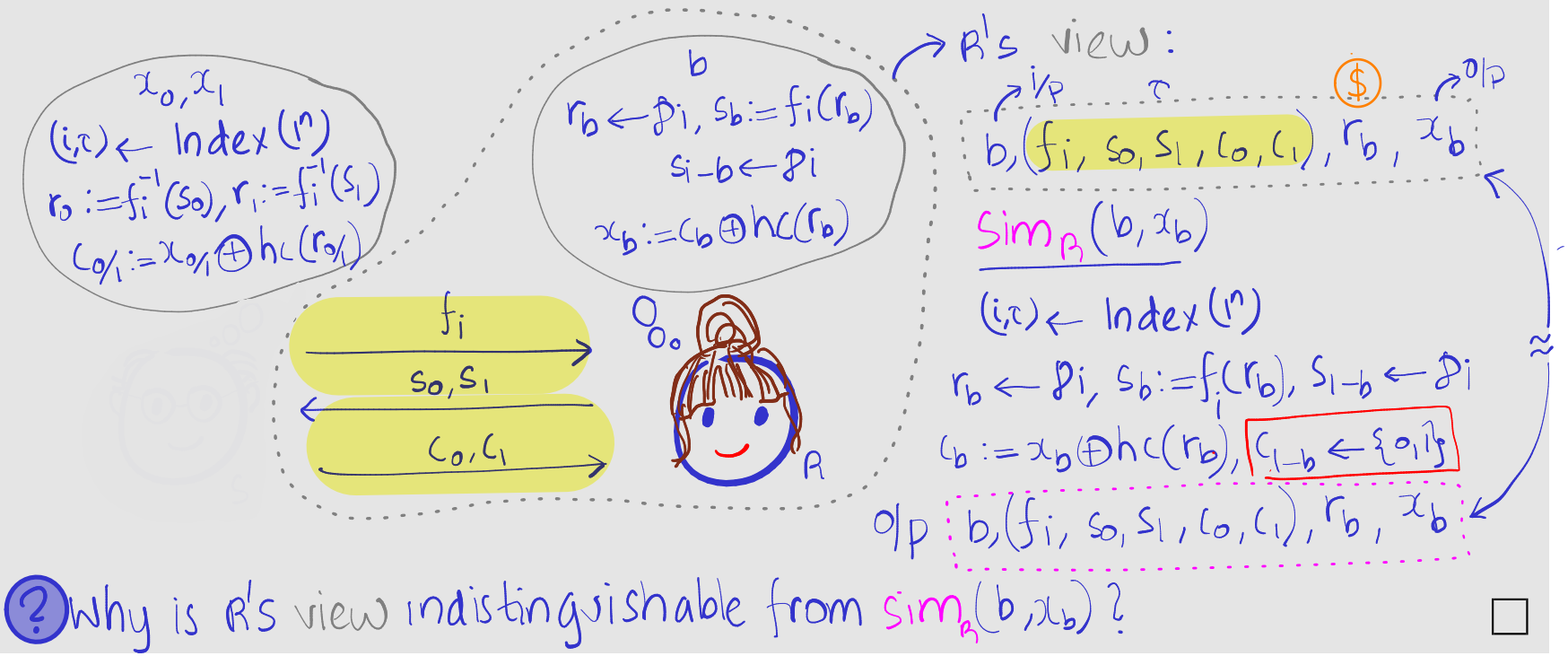

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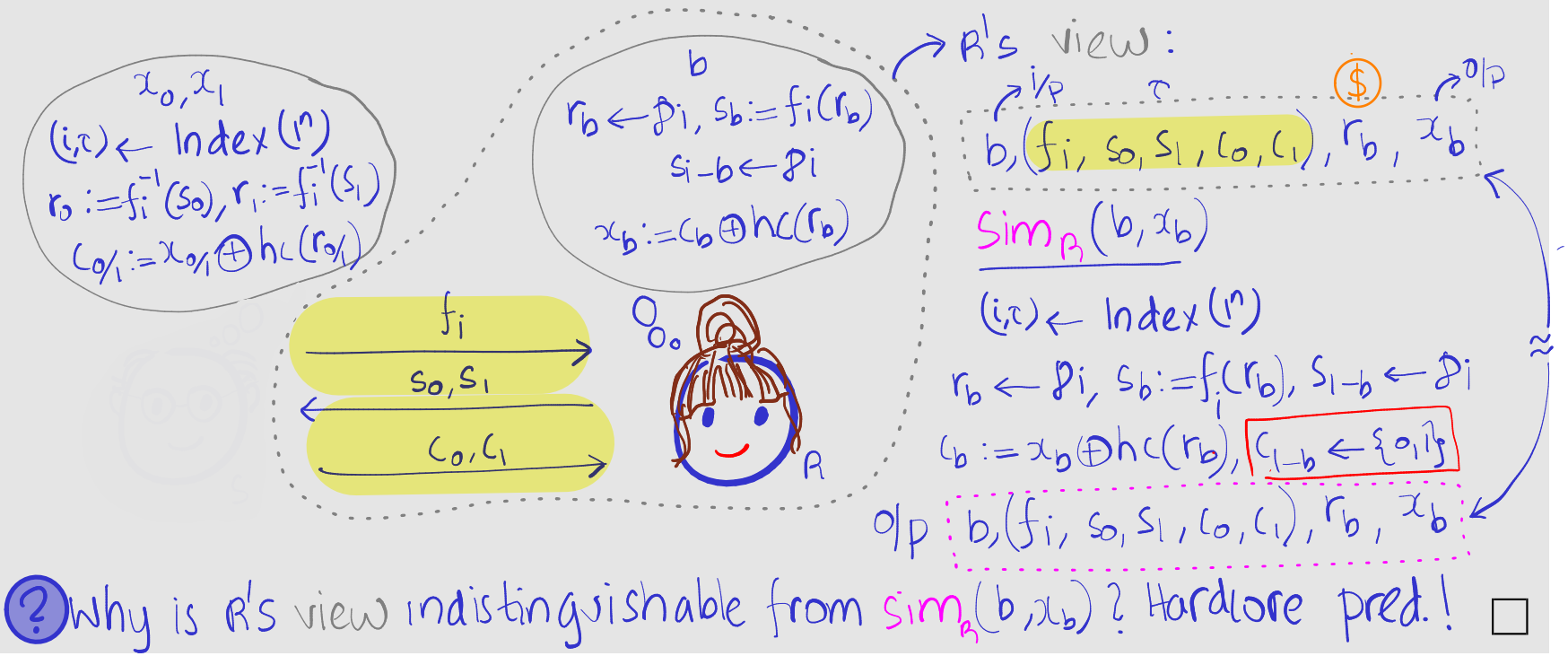

