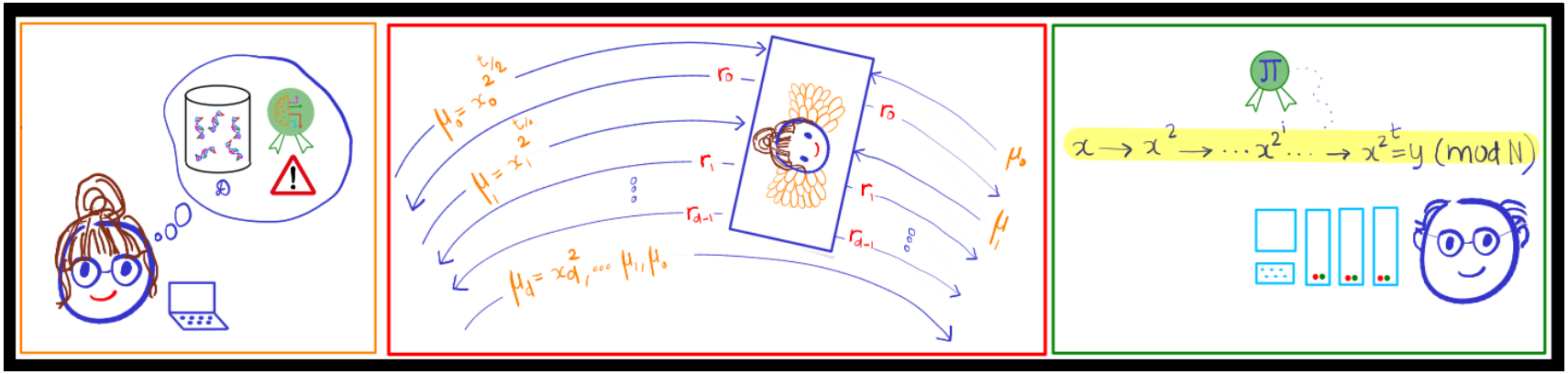


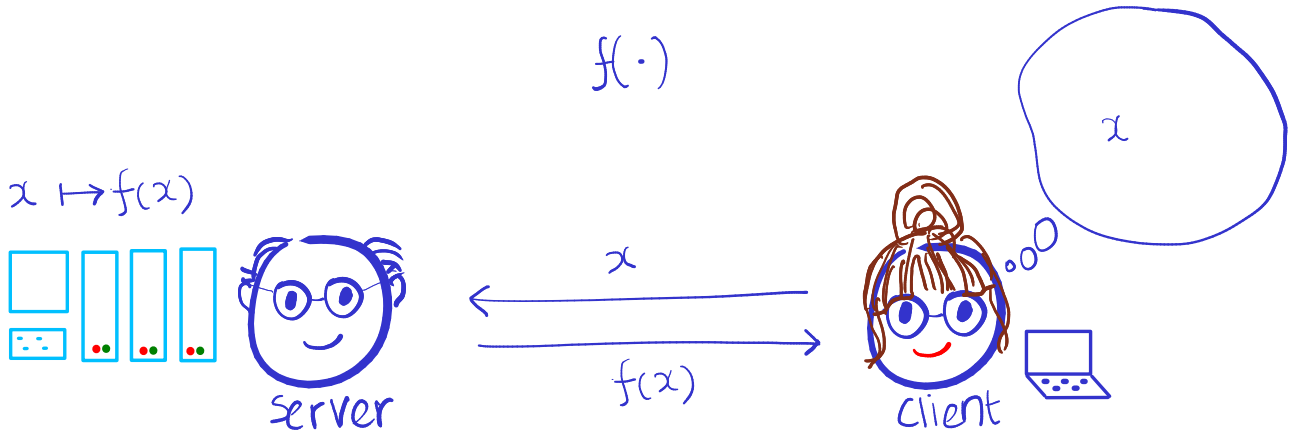
CS783: Theoretical Foundations of Cryptography

Lecture 20 (18/Oct/24)

Instructor: Chethan Kamath

Recall from Last Lecture

- Task 7: secure outsourcing in the client-server setting
- Task 7.a: private outsourcing in the client-server setting



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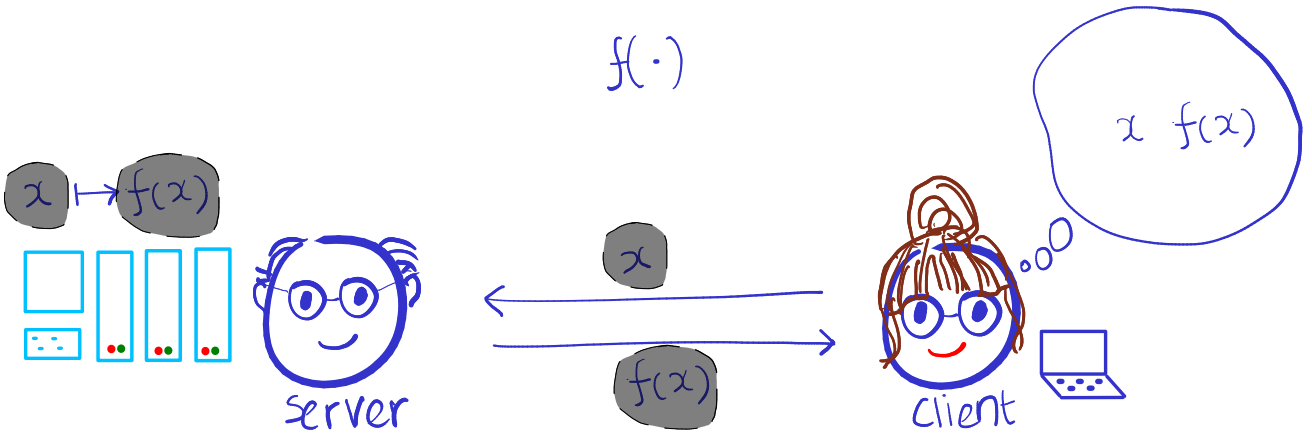

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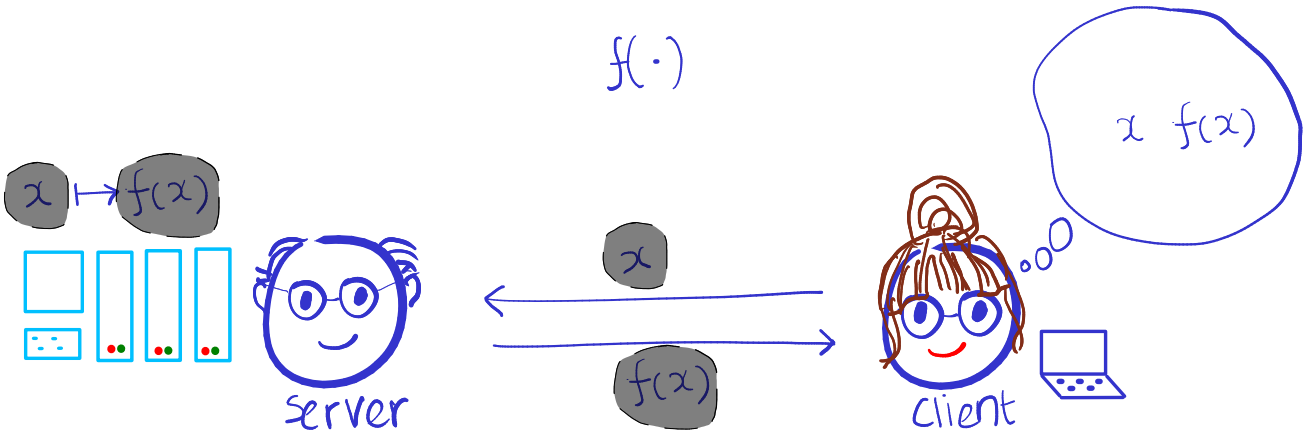

- Key tool: Fully homomorphic encryption (FHE)
  - FHE → private outsourcing of computation
  - In fact: FHE → 2PC of arbitrary functions

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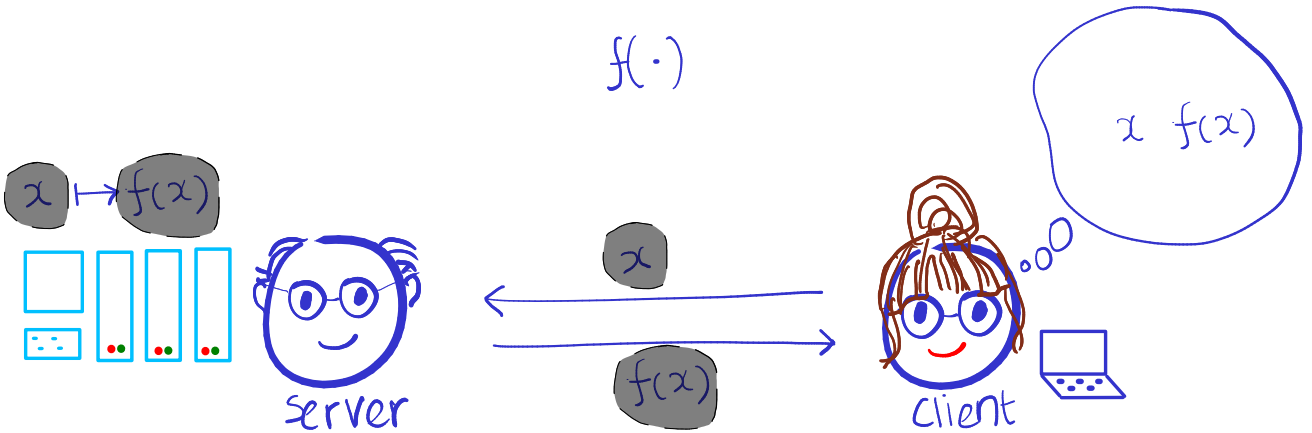

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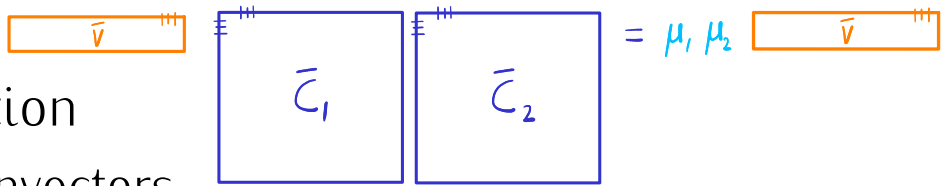
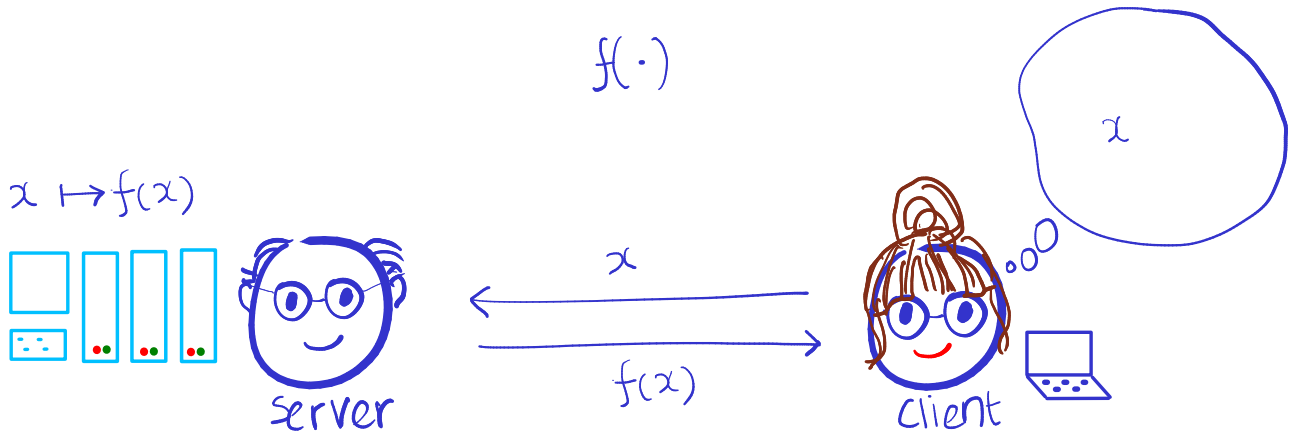


- GSW FHE from LWE assumption
  - Key idea: approximate eigenvectors
  - Similar idea used in approximate key exchange from LWE

Plan for Today's Lecture...

- Task 7: secure outsourcing in the client-server setting



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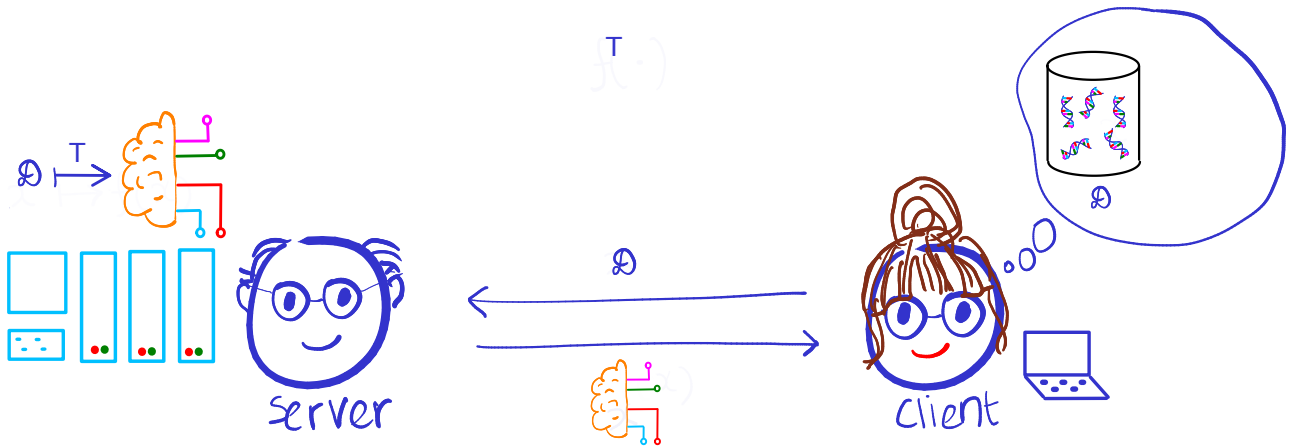

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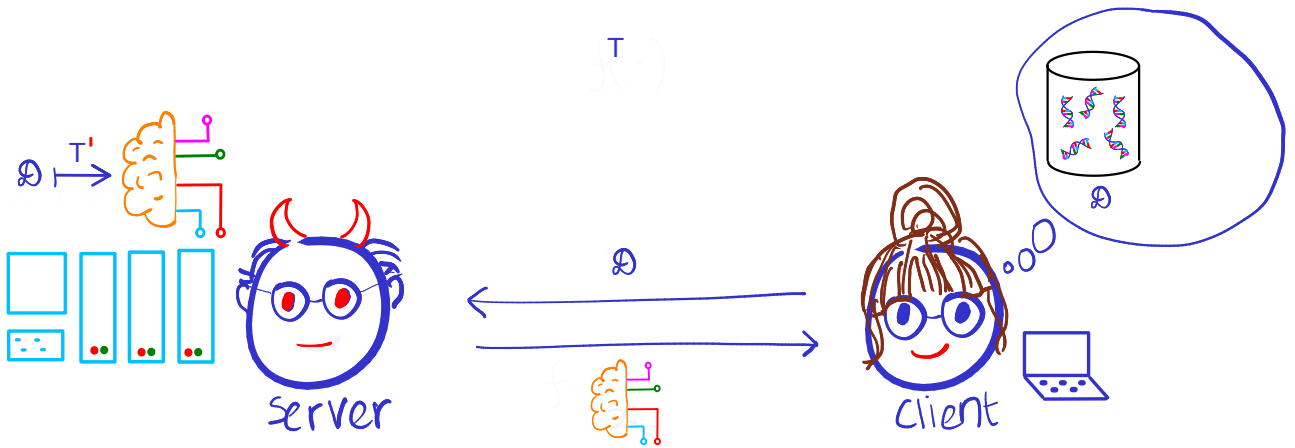
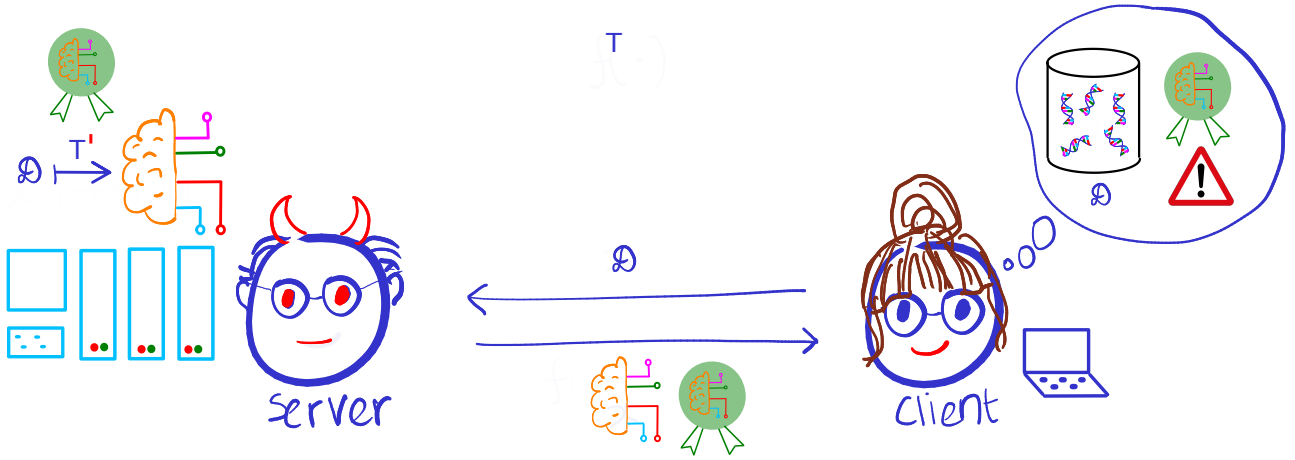
- Task 7.b: *verifiable* outsourcing in the client-server setting



Plan for Today's Lecture...

- Task 7: secure outsourcing in the client-server setting
- Task 7.b: *verifiable* outsourcing in the client-server setting
  - Verifying faster than recomputing



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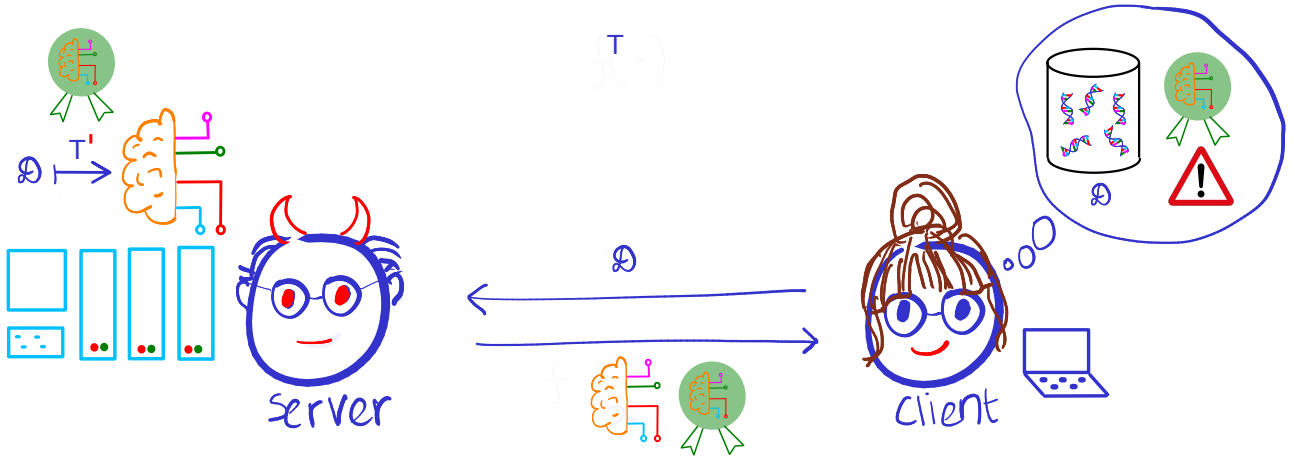
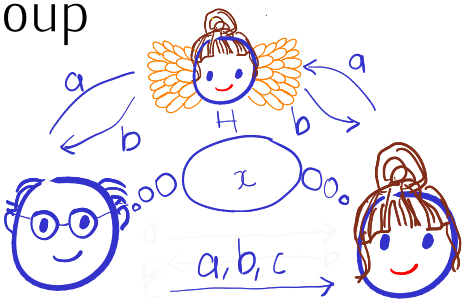


- Key tool: succinct non-interactive argument (SNARG)
- SNARG for repeated squaring function in RSA group
  - Pietrzak's interactive protocol
  - Remove interaction using Fiat-Shamir transform



Plan for Today's Lecture...

General *template*:

1. Identify the task → Task 7.b: verifiable outsourcing
2. Come up with precise threat model M (a.k.a security model)
   - Adversary/Attack: What are the adversary's capabilities?
   - Security Goal: What does it mean to be secure?
3. Construct a scheme Π
4. Formally prove that Π in secure in model M

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Task 7.b: verifiable outsourcing

malicious server

computational soundness

Plan for Today's Lecture...

General *template*:

1. Identify the task → Task 7.b: verifiable outsourcing
2. Come up with precise threat model M (a.k.a security model)
   - Adversary/Attack: What are the adversary's capabilities? → malicious server
   - Security Goal: What does it mean to be secure? → computational soundness
3. Construct a scheme Π → SNARG-based
4. Formally prove that Π in secure in model M
   → in random-oracle model

Plan for Today's Lecture

1. Verifiable Outsourcing of Computation
   - Succinct Non-Interactive Argument (SNARG)

2. SNARG for Repeated Squaring
   - Step I: Interactive Proof for Repeated Squaring
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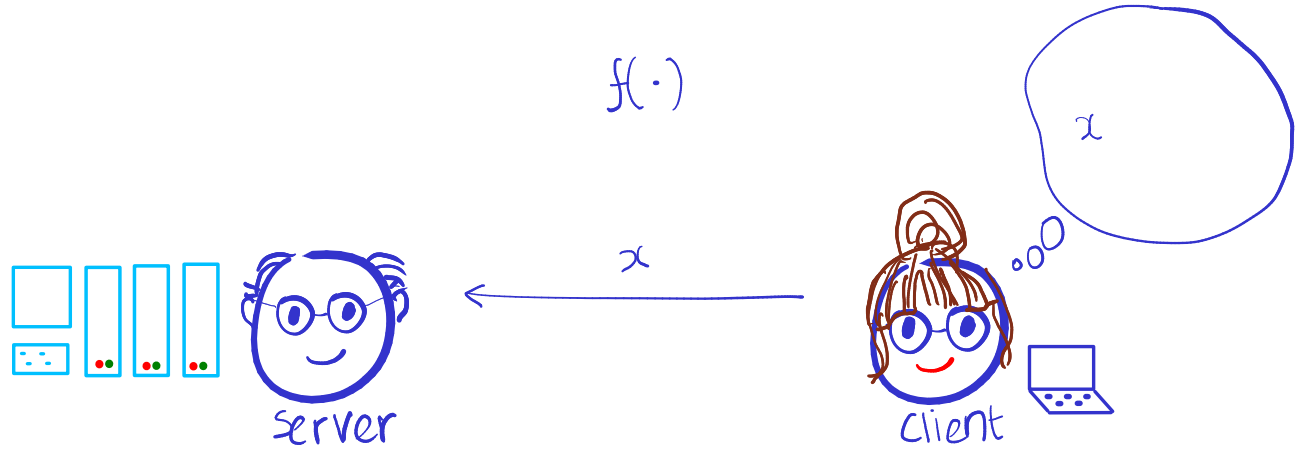
Verifiable Outsourcing: Setting and Requirements



- Setting:
  - Client is resource-constrained and server is powerful
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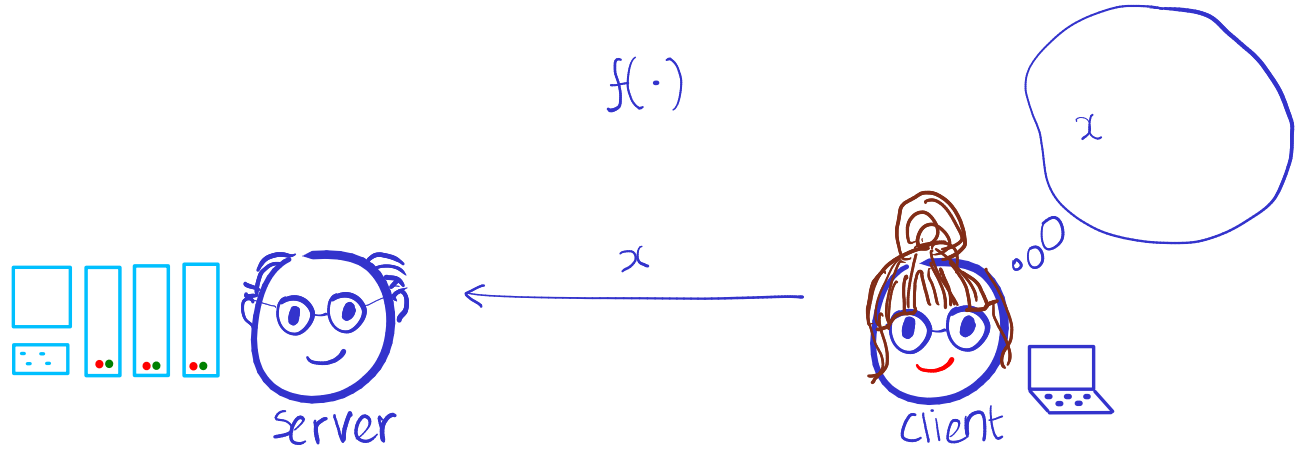

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DTIME(t) → computation

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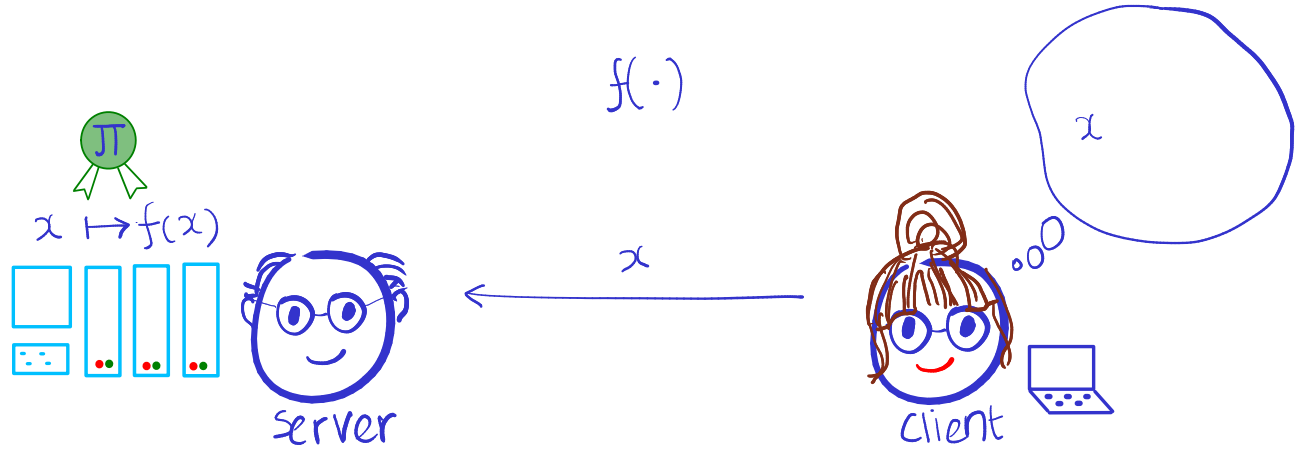

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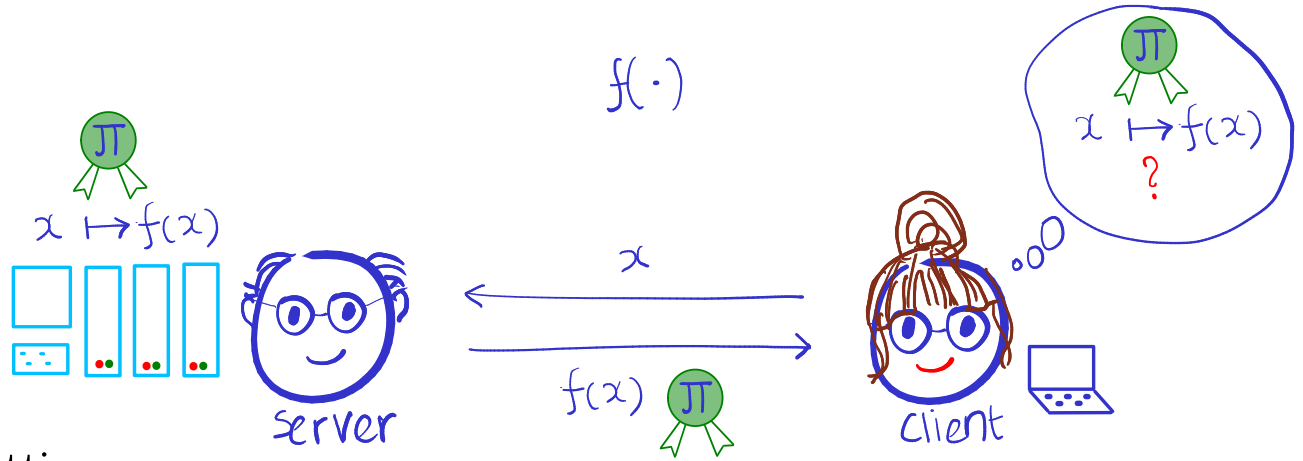

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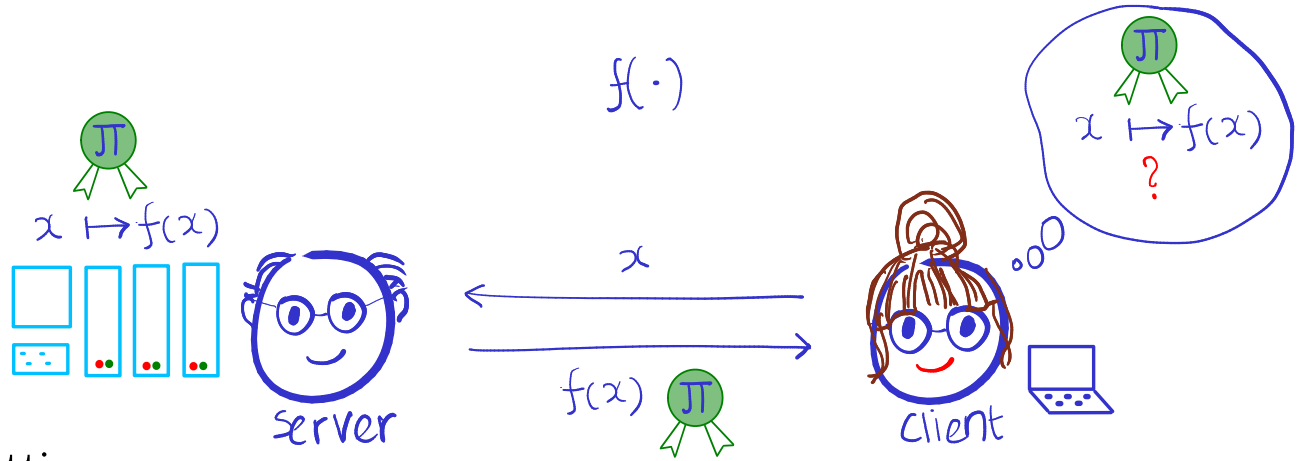

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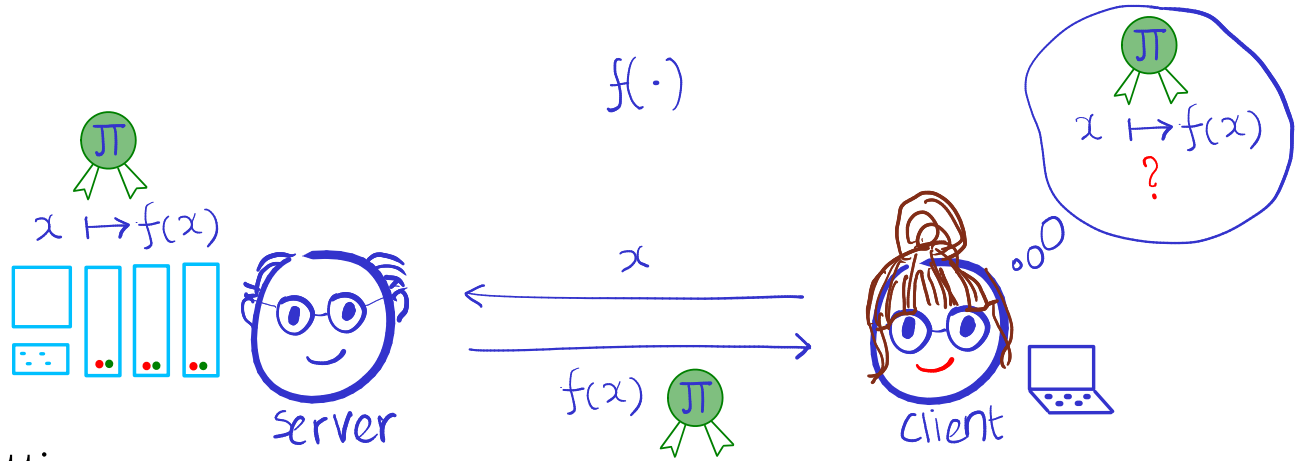

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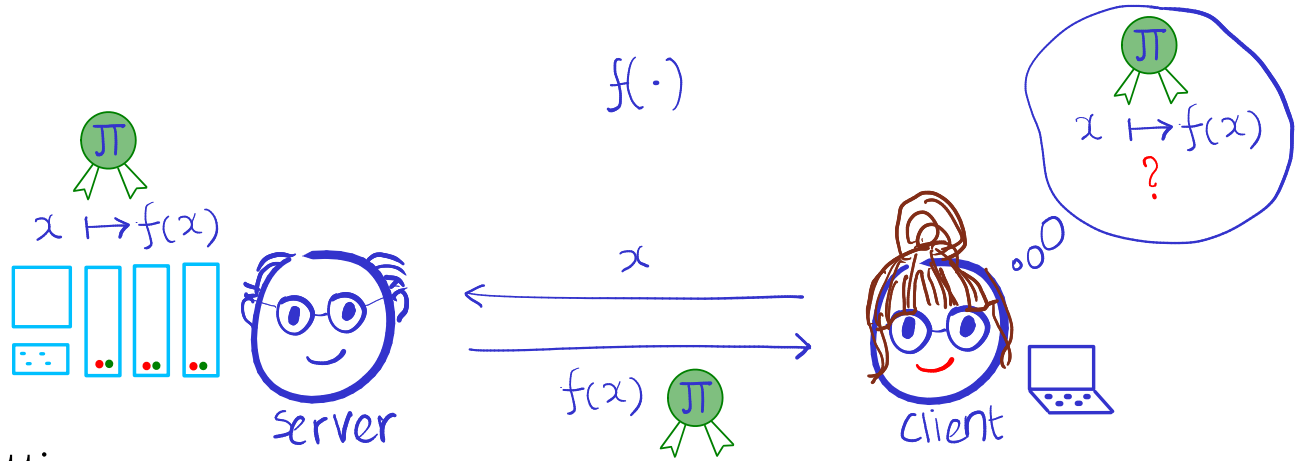

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- Requirements:
  - Complete: if $y = f(x)$ then client should *accept* π
  - Sound: if $y \neq f(x)$ then client should *reject* π
  - Efficient:
    - Server-side: overhead to compute π should be low
    - Client-side: verify π much faster than recomputing $f(x)$

ideally $O(t)$

ideally $O(\log(t))$

Plan for this Session

1. Verifiable Outsourcing of Computation
   - Succinct Non-Interactive Argument (SNARG)

2. SNARG for Repeated Squaring
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Recall Interactive Proofs

- Difference from NP proofs:
  - 1 Verifier V is *randomised*
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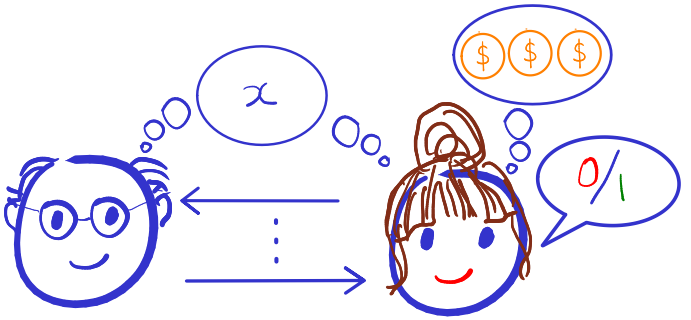

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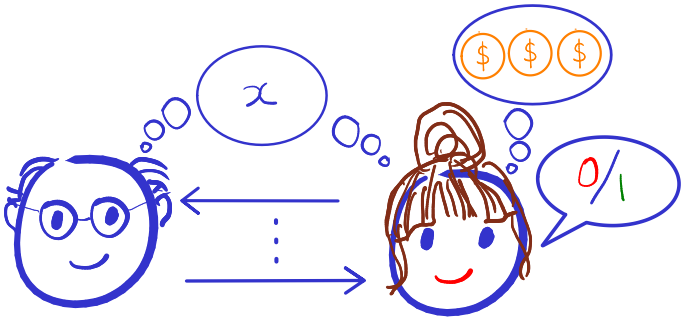

Defintion 1 (Interactive proof for a language $\mathcal{L}$)

An interactive protocol (P, V) *for* $\mathcal{L}$ *that is:*

completeness error $\varepsilon_c(n)$

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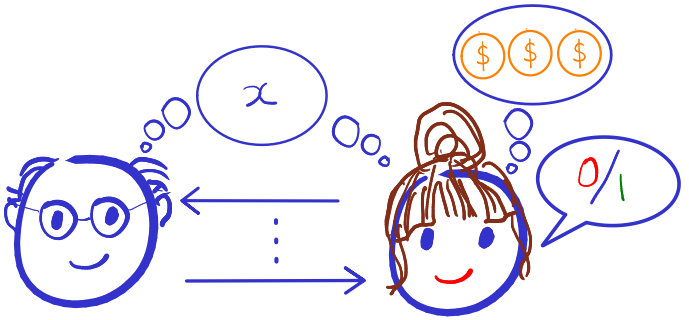

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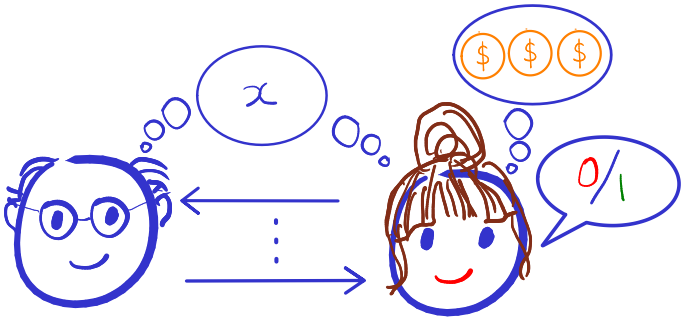

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- We saw (zero-knowledge) IP for GI and GNI
  - ? What was the communication complexity for those IPs?