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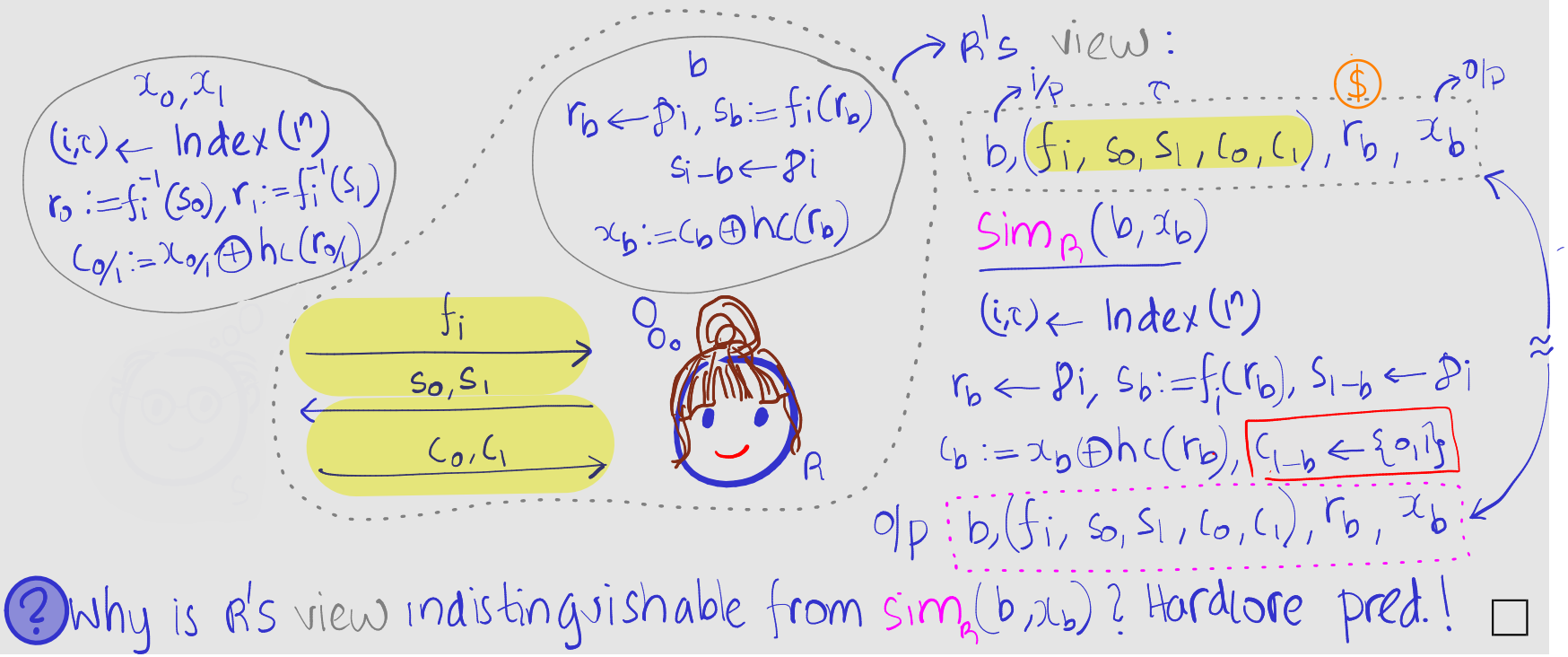

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Exercise 3

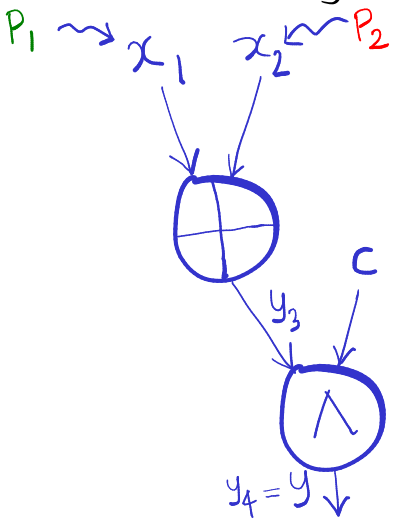
What happens if the sender (resp., receiver) is malicious?

Plan for Today's Lecture

1. Computing $\wedge$ with Perfect Privacy is Impossible
2. New Cryptographic Primitive: Oblivious Transfer (OT)
3. Goldreich-Micali-Wigderson (GMW) Protocol

Recall Π (Protocol 1) from Last Lecture

- Every linear function f over $\mathbb{F}_2 = (\mathbb{Z}_2, \oplus, \wedge)$ can be represented by a Boolean circuit consisting of $\oplus$ and $\wedge_c$



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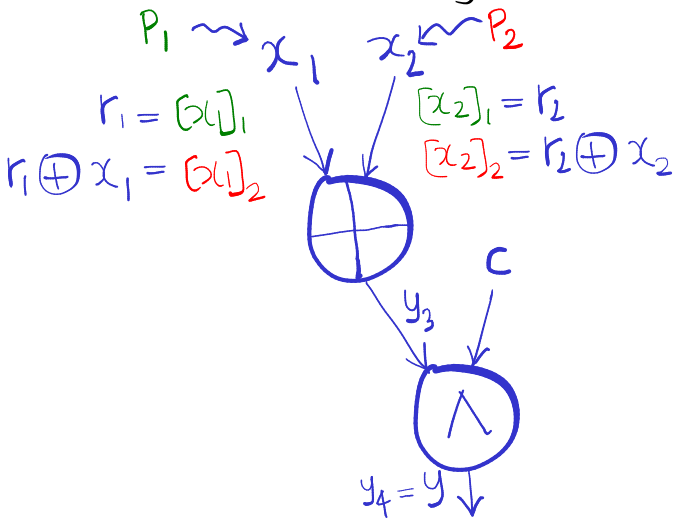

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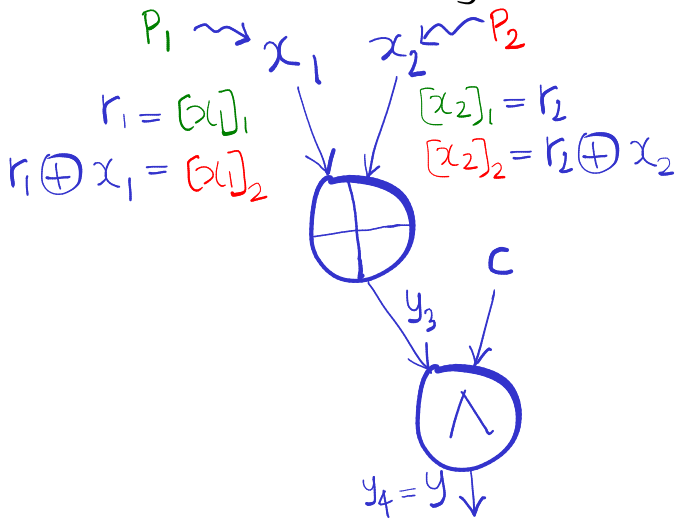