(size of transcript)

Let's Define Succinct, Non-Interactive Proofs for $\mathcal{L}_f$...

- Recall our requirements for verifiable outsourcing:
  - → Non-interactive: server sends π in one go
  - Complete: if $y = f(x)$ then client should *accept* π
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Exercise 1 (Problem with Defintion 2? It is too strong)

Show that if $\mathcal{L}_f \in \mathsf{DTIME}(t)$ *has succinct non-interactive proof as per Defintion 2 then* $\mathsf{DTIME}(t) \subseteq \mathsf{NTIME}(o(t))$

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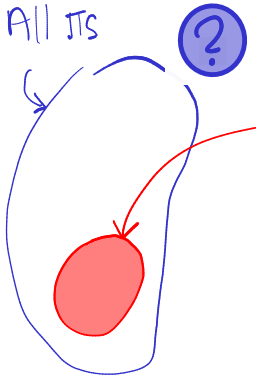
- The way around?

Let's Define Succinct, Non-Interactive Proofs for $\mathcal{L}_f$...



The way around? Relax to computational soundness=argument

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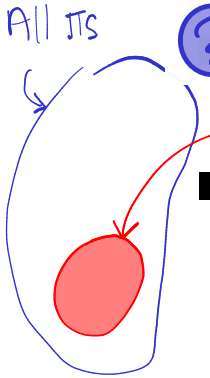
Let's Define Succinct, Non-Interactive Proofs for $\mathcal{L}_f$...



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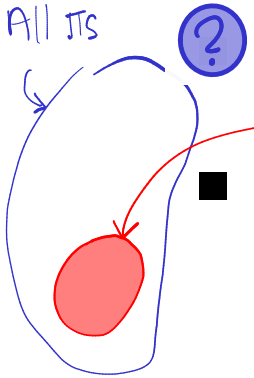

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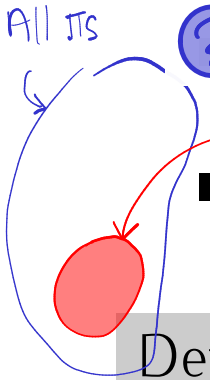

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A non-interactive protocol $(\mathsf{P}^H, \mathsf{V}^H)$ *for* $\mathcal{L}_f$ *that is:*

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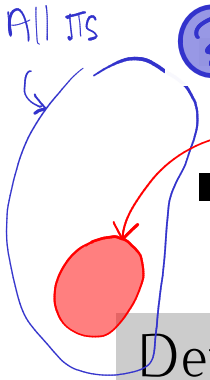

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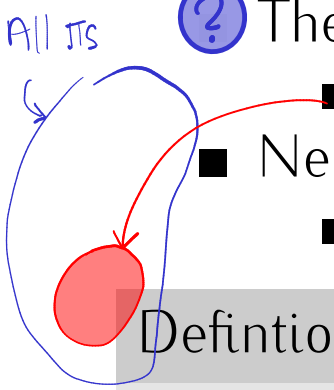

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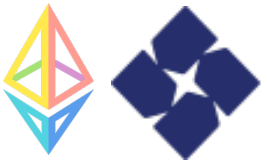
How to carry out verifiable outsourcing using SNARGs?

Where are SNARGs Useful?

- Compute as a service (same as *private* outsourcing)
- Blockchain: Ethereum, Starkware etc have deployed SNARGs



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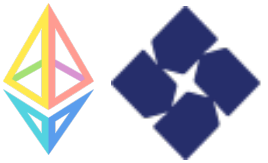

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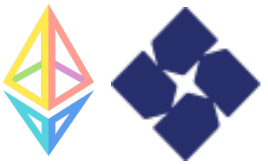

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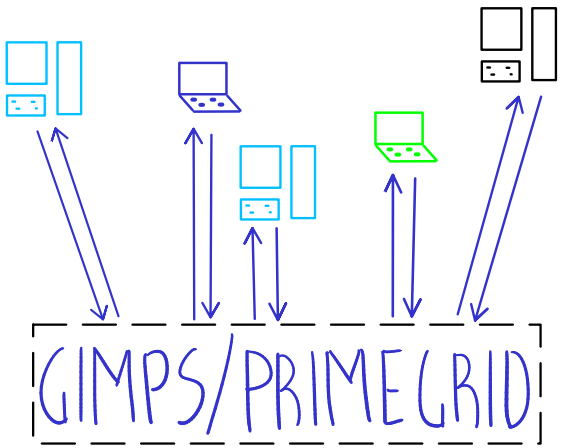

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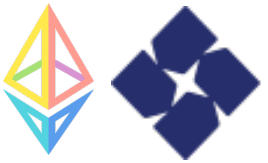

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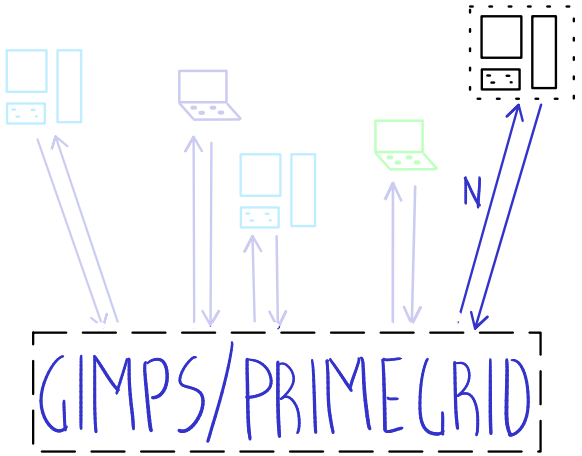

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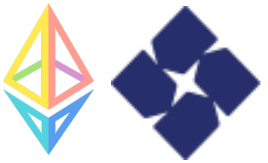

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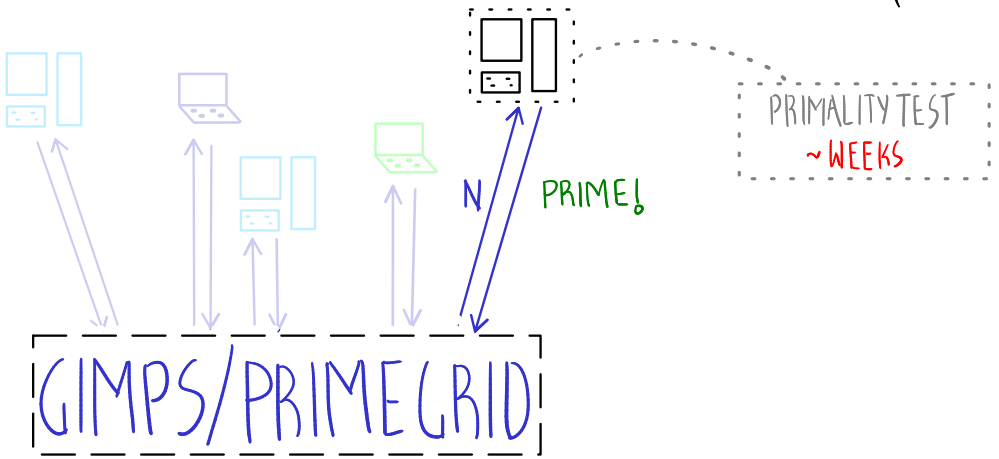

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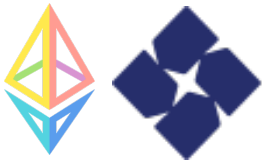

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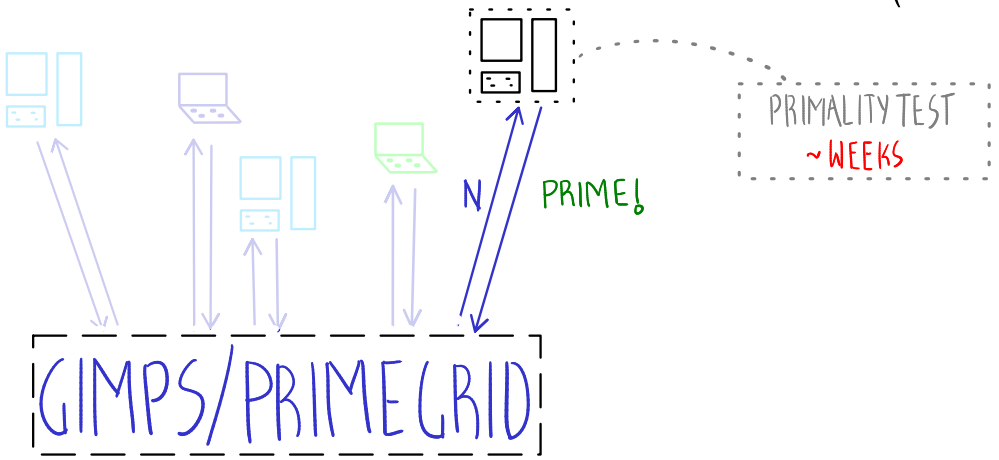

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WIKIPEDIA:LARGEST-KNOWN PRIME NUMBER

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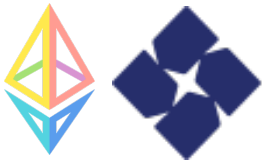

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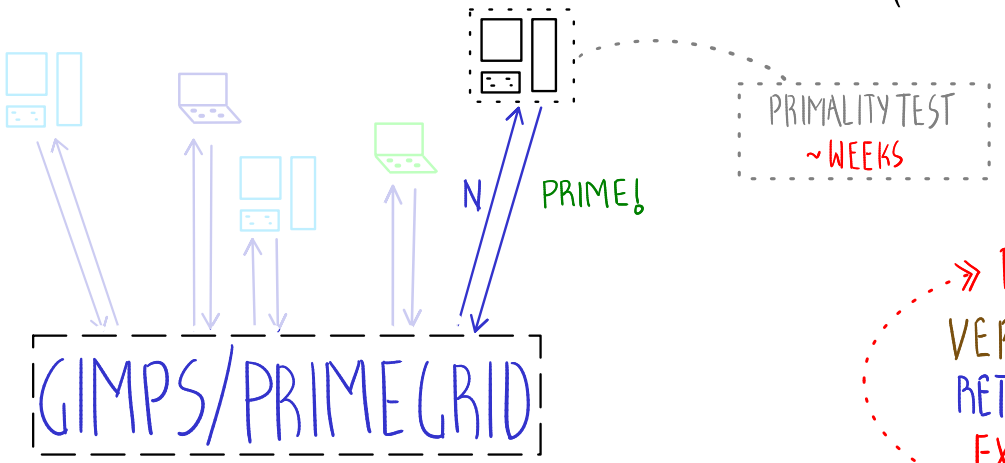

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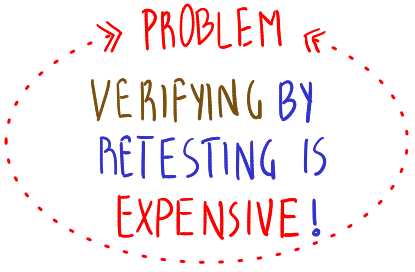

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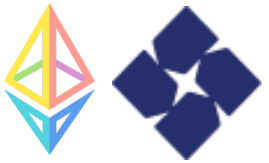

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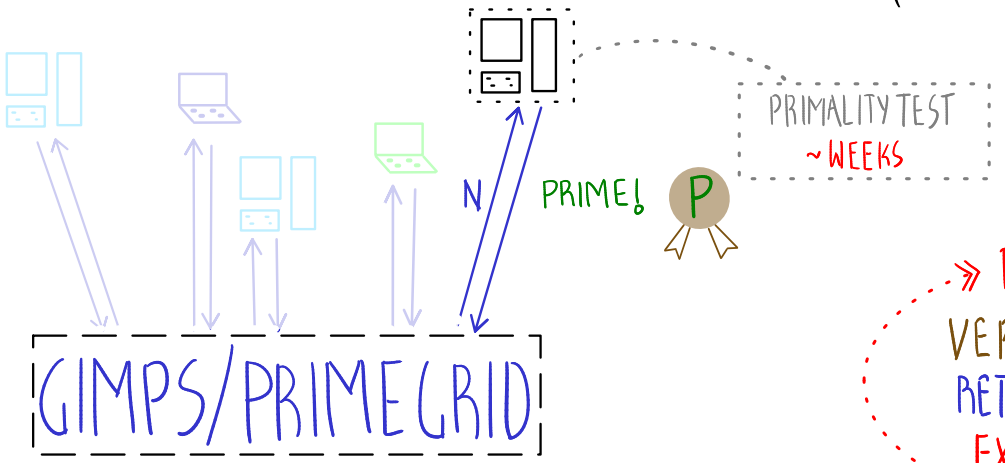

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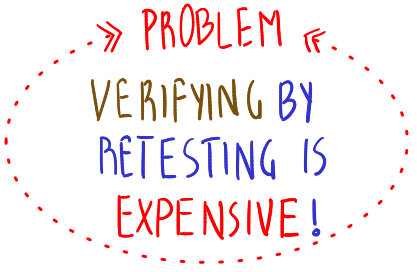

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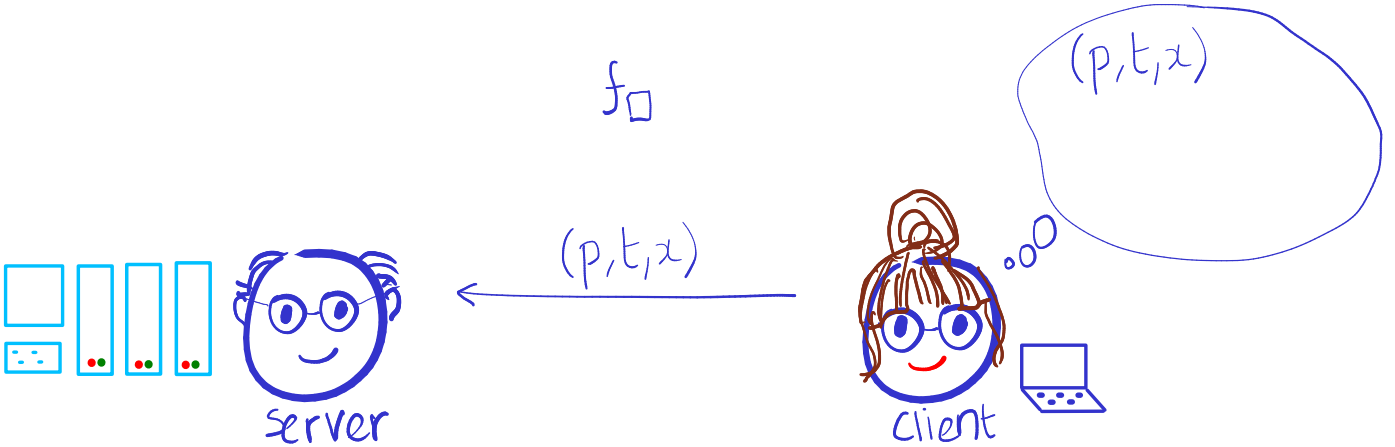
Plan for this Session

1. Verifiable Outsourcing of Computation
   - Succinct Non-Interactive Argument (SNARG)

2. SNARG for Repeated Squaring
   - Step I: Interactive Proof for Repeated Squaring
   - Step II: Remove Interaction via Fiat-Shamir Transform

Repeated Squaring Function over RSA Group...



- Repeated squaring function modulo prime p:

$$f_\square(p, t, x) := x^{2^t} \bmod p$$

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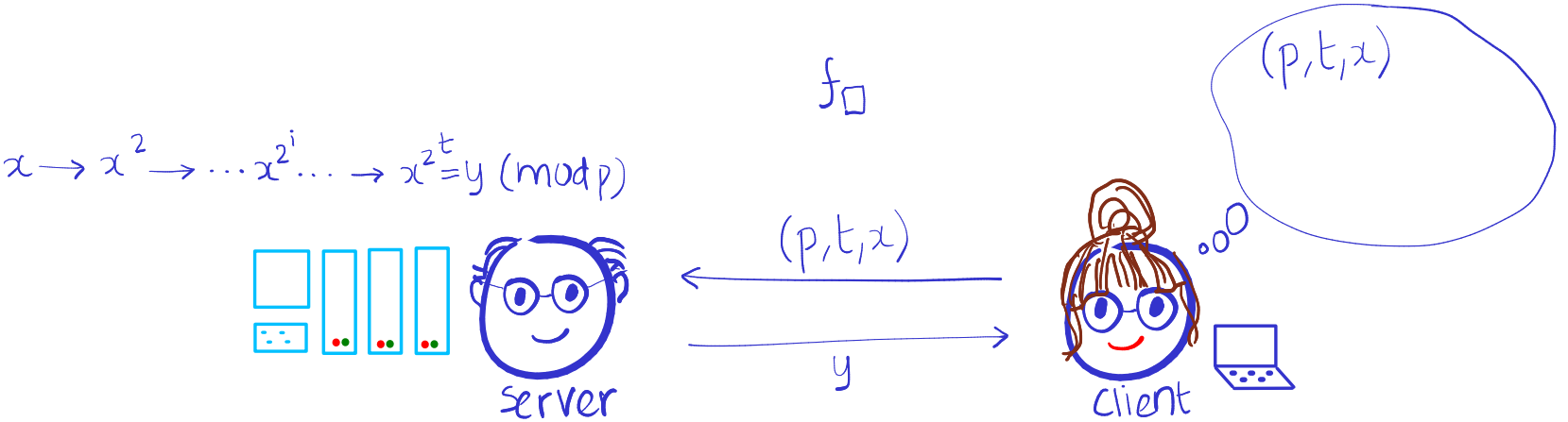

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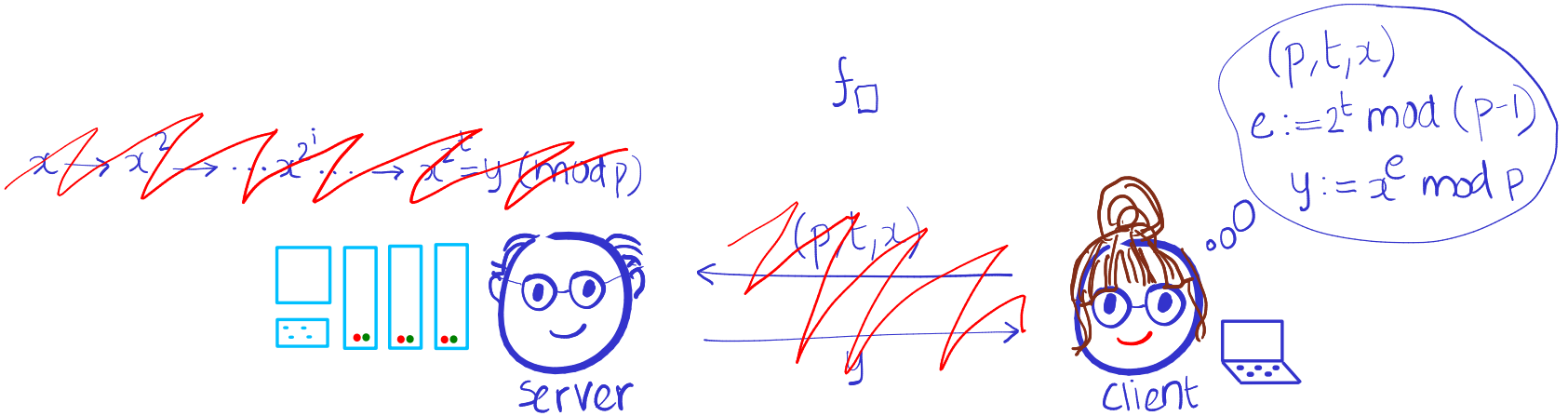

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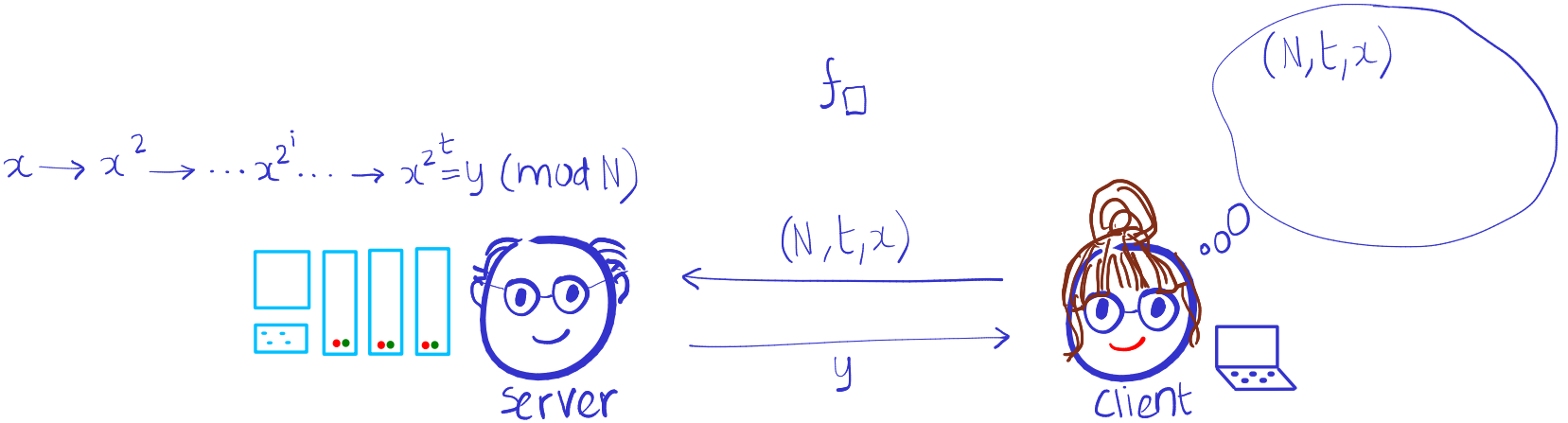

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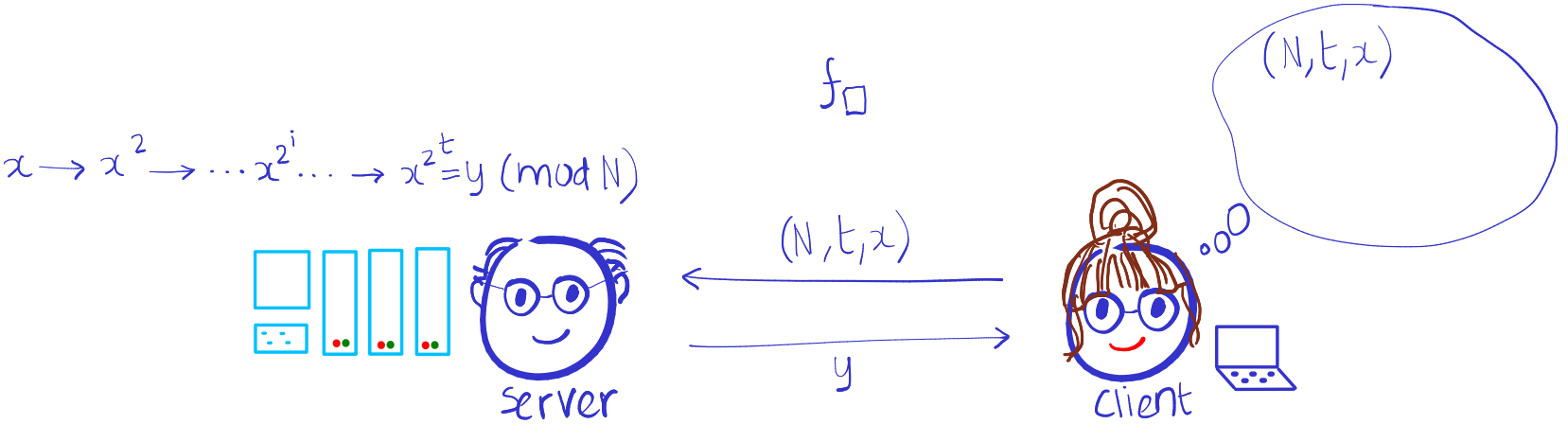

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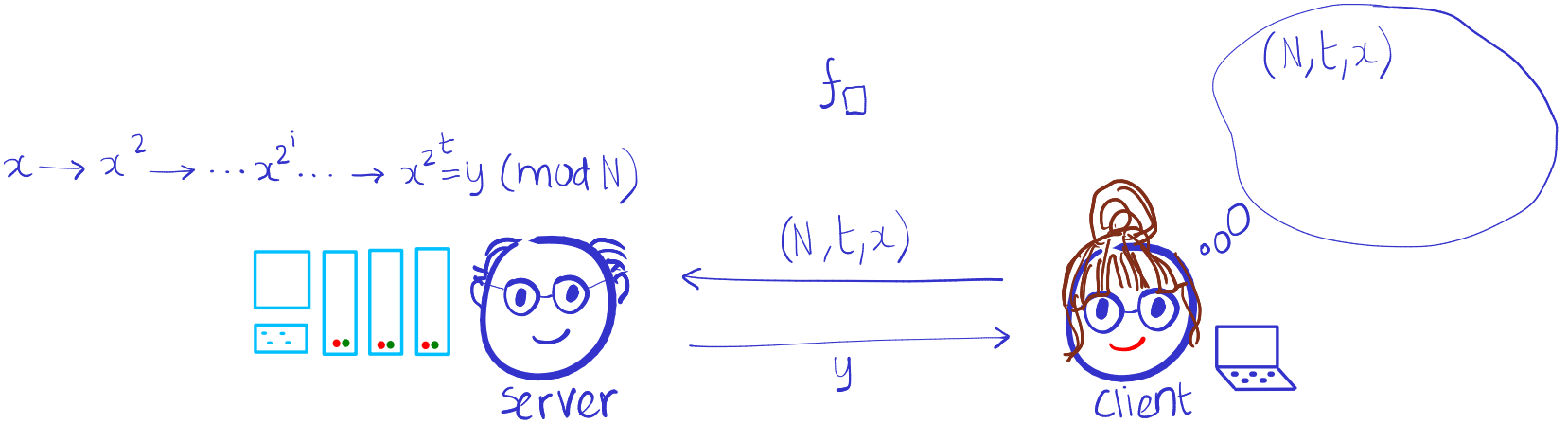

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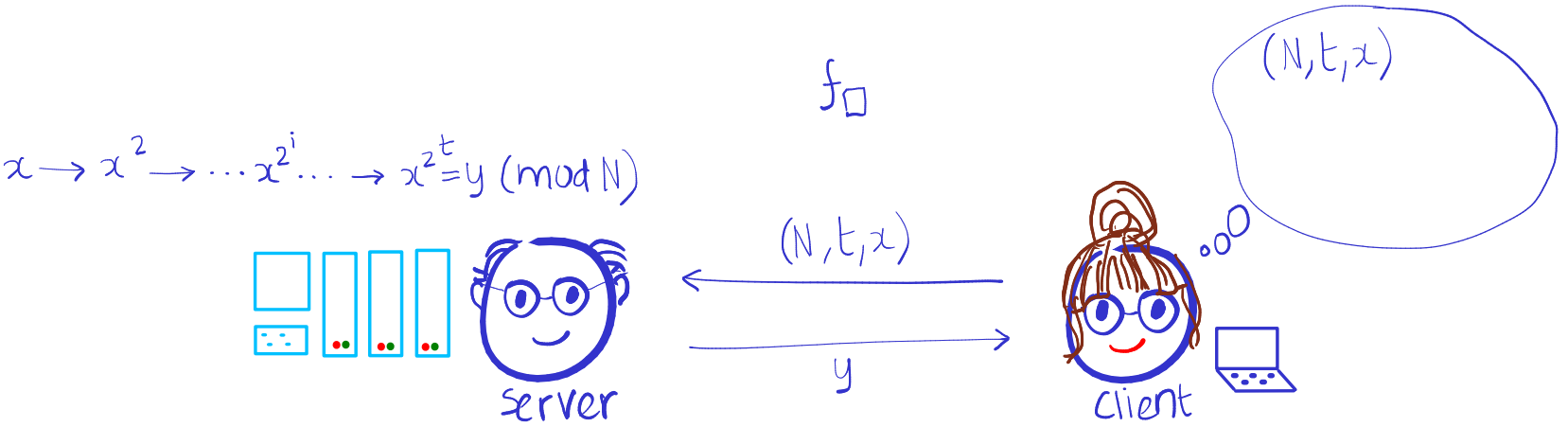

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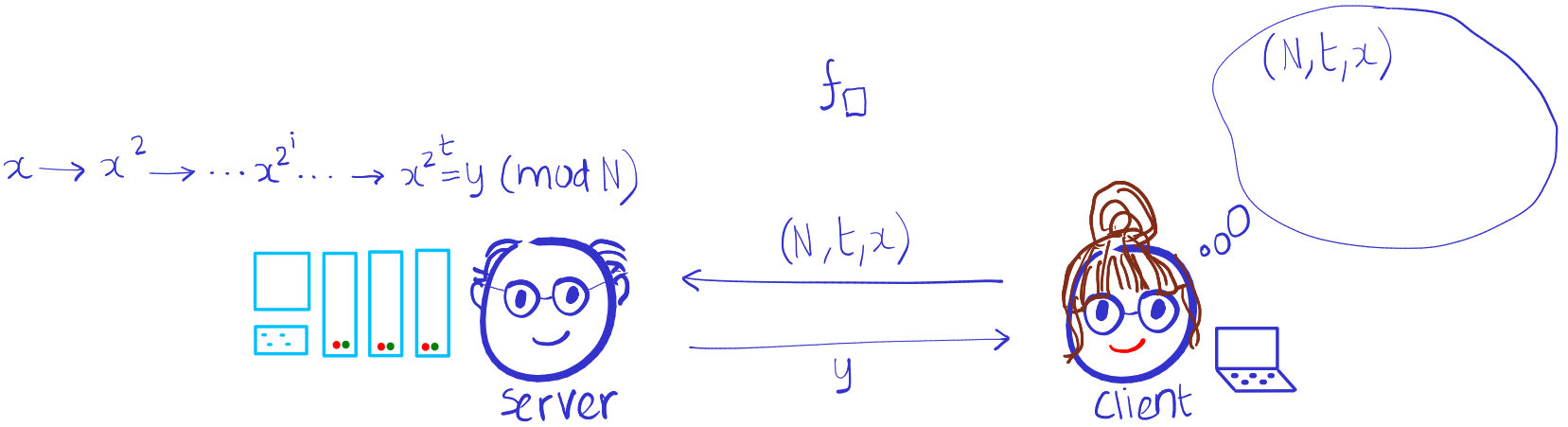

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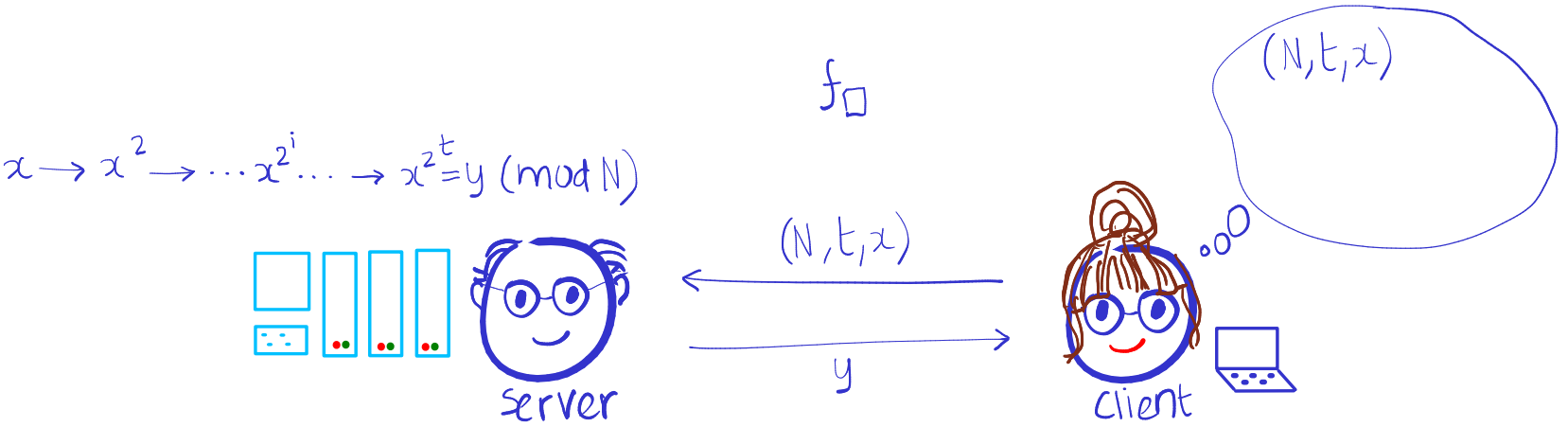

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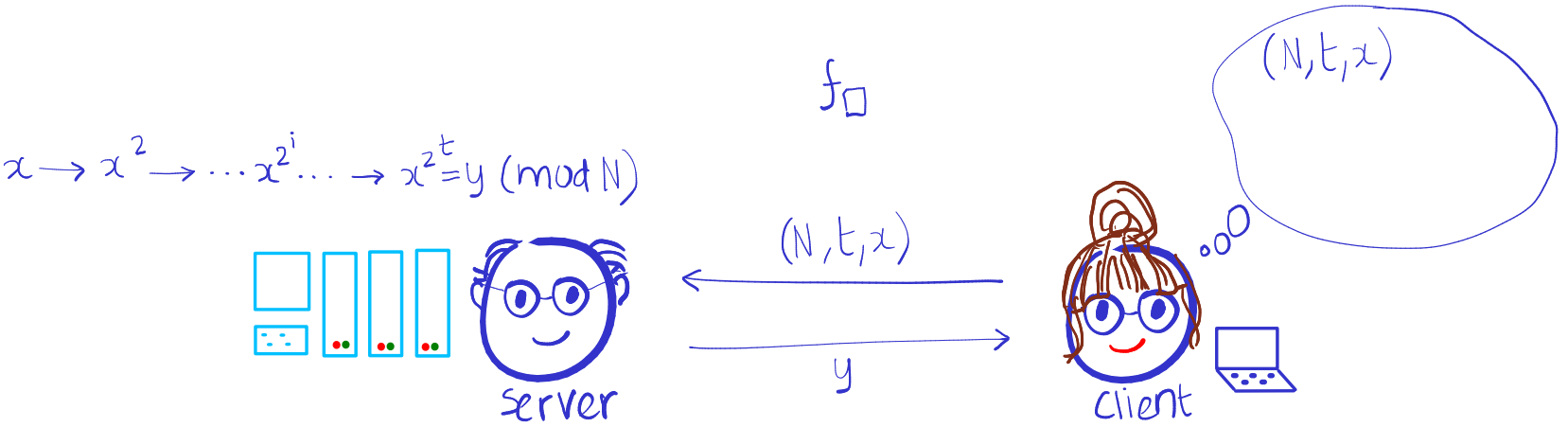

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  - Even if server has $\mathbf{poly}(n)$ amount of parallelism!
  - So-call RSW assumption: useful in "timed" cryptography
    - Time-lock puzzles
    - Verifiable delay functions

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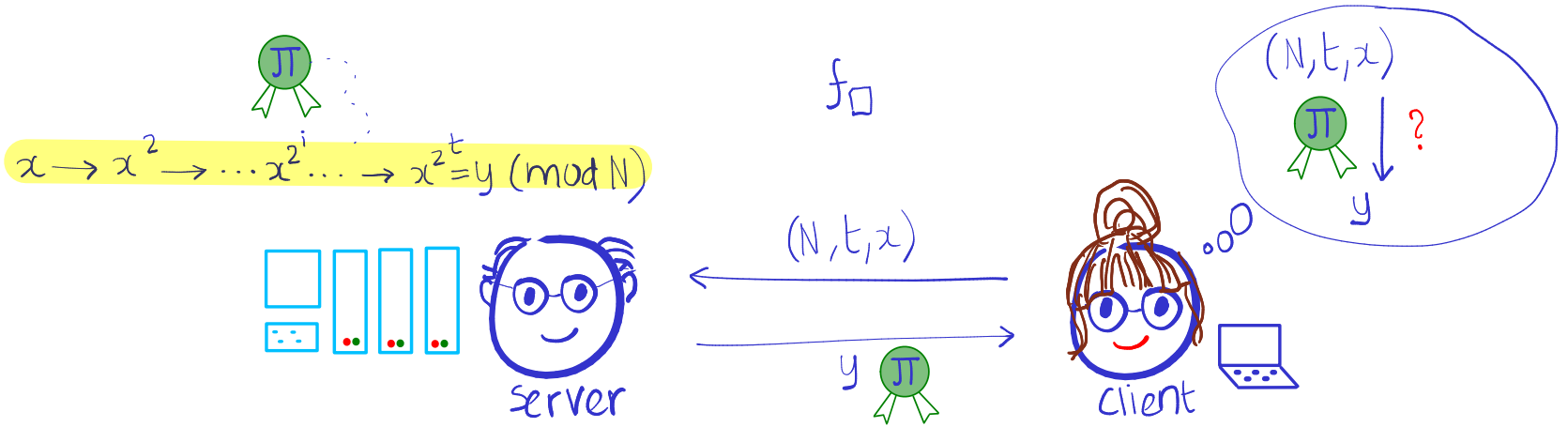

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  - Server could factor N and return (p, q)
  - ⚠ Recall: we need honest prover to not have too much overhead
- Without factors, fastest way to compute $f_\square(N, t, x)$ believed to be by repeated squaring
  - Even if server has $\mathbf{poly}(n)$ amount of parallelism!
  - So-call RSW assumption: useful in "timed" cryptography
    - Time-lock puzzles
    - Verifiable delay functions
- A proof π would be useful here

Plan for this Session

1. Verifiable Outsourcing of Computation
   - Succinct Non-Interactive Argument (SNARG)

2. SNARG for Repeated Squaring
   - Step I: Interactive Proof for Repeated Squaring
   - Step II: Remove Interaction via Fiat-Shamir Transform

Step I: Harness the Power of IP

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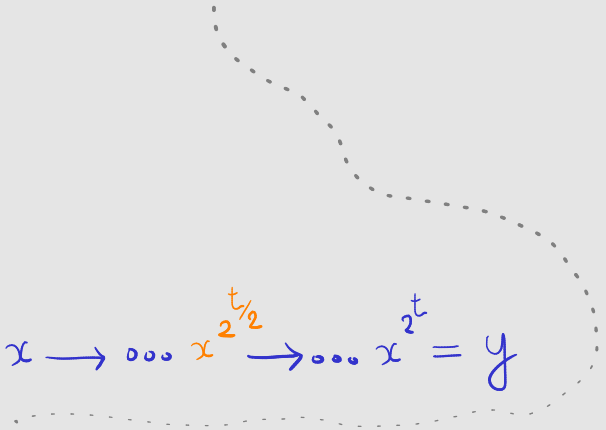
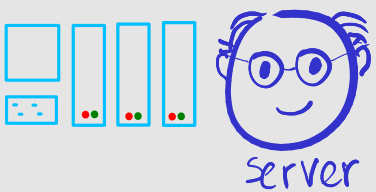
$t = 2^a$