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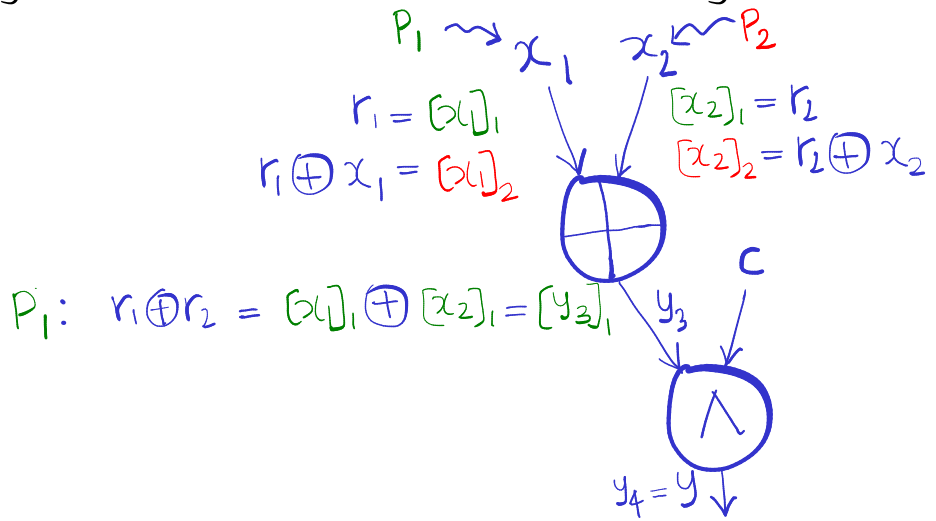

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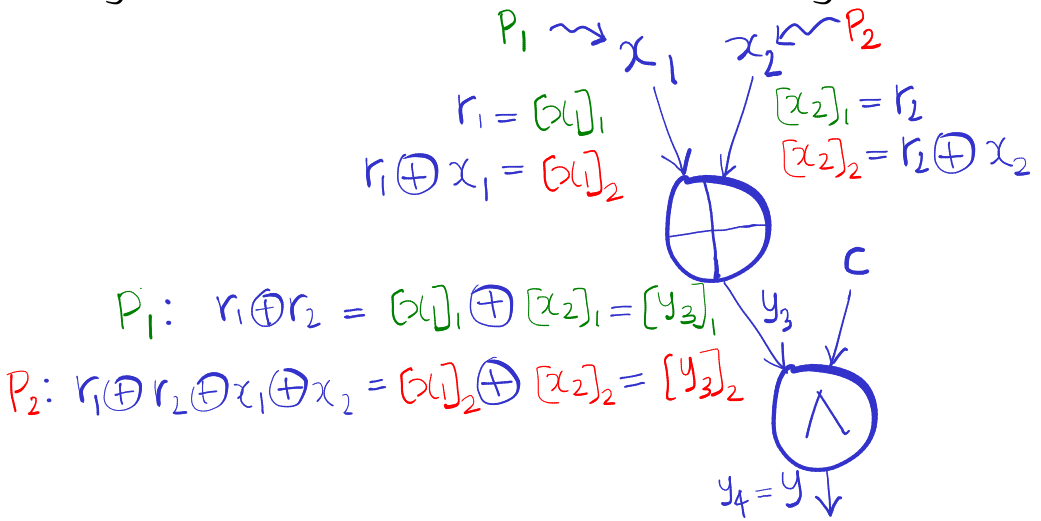

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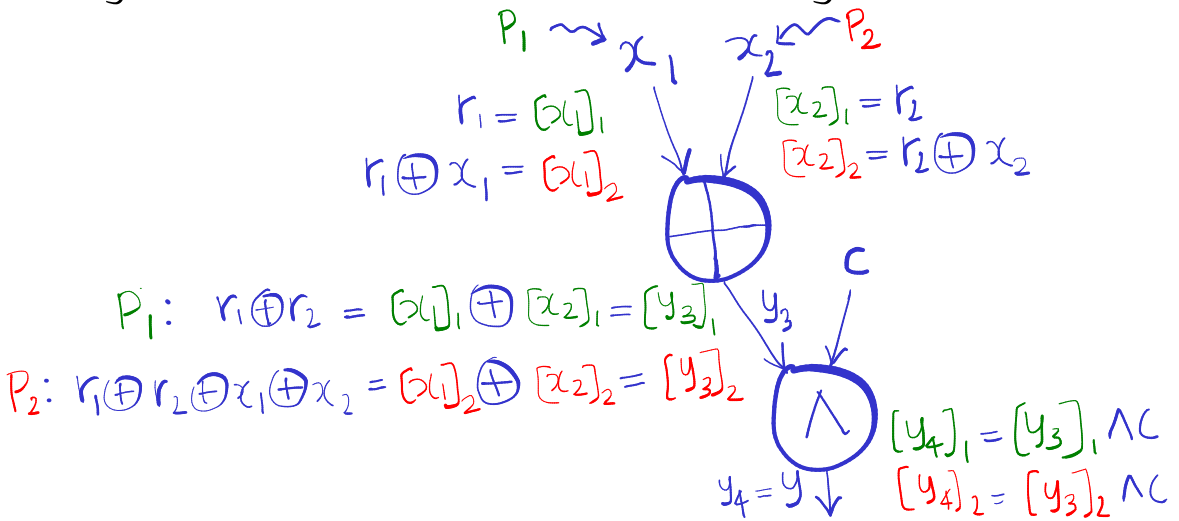

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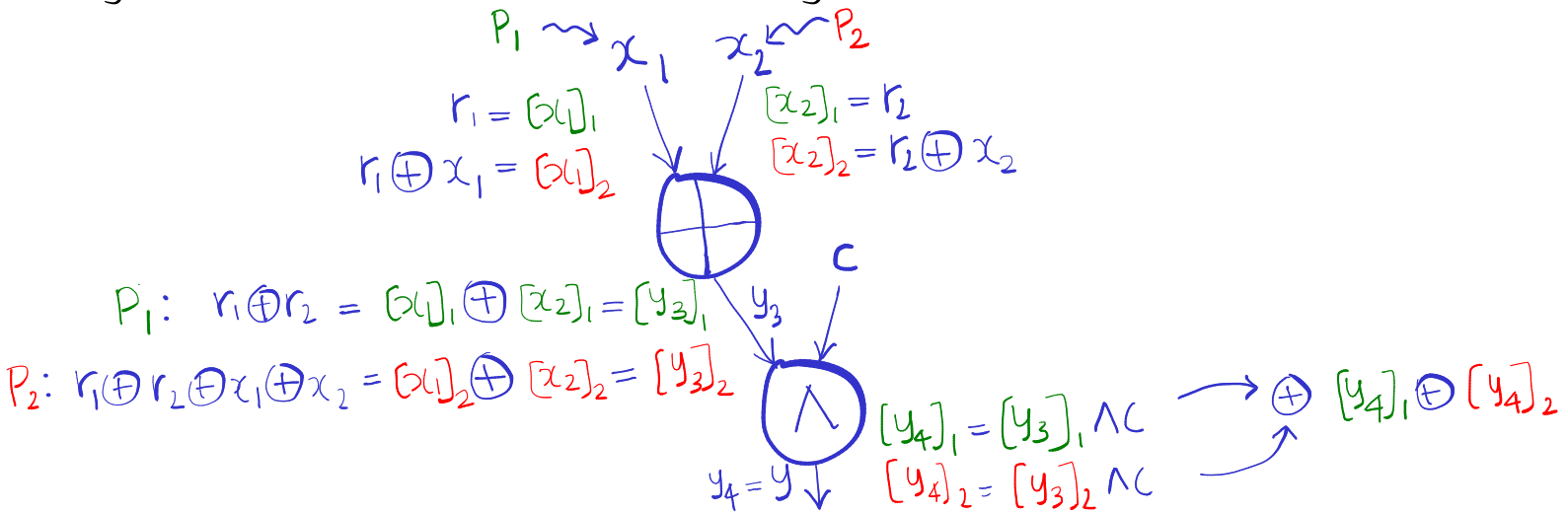