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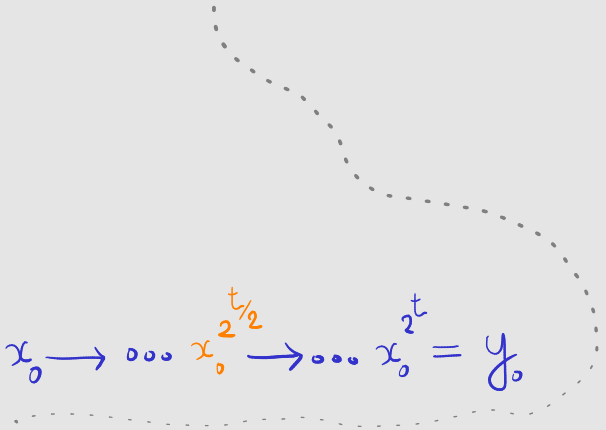

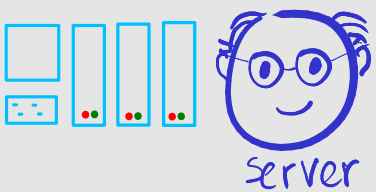

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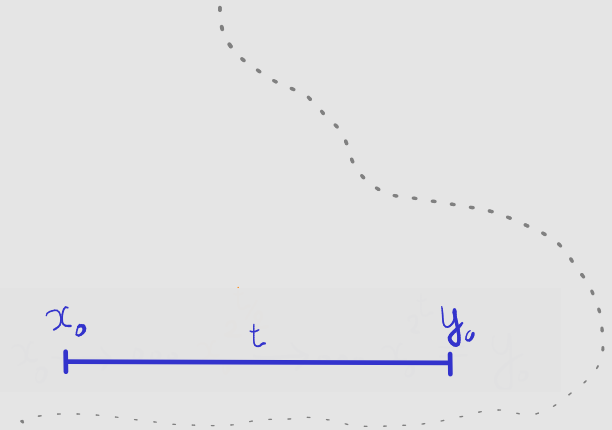

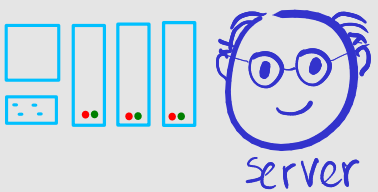

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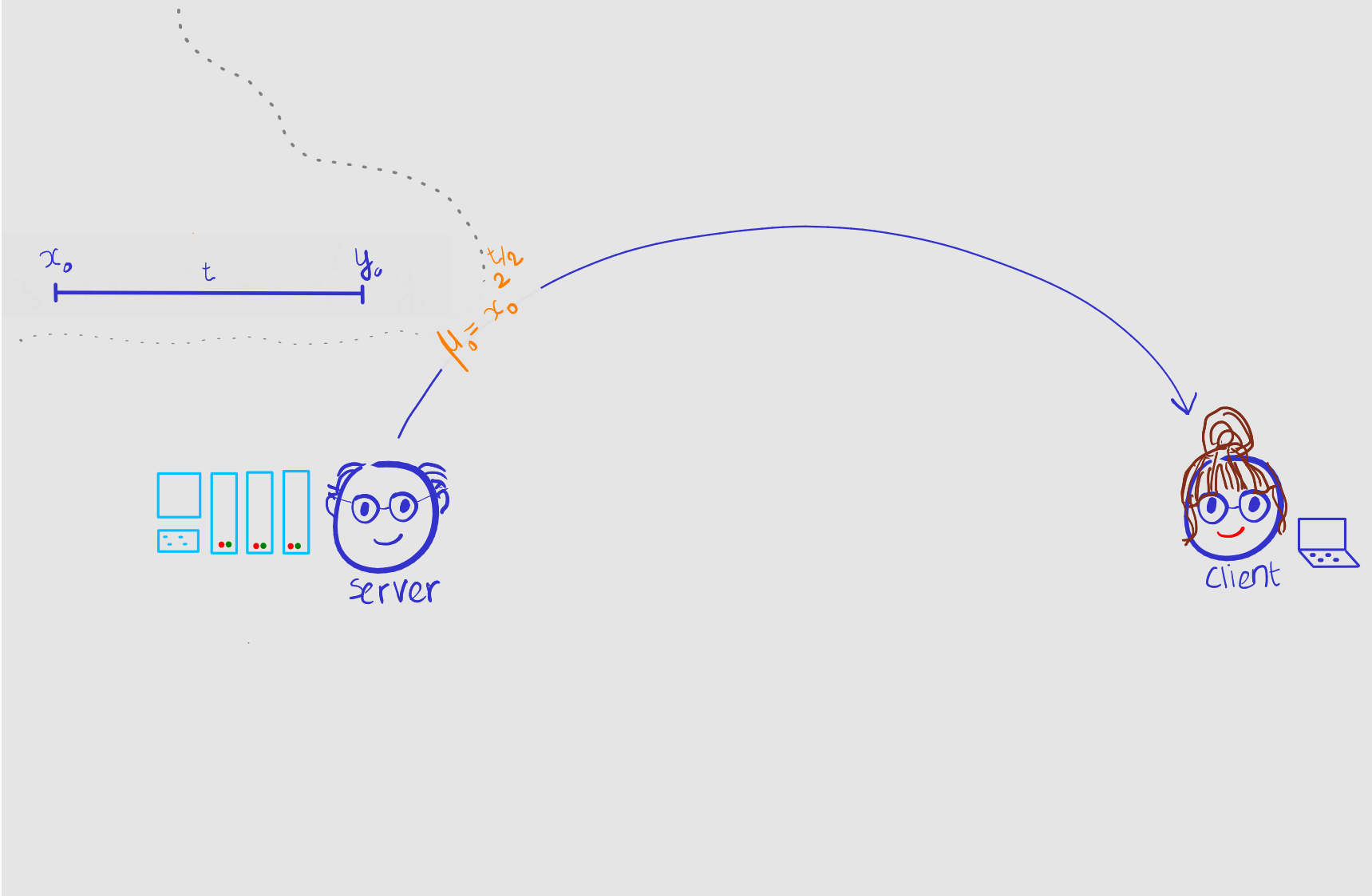

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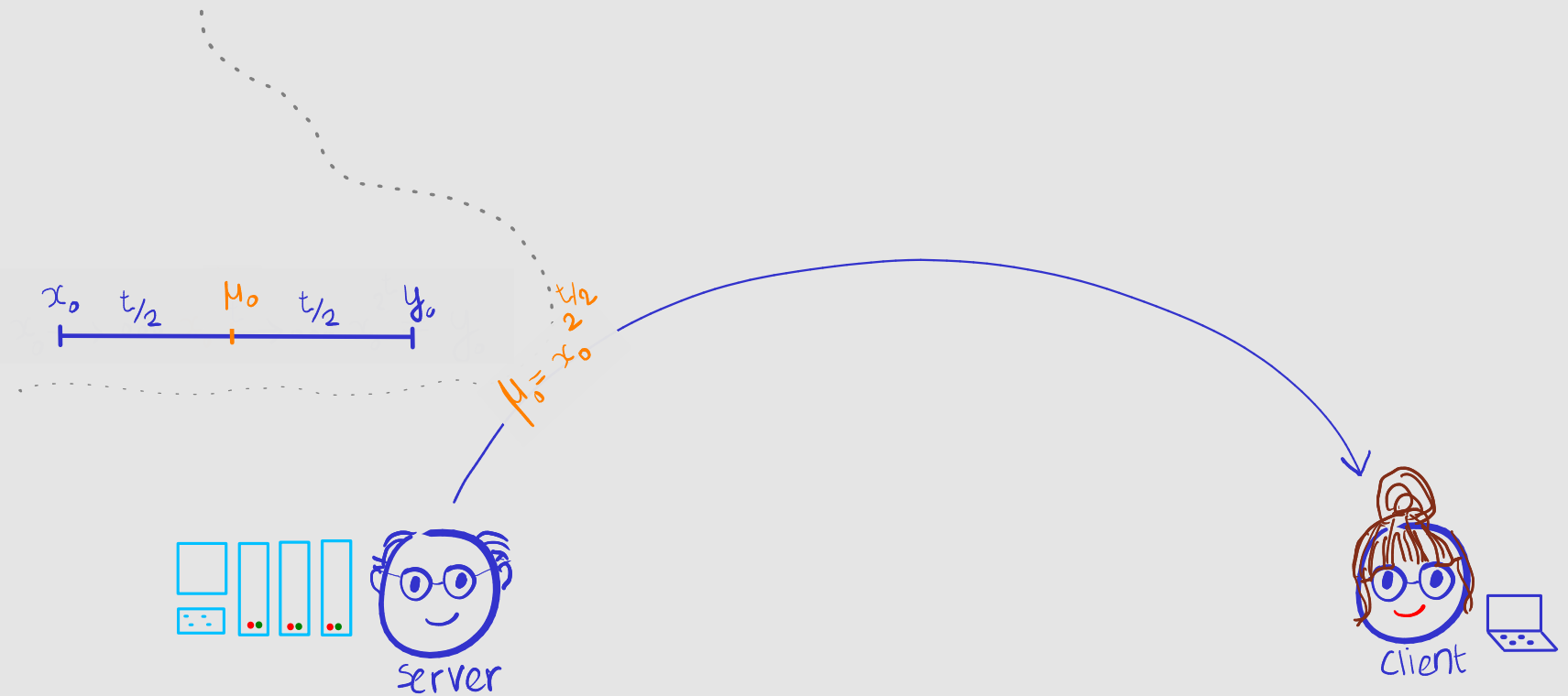

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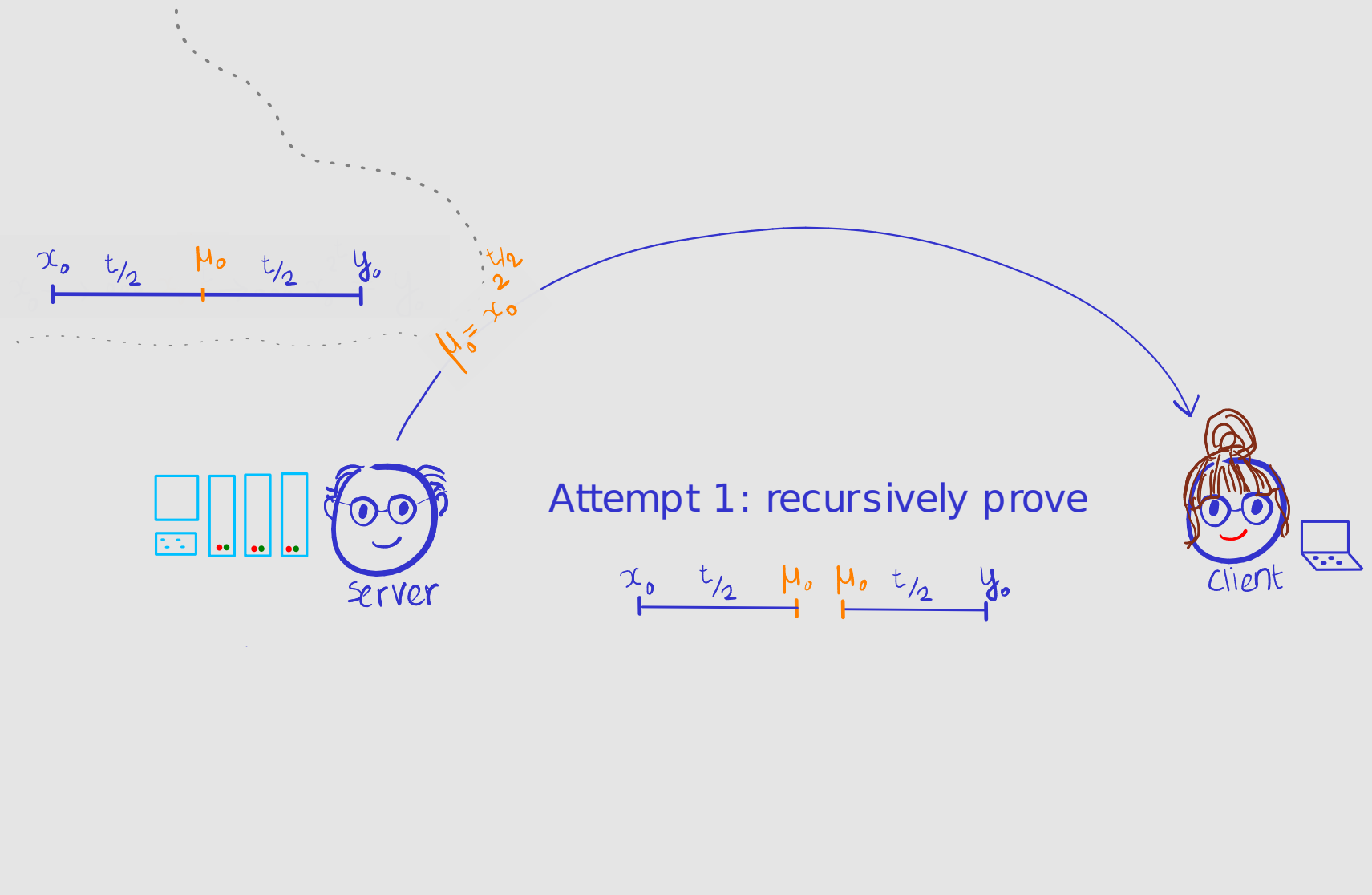

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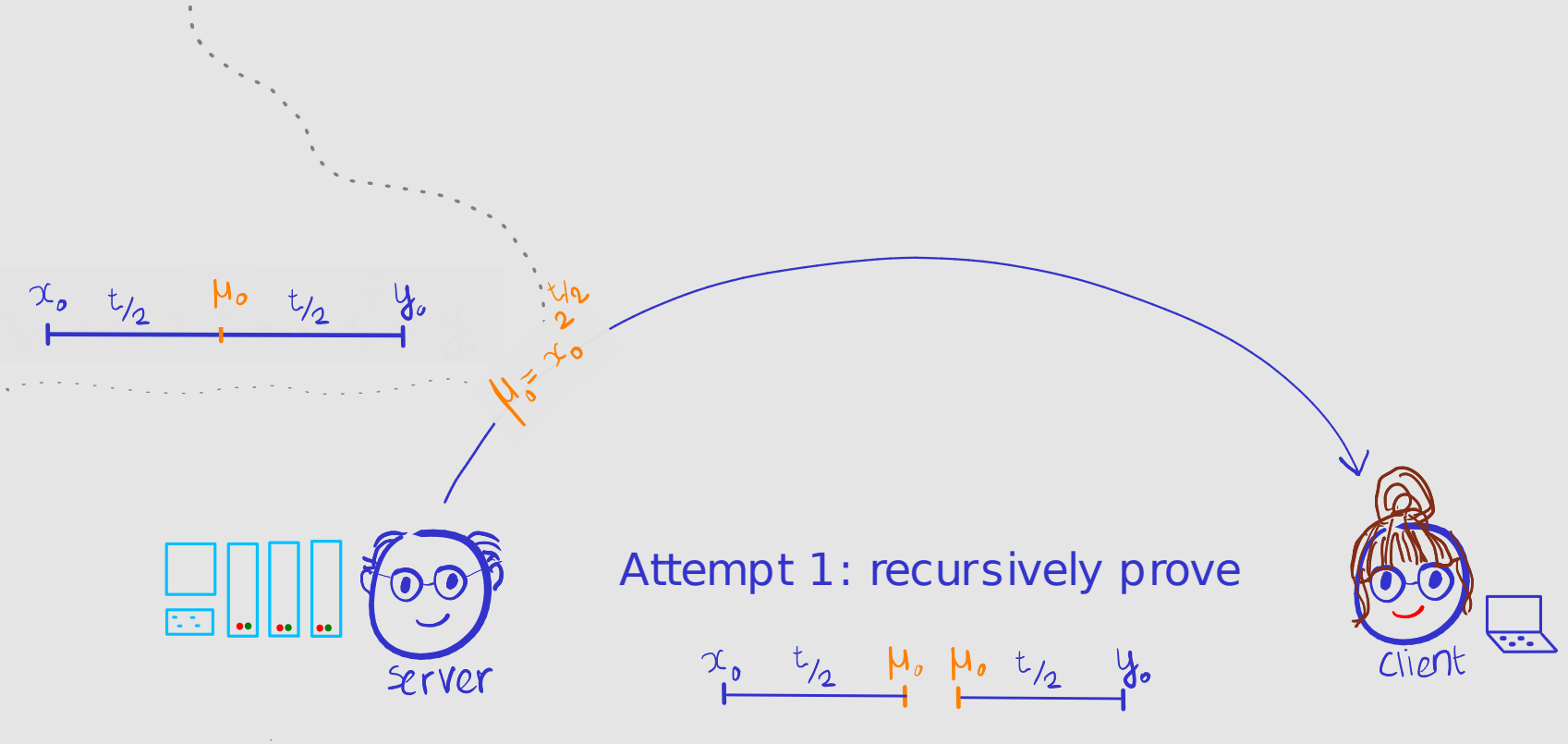

Attempt 1: recursively prove

Problem: recursion blows up!