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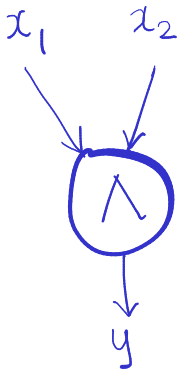

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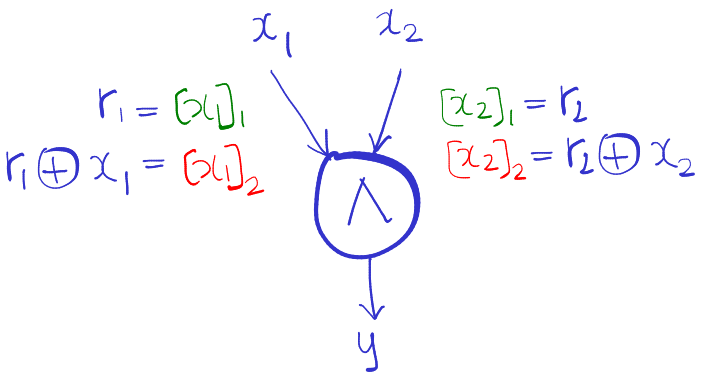

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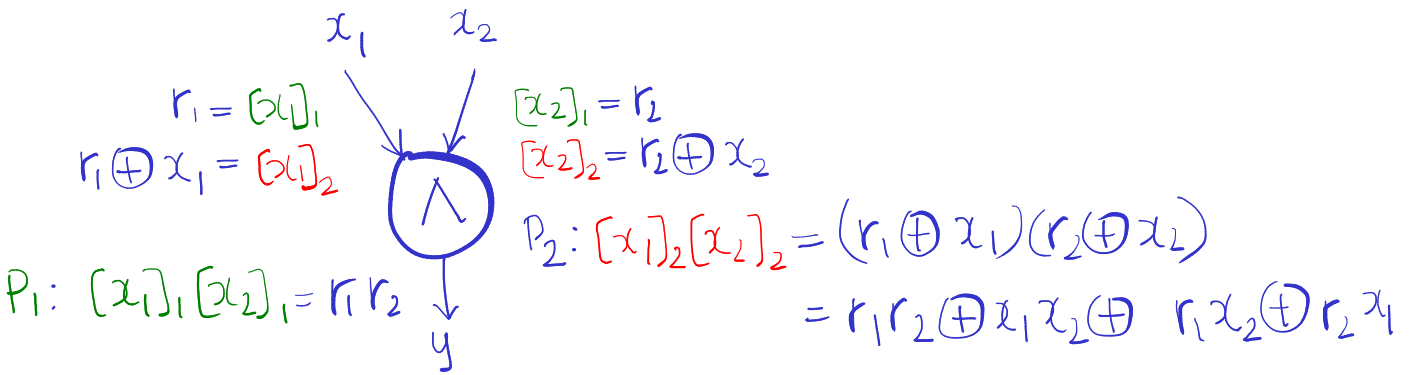

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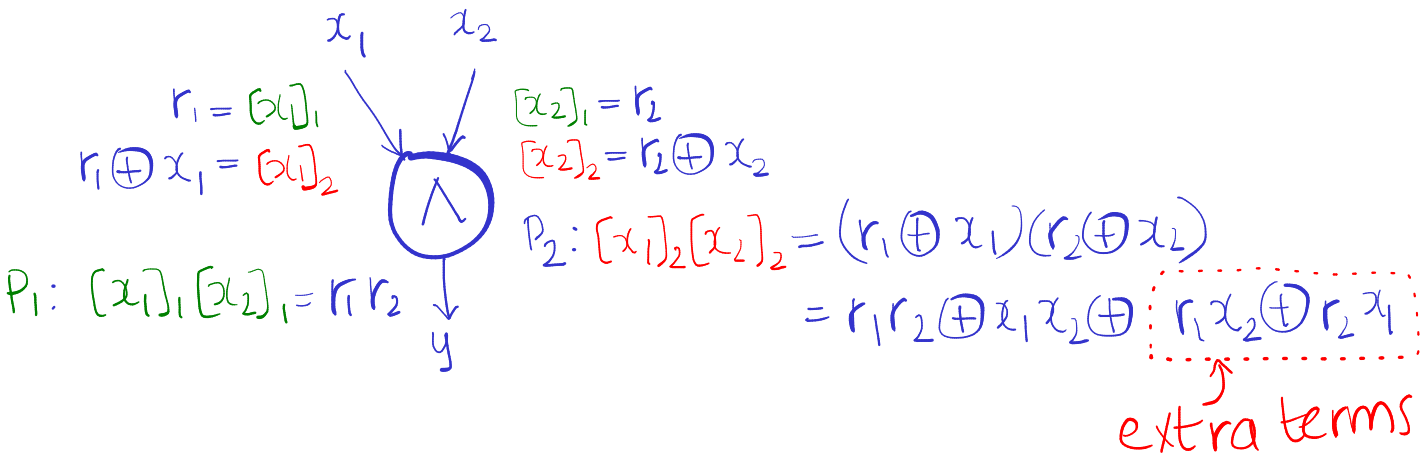

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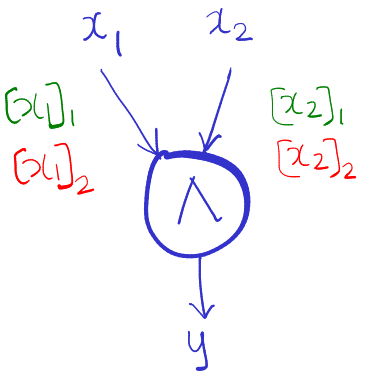

1-out-of-4

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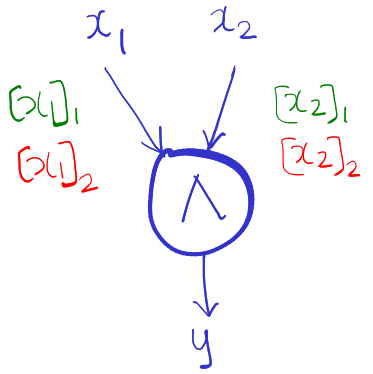

1-out-of-4 $\hat{\wedge}$ OT

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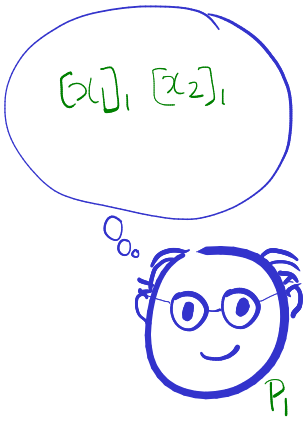

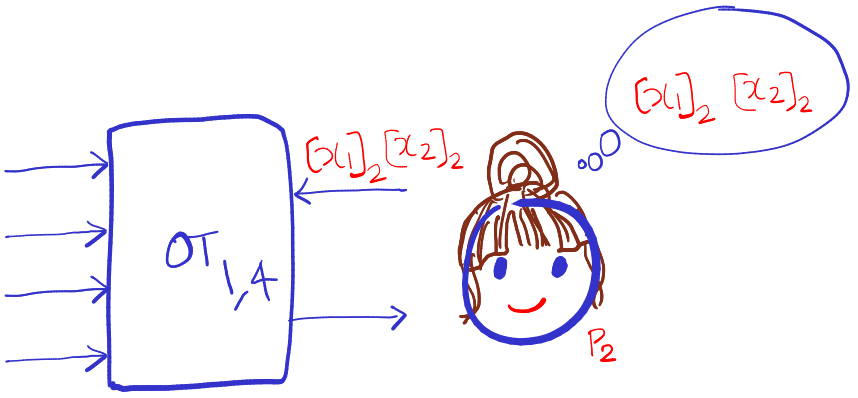

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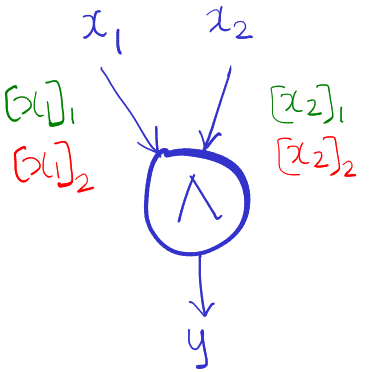

1-out-of-4 $\hat{\wedge}$ OT

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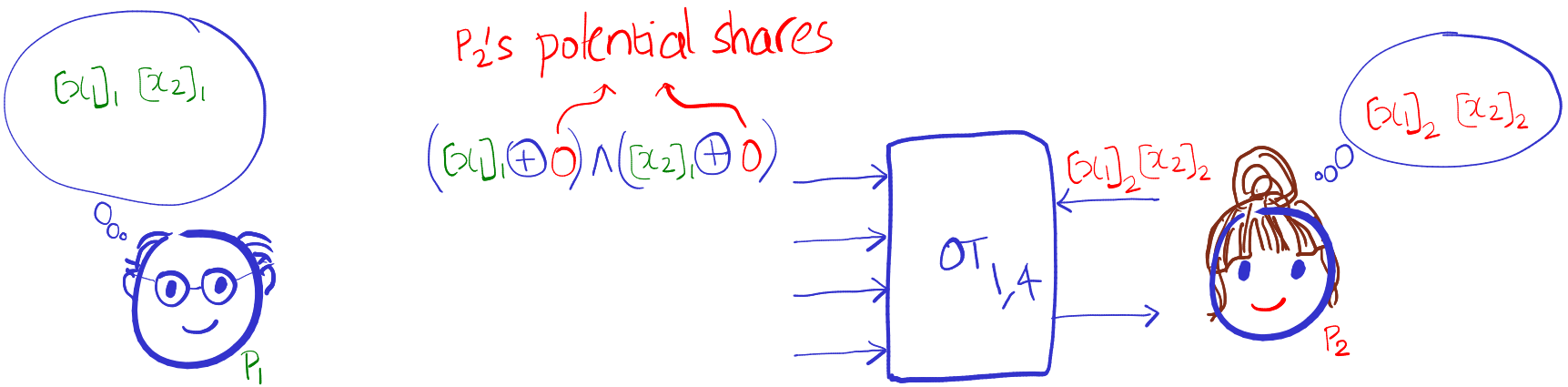

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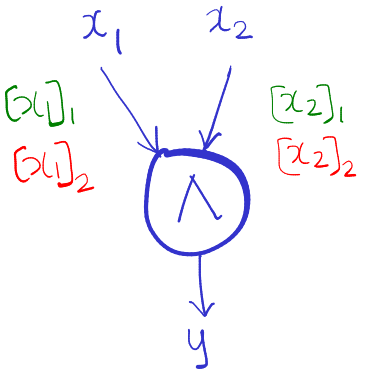

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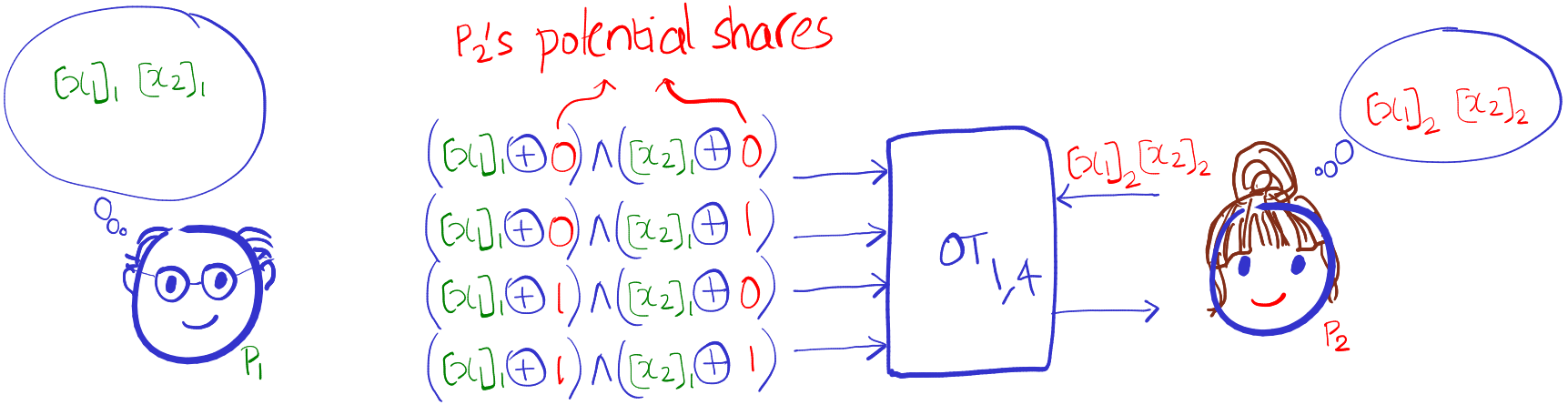