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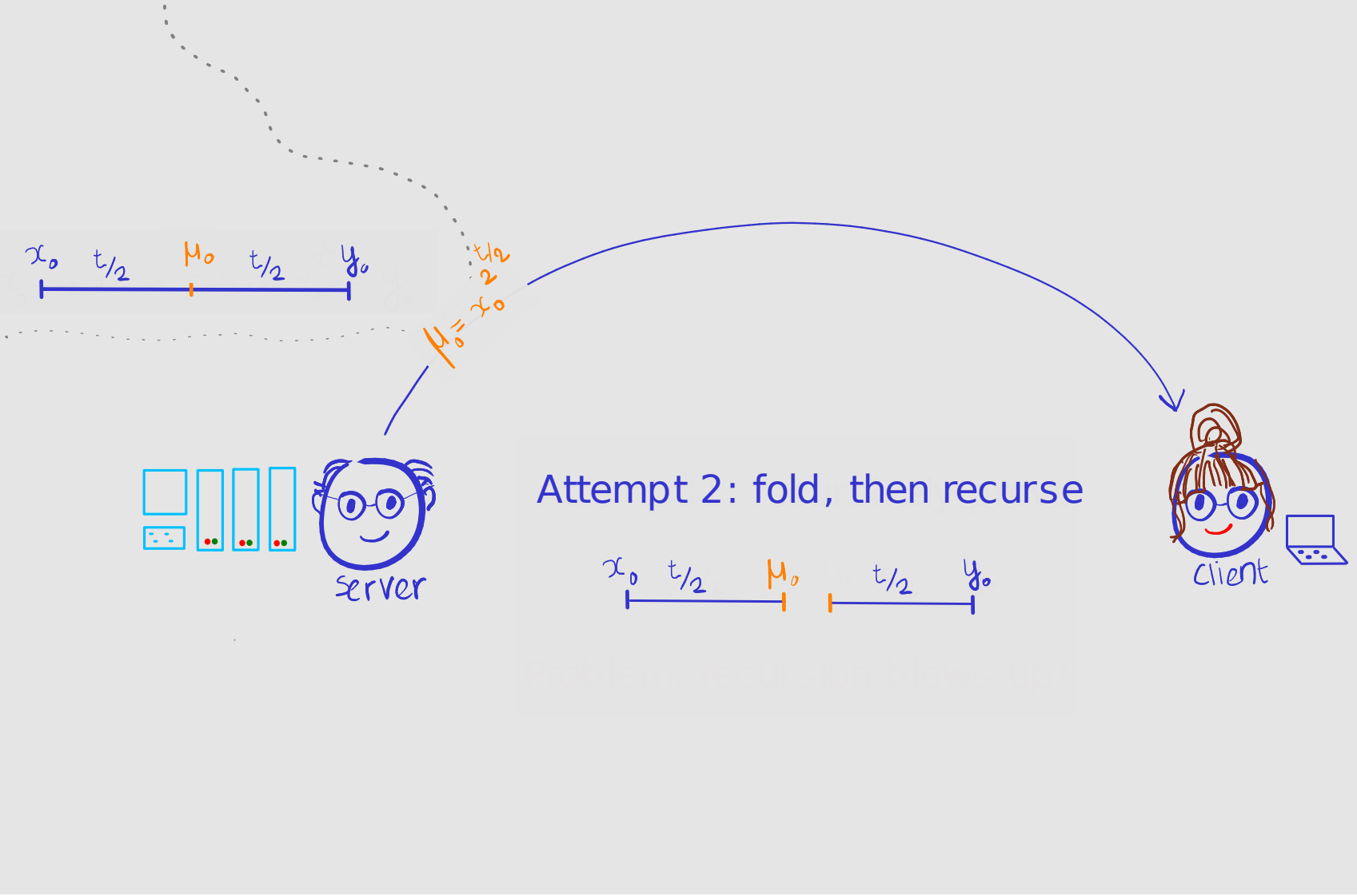

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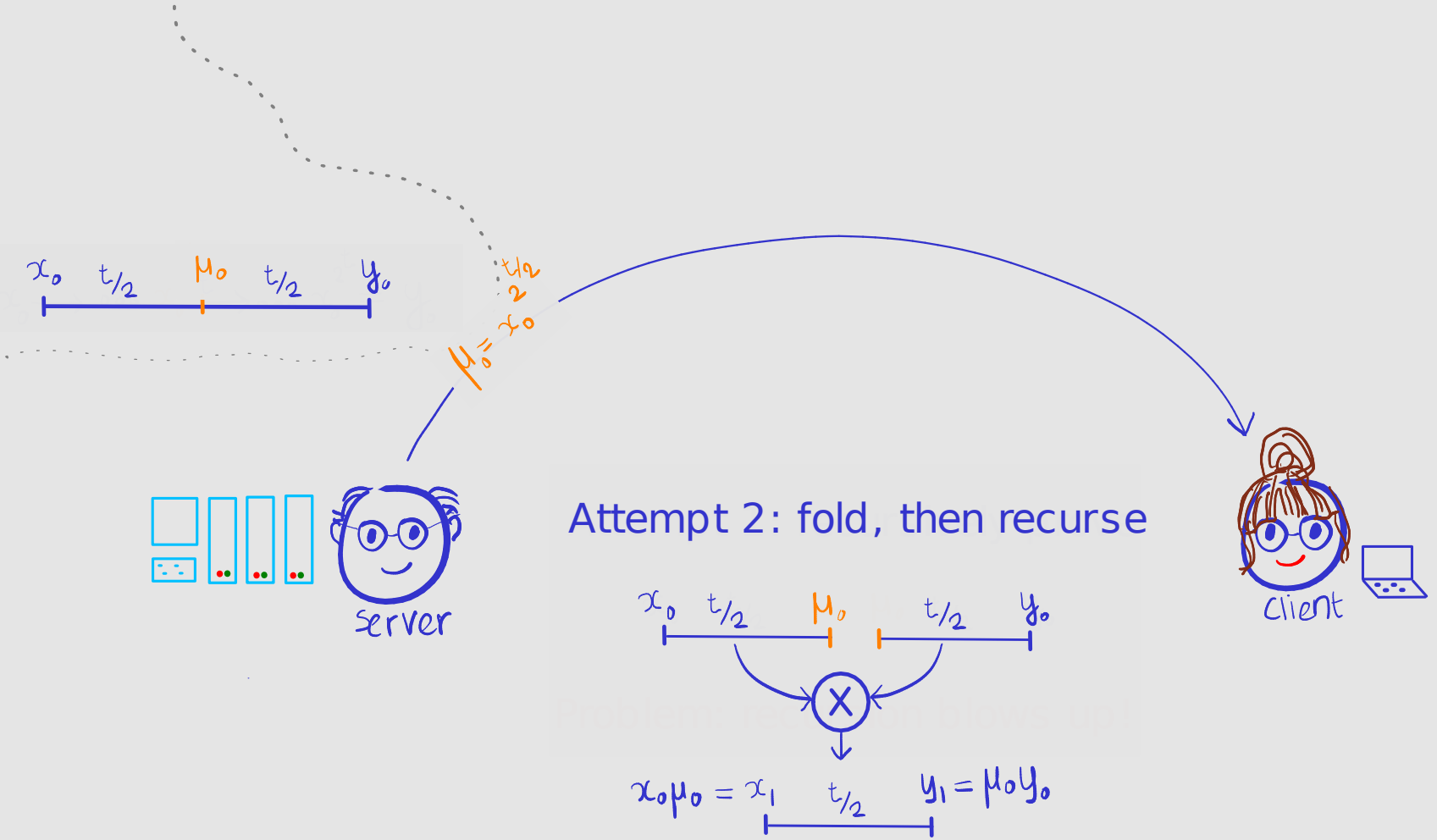

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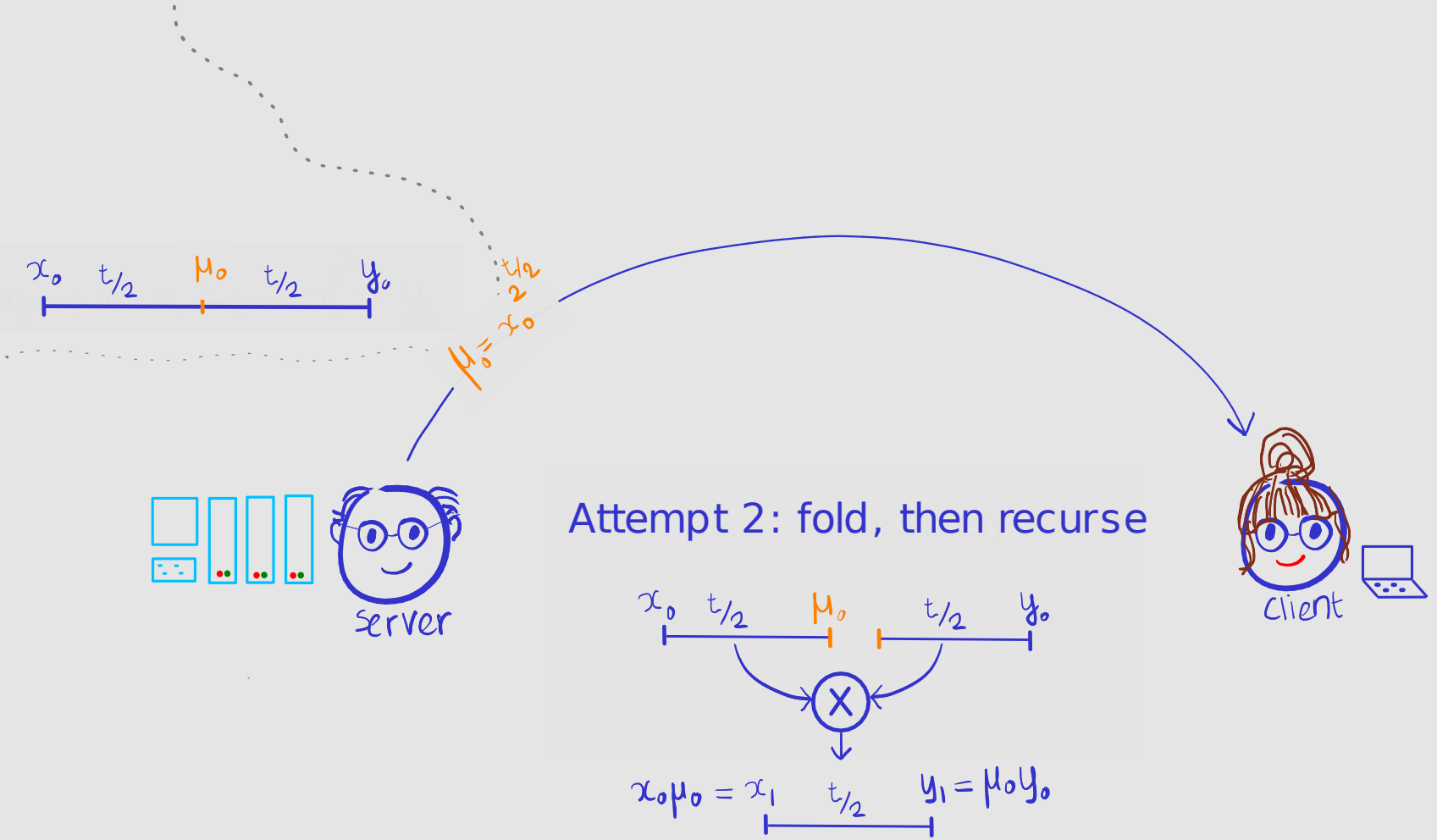

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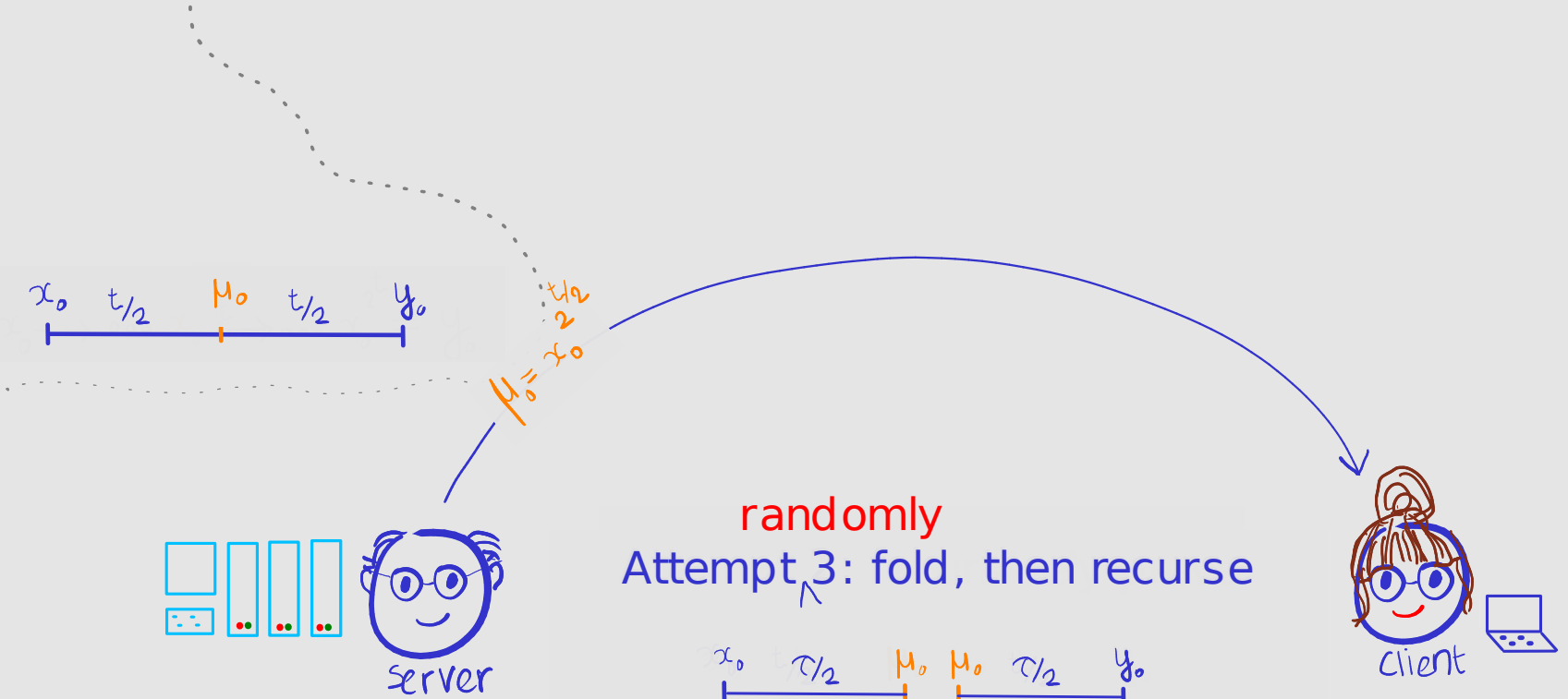

Problem: Bob can cheat. How?

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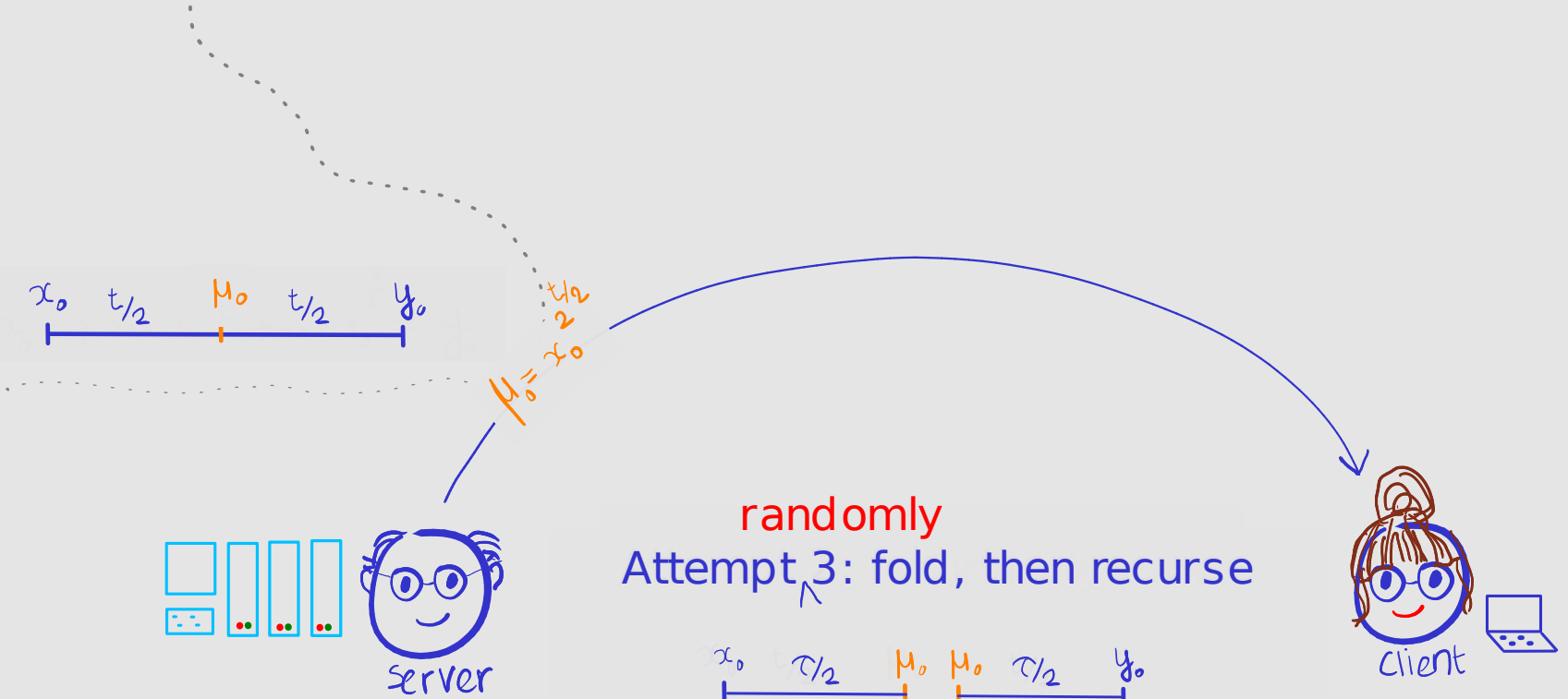

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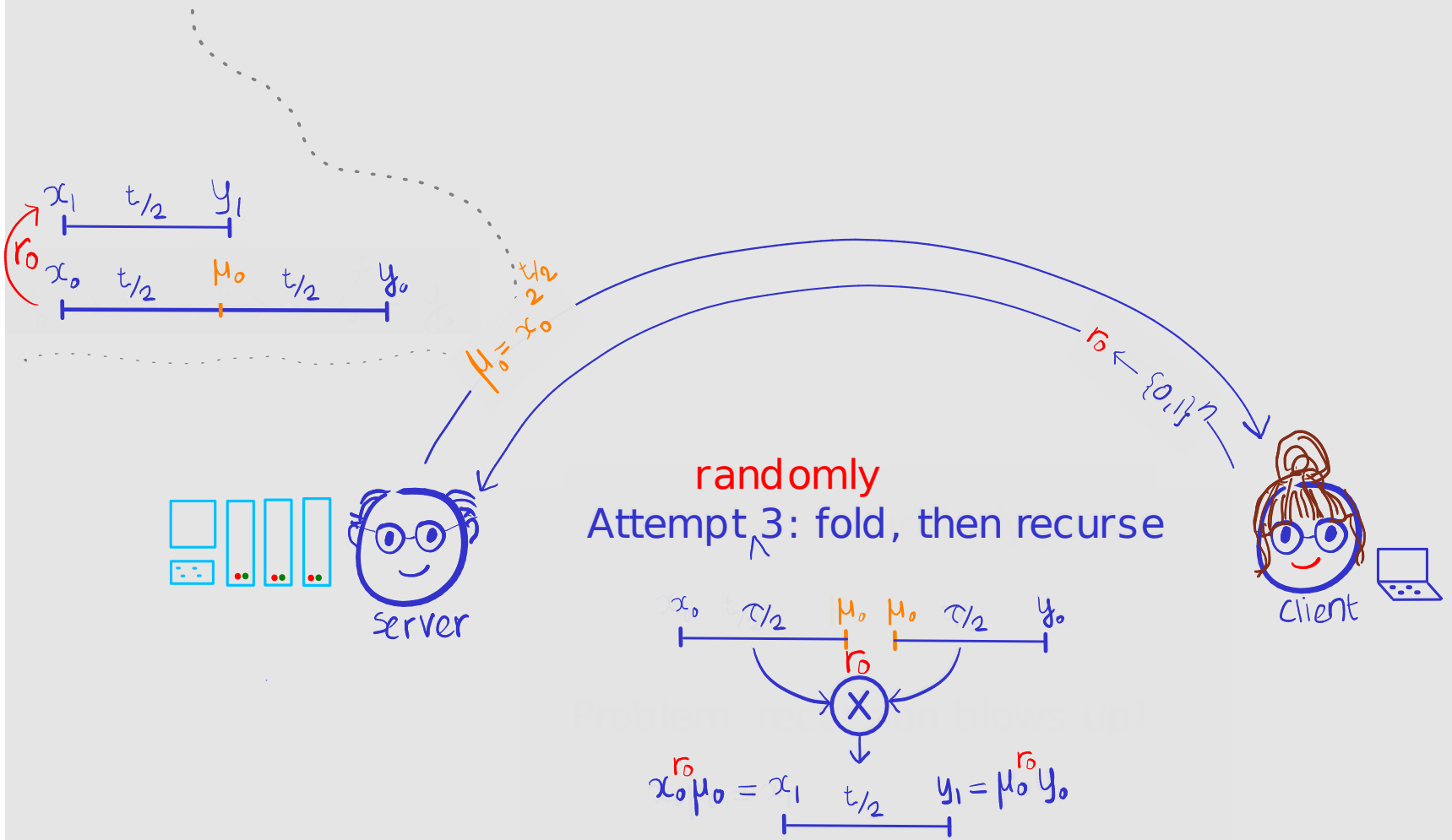

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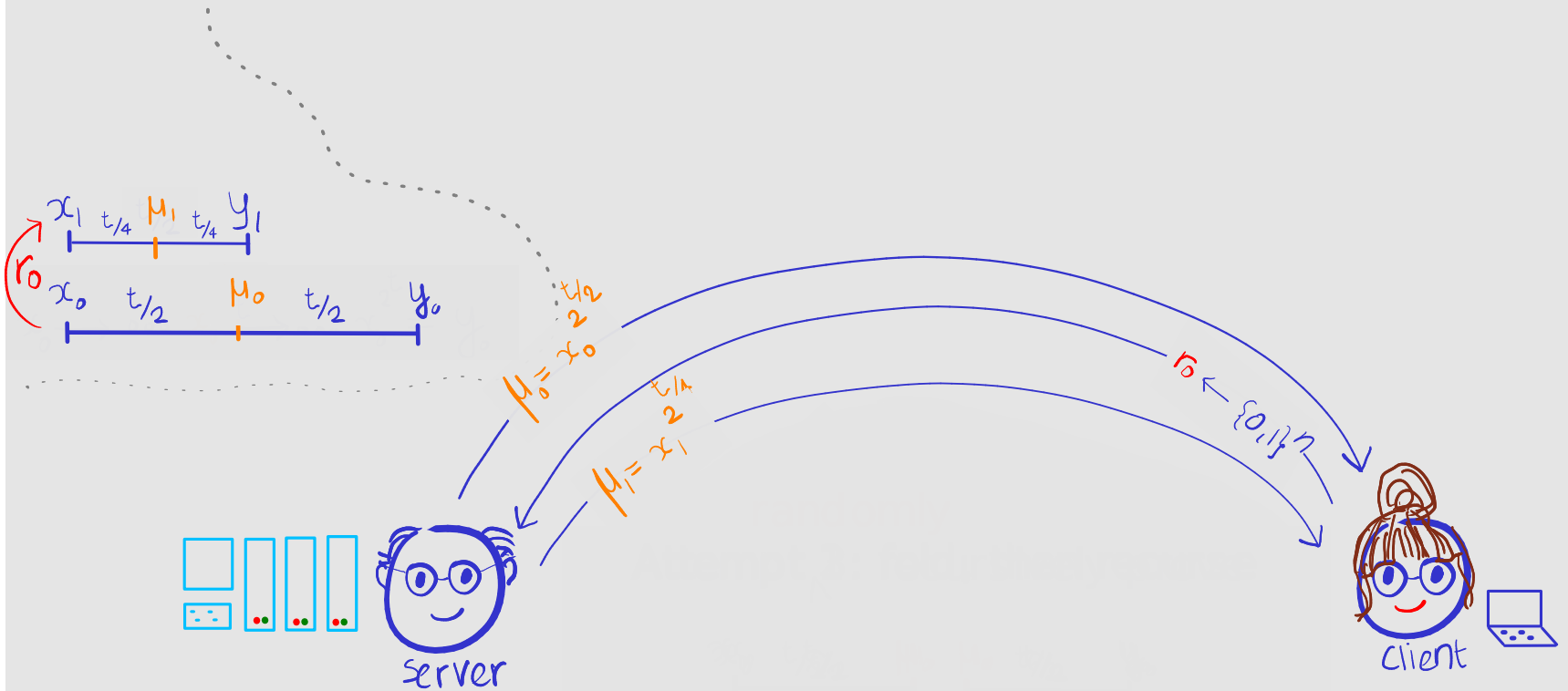

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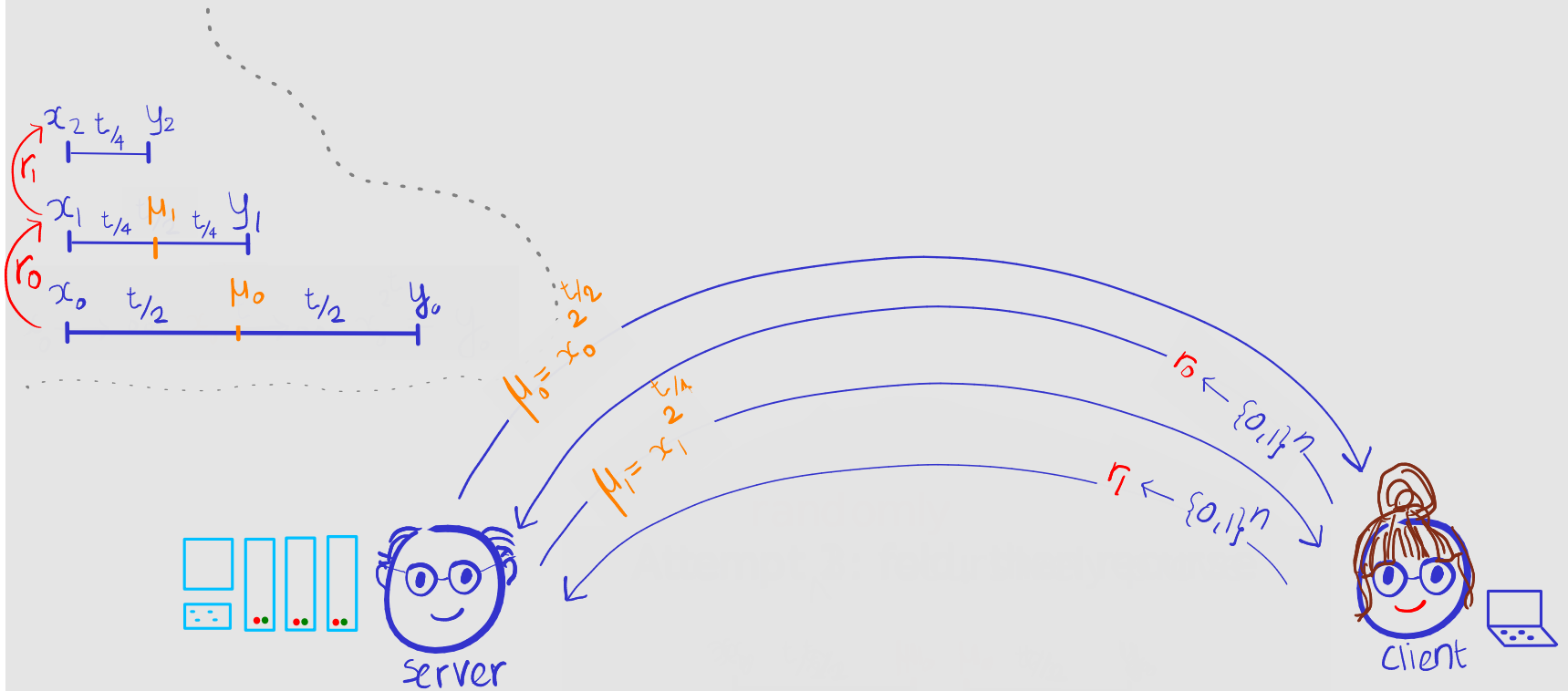

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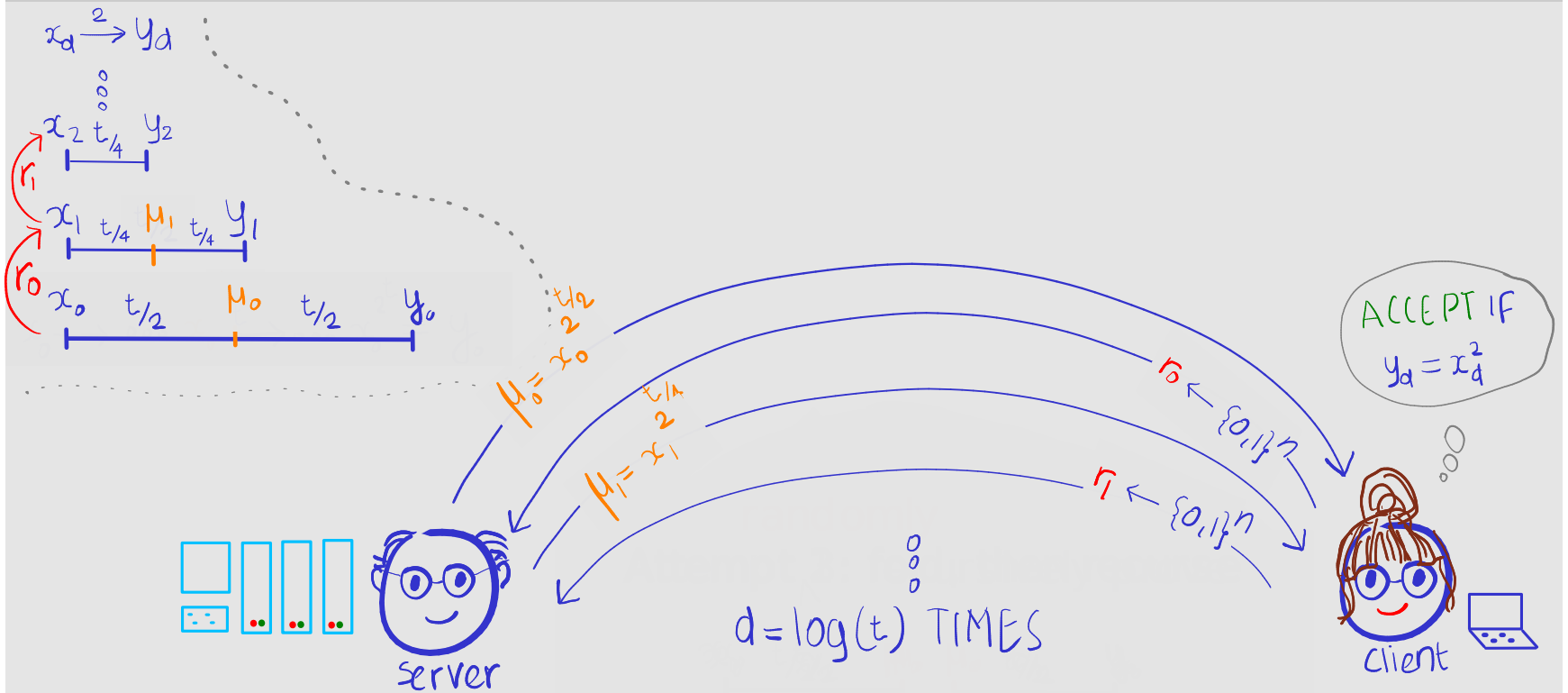

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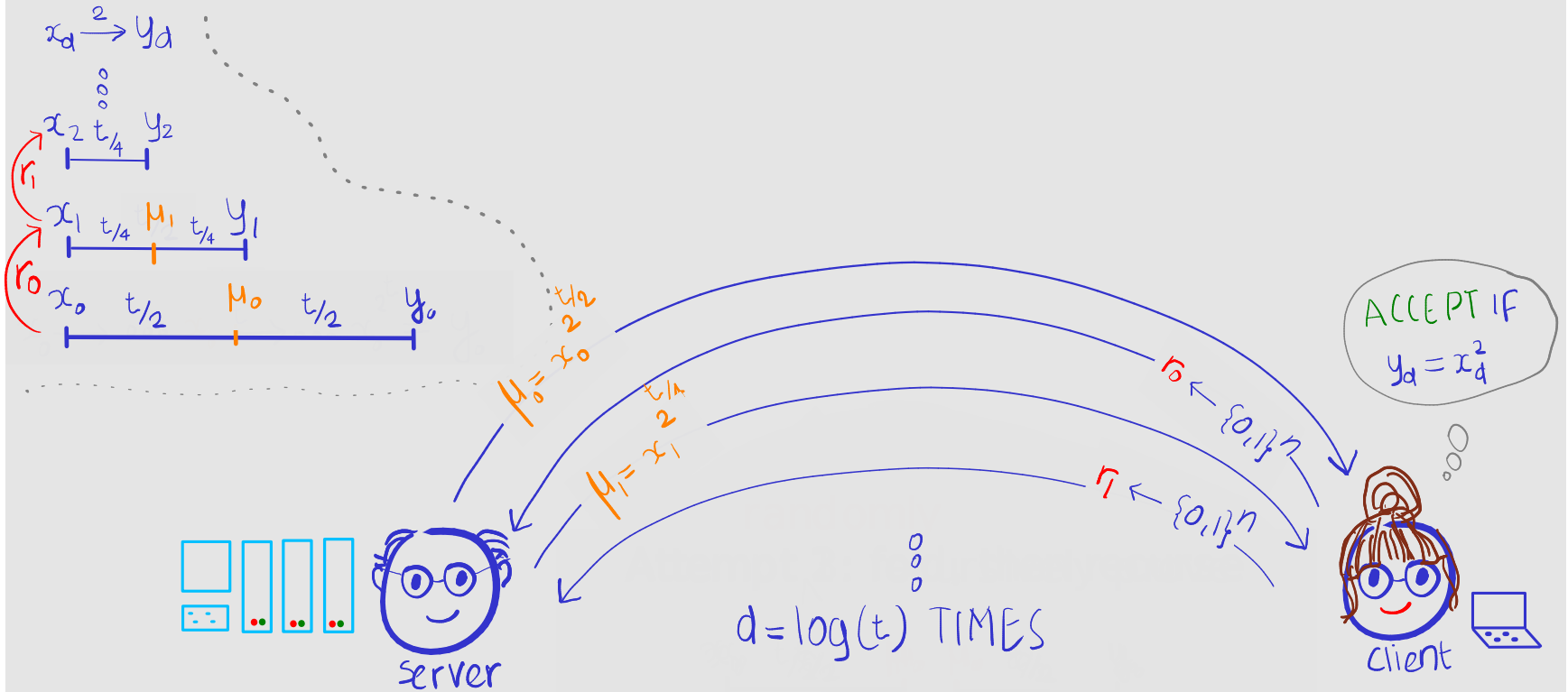

? Why is the protocol complete?

Why is Π Sound?

Theorem 1

Π *is an interactive proof for the language*

$$\mathcal{L}_{\mathsf{f}_\square} := \left\{ (x, y, t, N) : y = x^{2^t} \bmod N \right\}$$

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Proof Sketch.

Statistical soundness (for first round) can be argued using:

1. Claim: if $y_0 \neq x_0^{2^t}$ then $y_0 \neq \mu_0^{2^{t/2}}$ *or* $\mu_0 \neq x_0^{2^{t/2}}$ must hold ?

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$(x_0, y_0, t) \notin \mathcal{L}_{\mathsf{f}_\square}$

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x_1 y_1

$\hookrightarrow (x_1, y_1, t/2) \notin \mathcal{L}_{f_\square}$

w.h.p. over choice of r. $\square$

Exercise 2

Prove Claims 1 and 2. Argue overall soundness.

Plan for this Session

1. Verifiable Outsourcing of Computation
   - Succinct Non-Interactive Argument (SNARG)

2. SNARG for Repeated Squaring
   - Step I: Interactive Proof for Repeated Squaring
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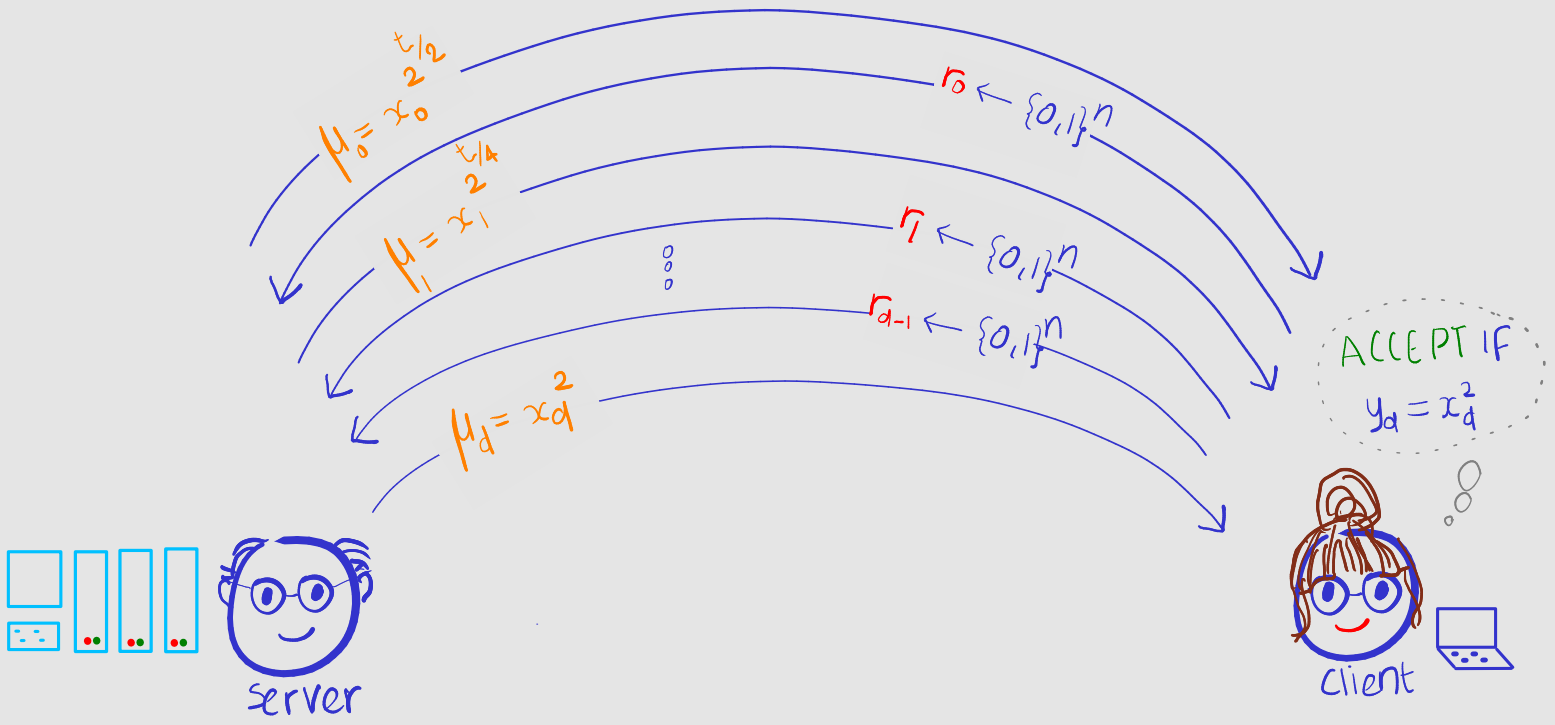