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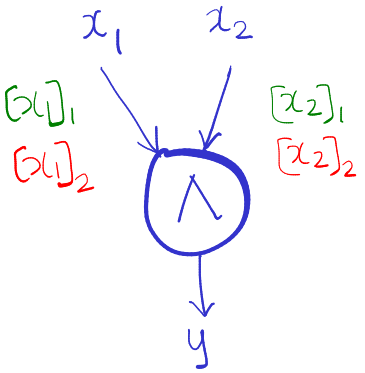

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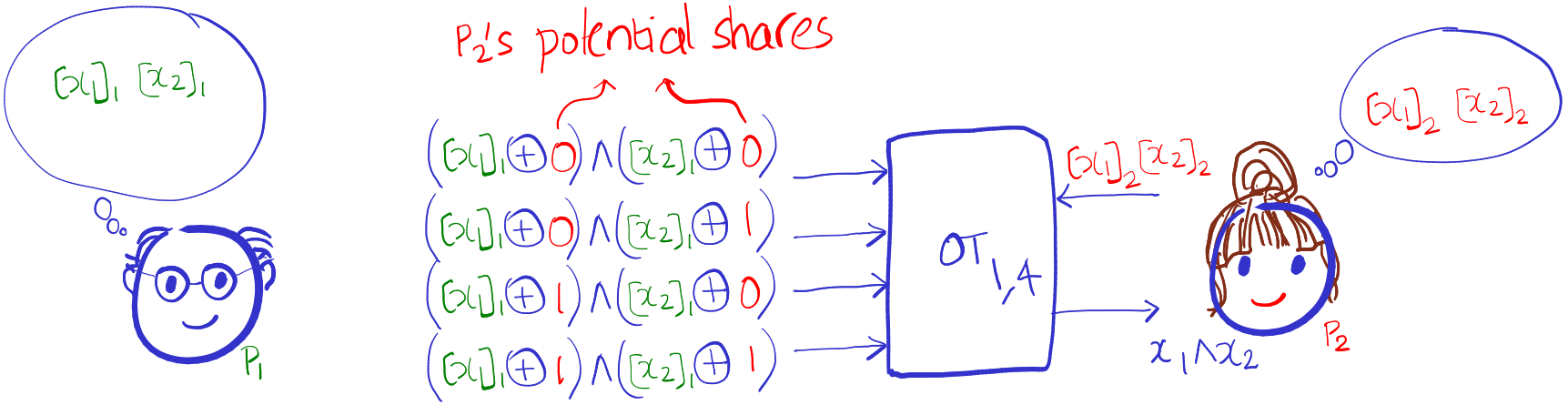

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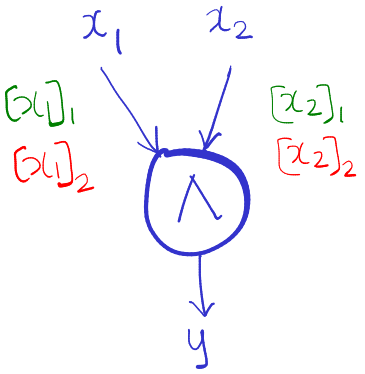

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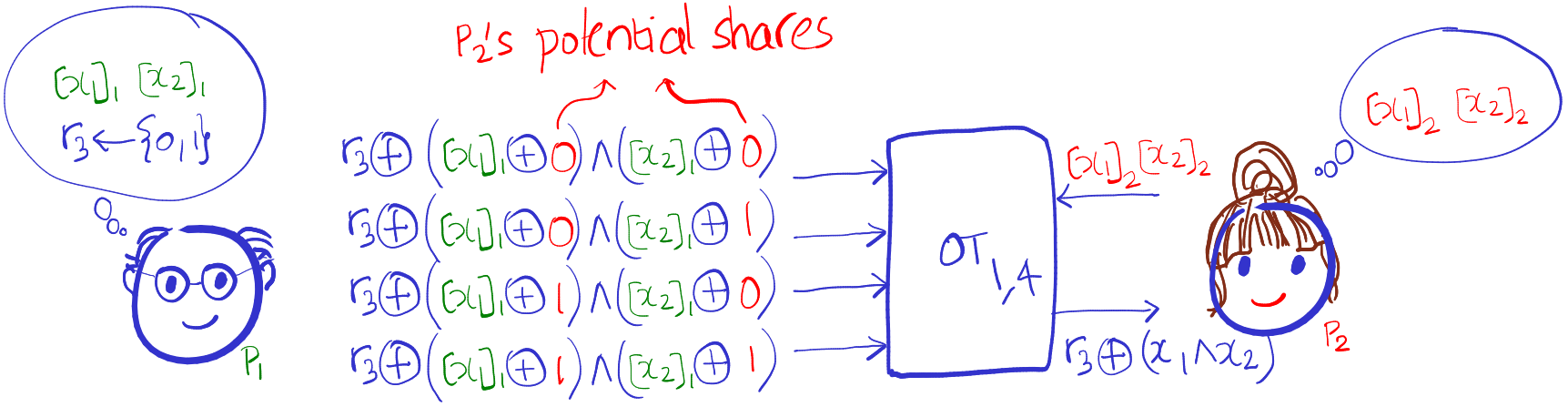

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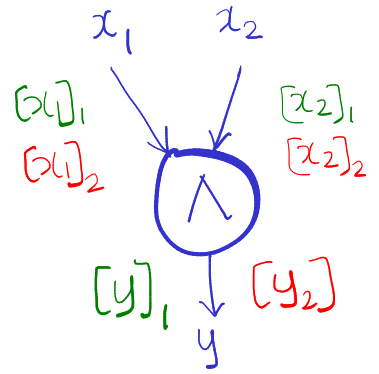

1-out-of 4

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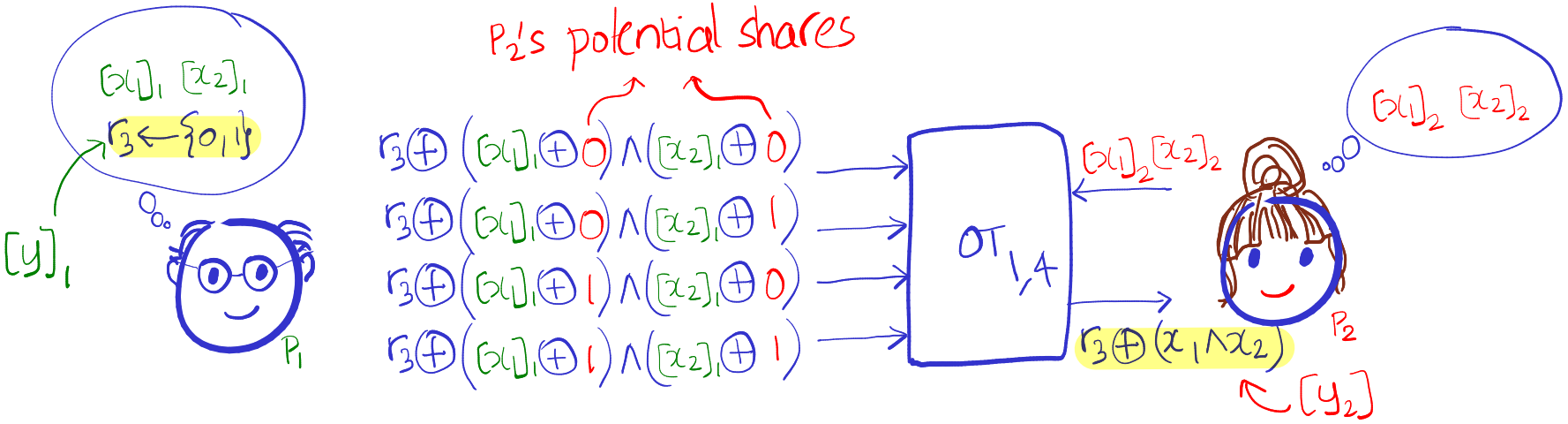
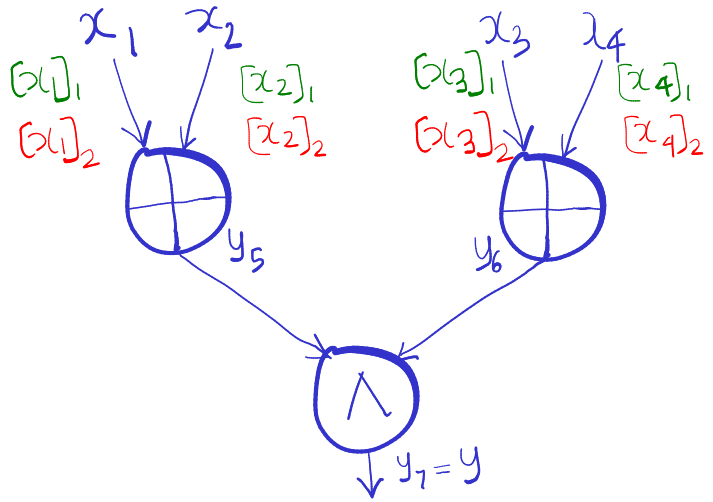


GMW Protocol...



Protocol 2 (for $f : \{0,1\}^{2n} \to \{0,1\}$ represented by circuit C)

1. *Each party secret-shares its input bits with the other party*

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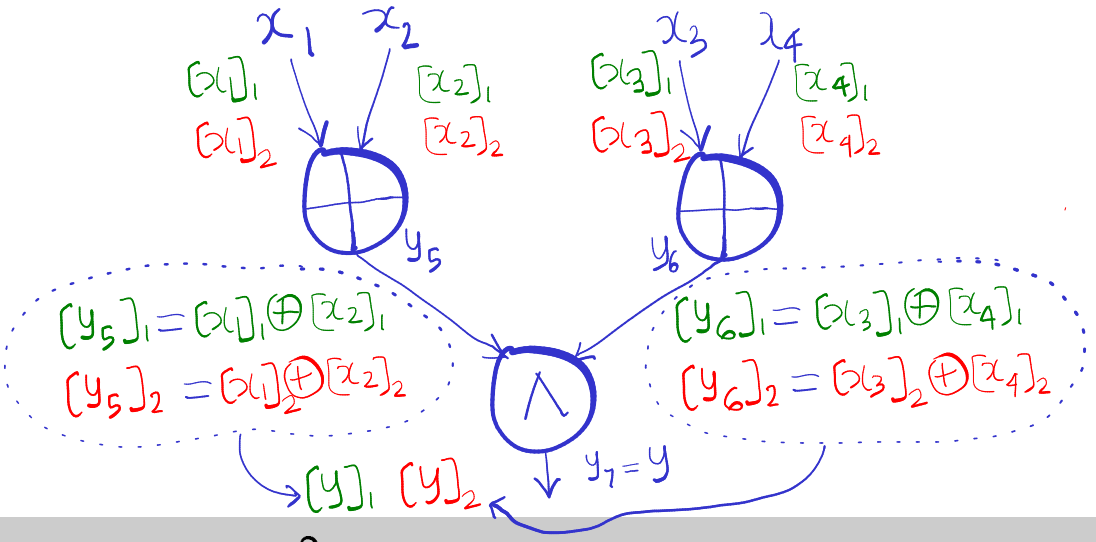

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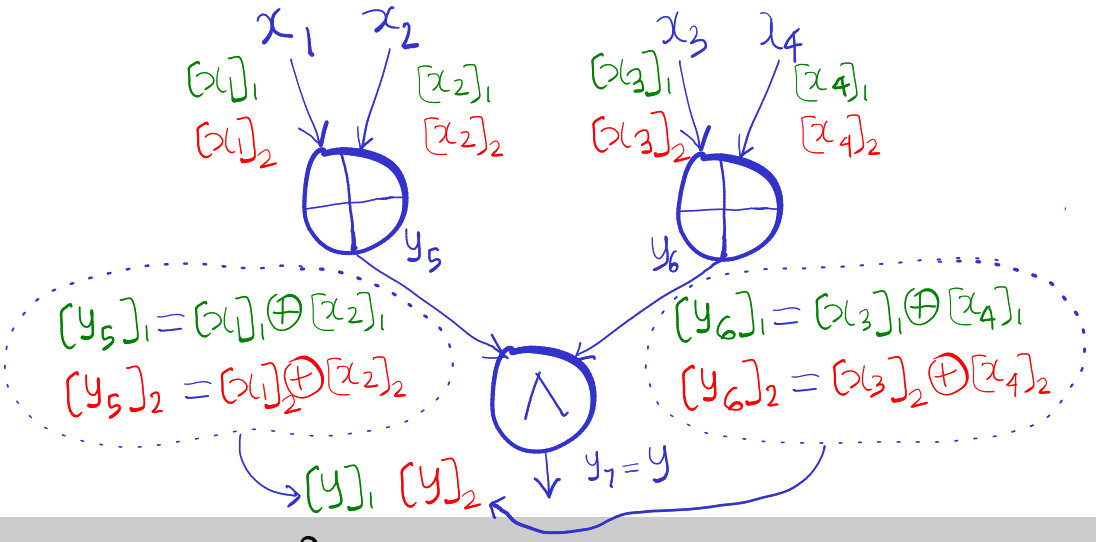

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GMW Protocol...

Theorem 2

OT secure $\Rightarrow$ *GMW protocol computes* f *with computational privacy.*

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Exercise 4

- *Prove* P_1*'s privacy: OT simulator* $\rightarrow$ *simulator* Sim_{P_2} *for* P_2.
- *Prove* P_2*'s privacy: simulator* Sim_{P_1} *for* P_1.

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  - Counter-example: $\wedge$

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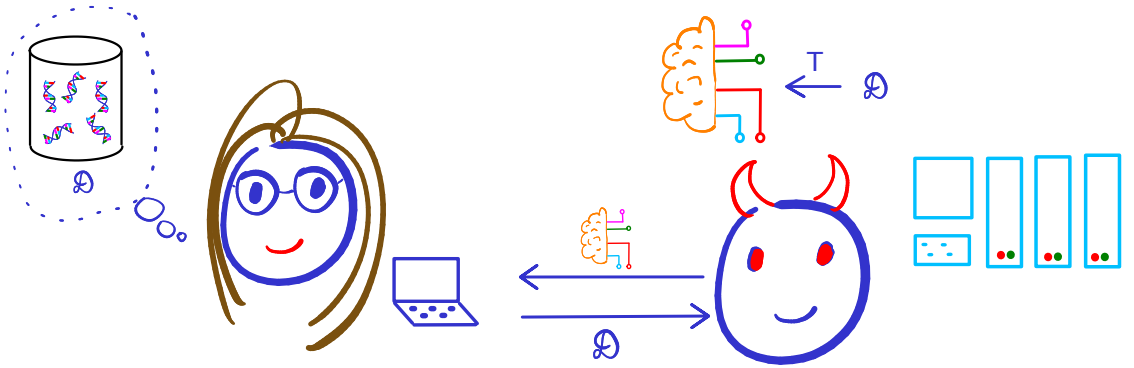
- For privacy against malicious parties, use zero knowledge proof (ZKP)
  - Semi-honest-secure MPC + ZKP $\rightarrow$ malicious-secure MPC

Next Two Lectures

- Outsourcing in client–server model



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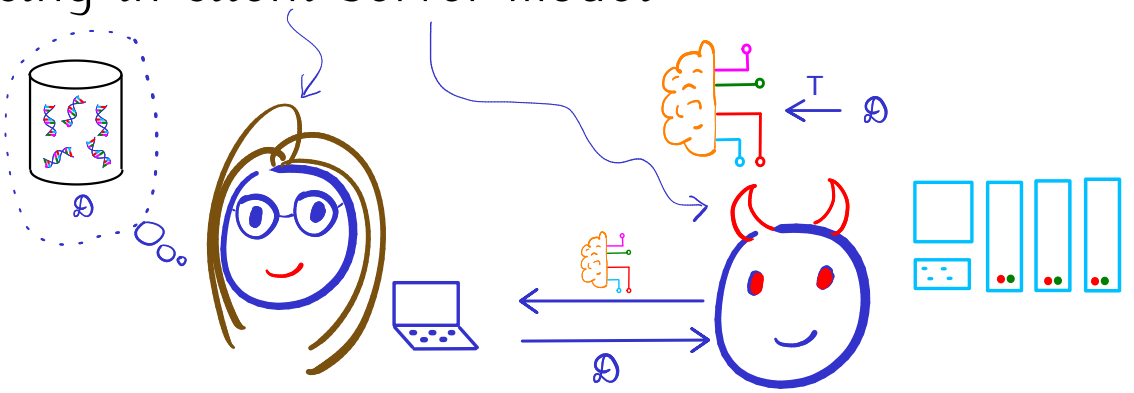

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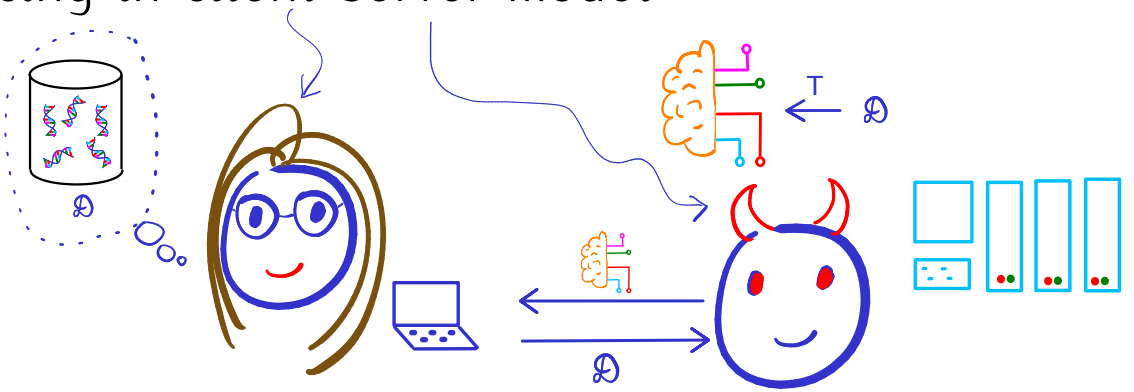

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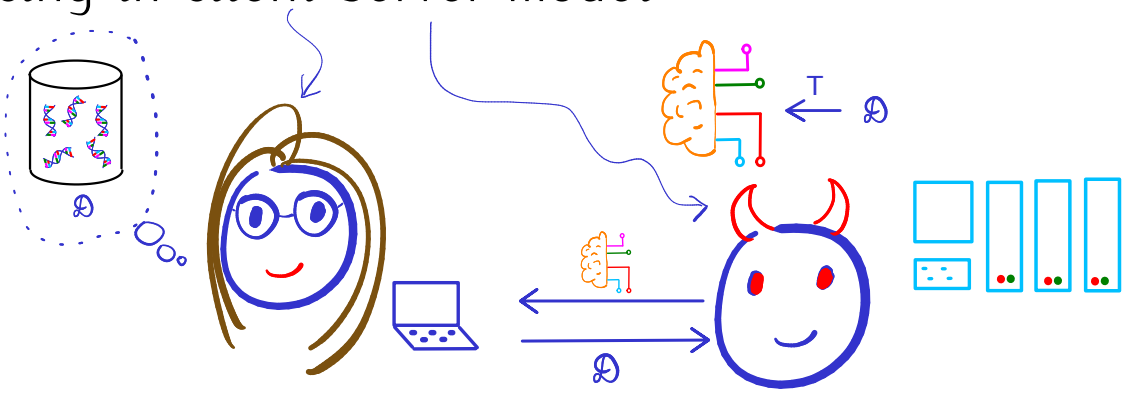

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- Verifiable outsourcing
  - Using succinct non-interactive argument (SNARG)
  - SNARG for repeated squaring problem in ROM
    - Pietrzak's protocol
  - SNARG for **NP** in ROM (if time permits)
    - Kilian's protocol

References

1. Most of this lecture is based on (slides of) Lecture 18 from Vinod Vaikuntanathan's MIT6875. For a more formal description of the protocols, see [Gol04, §7.3].
2. Claim 1 is folklore. The proof presented here is due to Rotem Oshman (and taken from the slides above)
3. Oblivious transfer was introduced by Rabin [Rab81] (although Wiesner introduced a similar primitive in [Wie83]). Its construction from TDP is from [EGL82].
4. The GMW protocol is from [GMW87]


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