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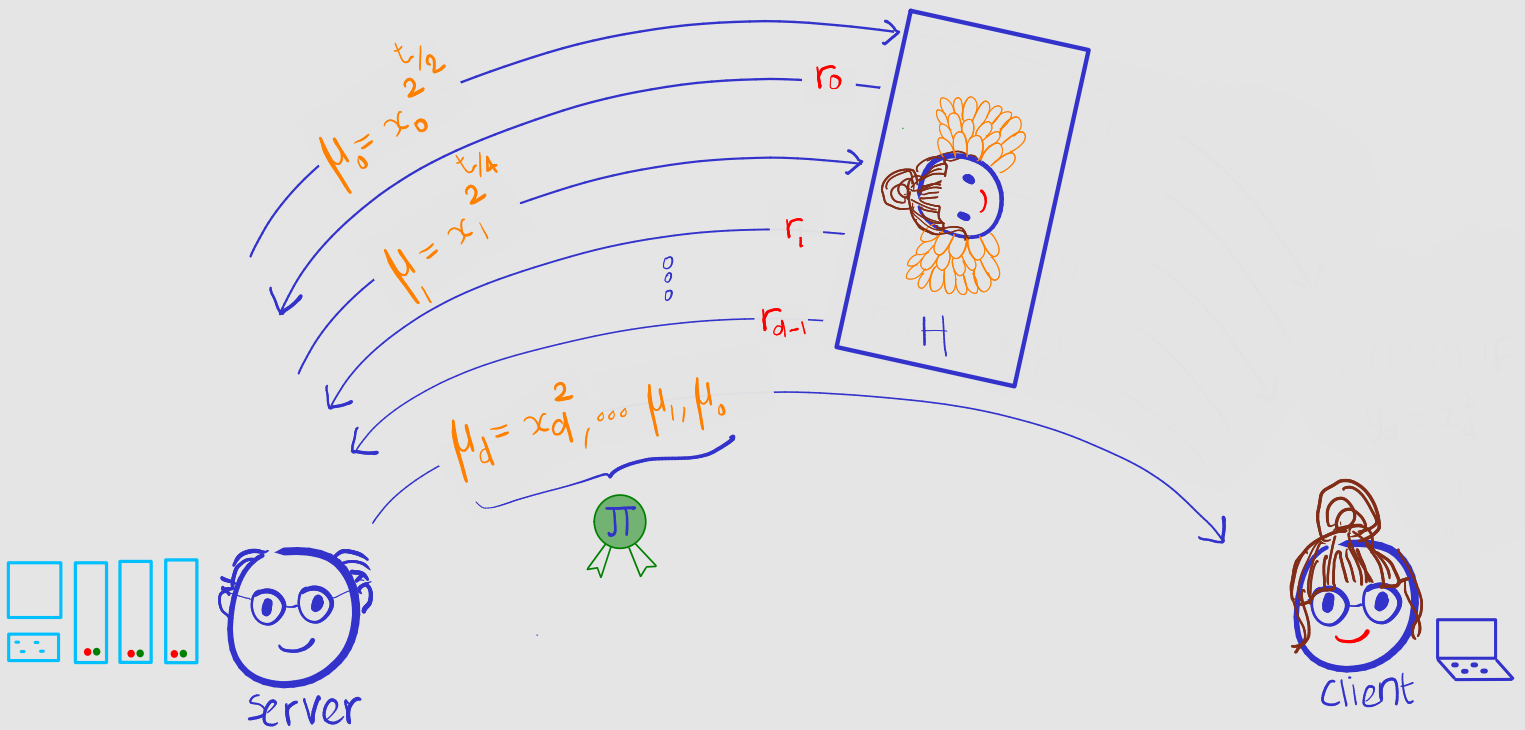

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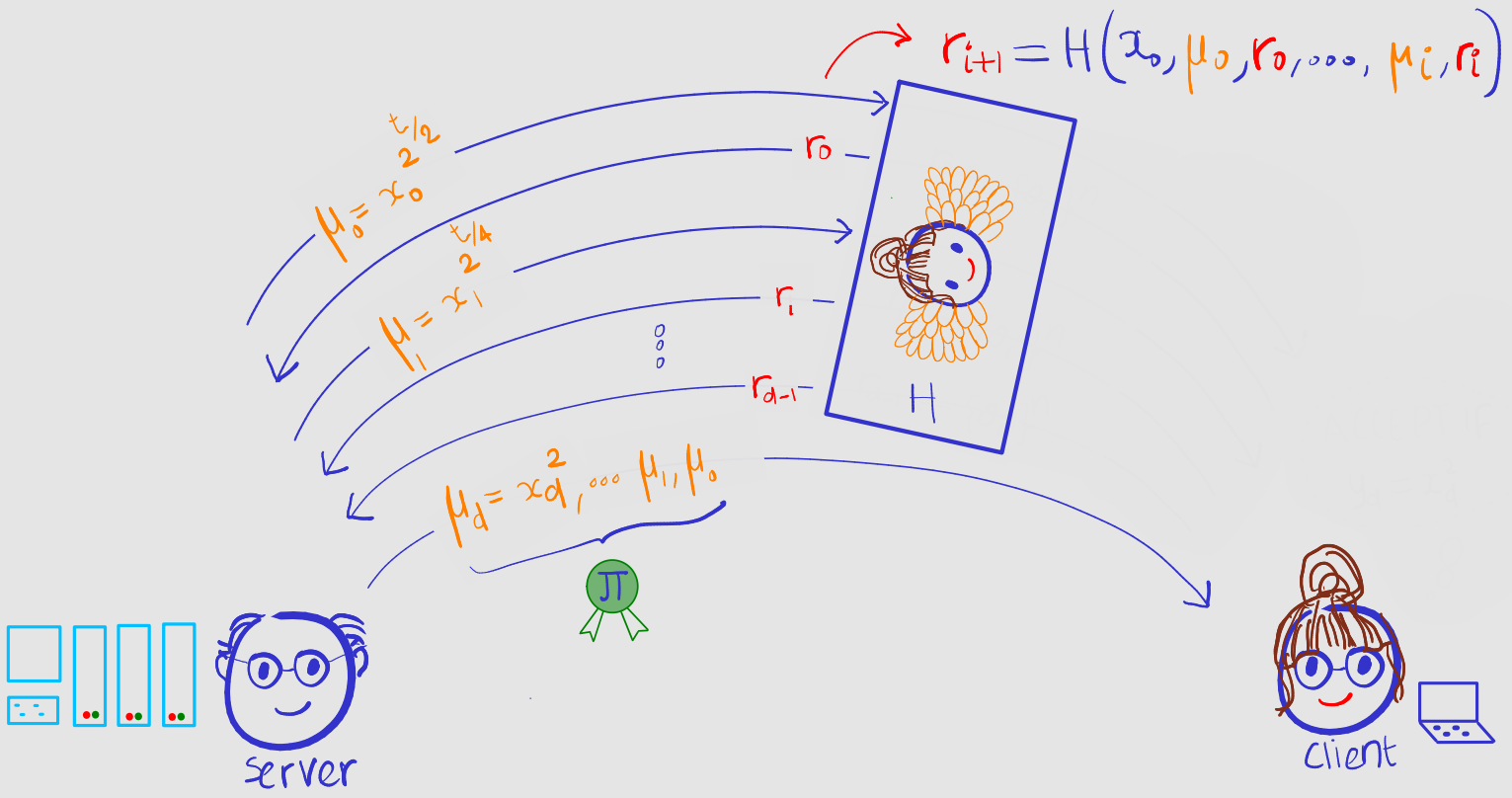

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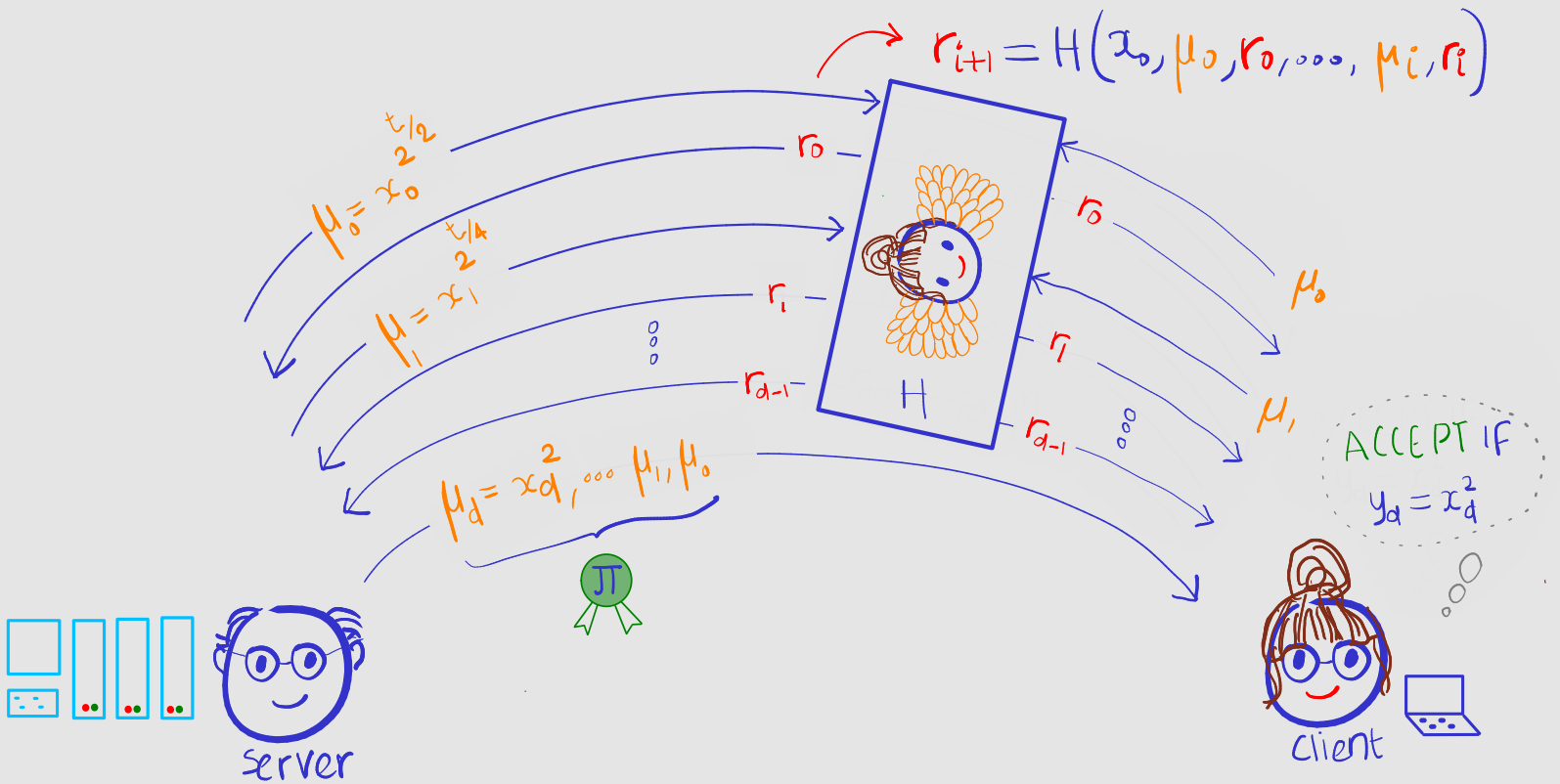

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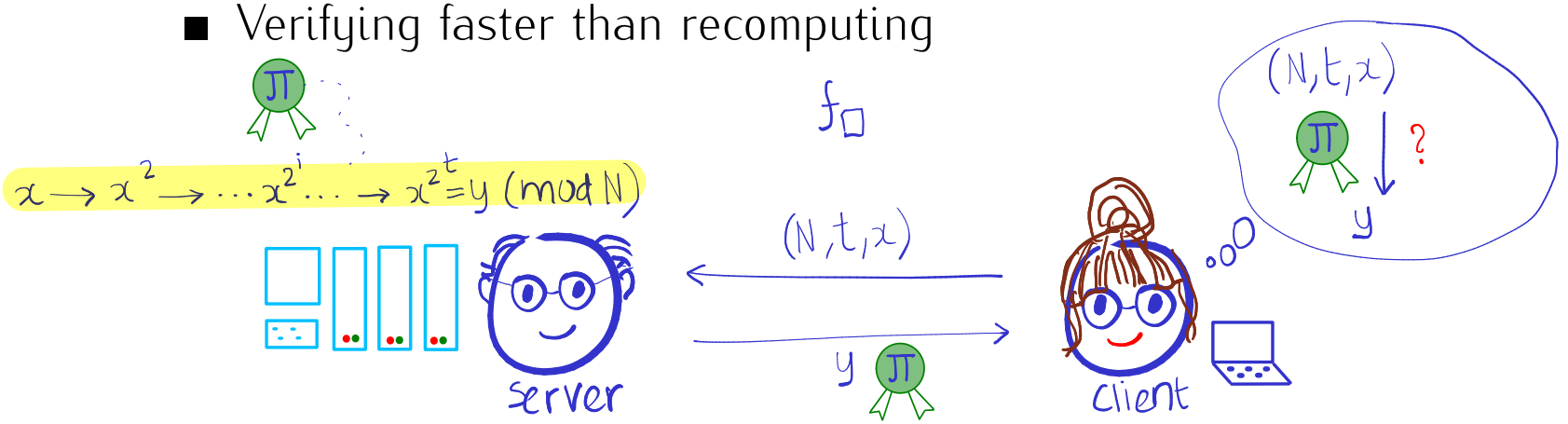
Exercise 3

Work out the proof of soundness in random-oracle model.

To Recap Today's Lecture

- Task 7: secure outsourcing in the client-server setting
- Task 7.b: verifiable outsourcing in the client-server setting
  - Verifying faster than recomputing



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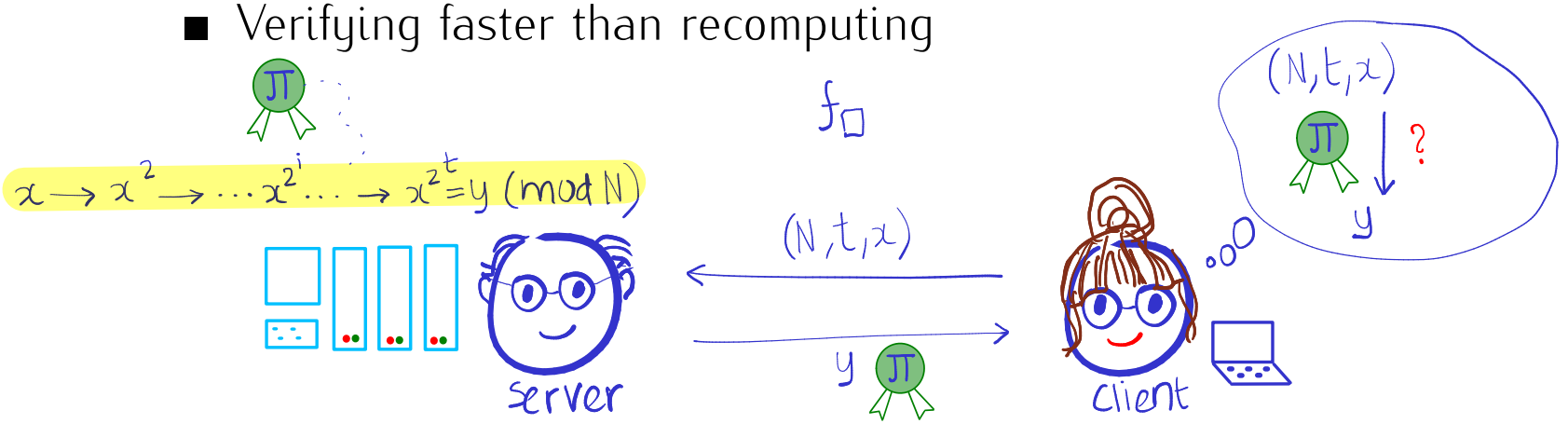

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- SNARG for repeated squaring problem in RSA group

  - Pietrzak's interactive protocol
  - Main ideas: 1) downward self-reducibility 2) folding
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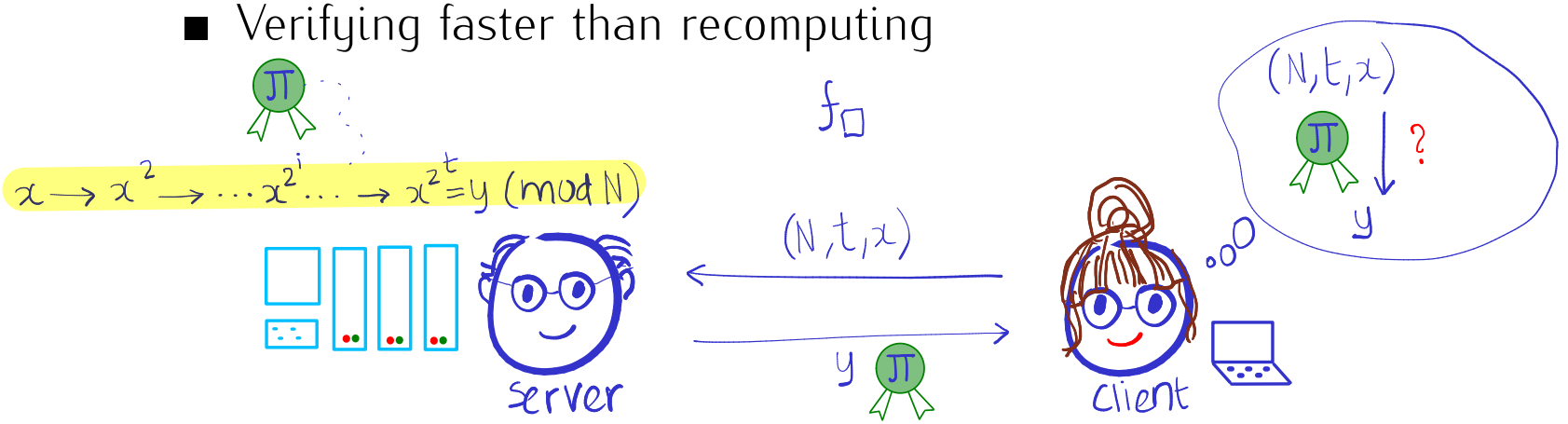

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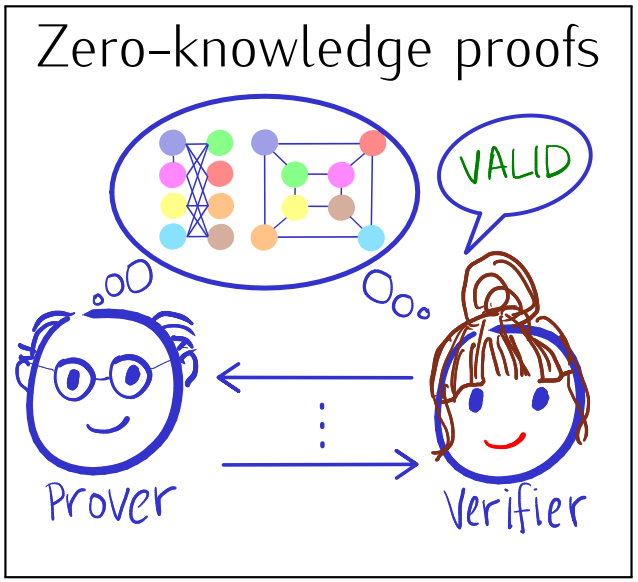
- SNARGs for **NP** (in ROM)
  - Coming Spring: Introduction to Probabilistic Proof Systems

To Recap Module III

- We saw several avatars of secure computation



To Recap Module III

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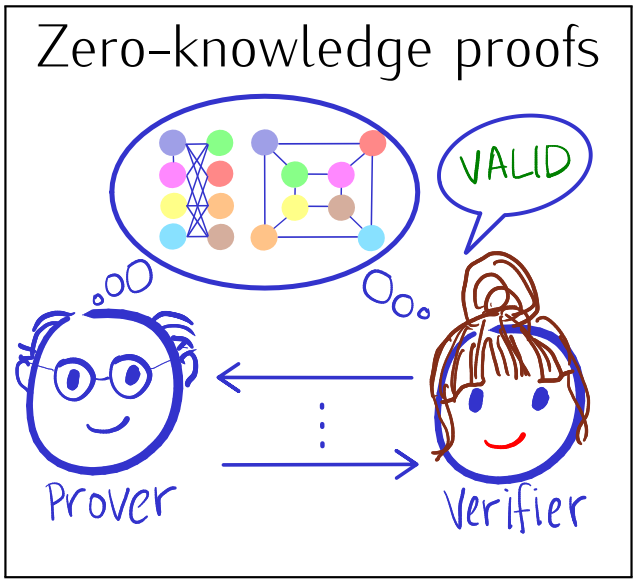

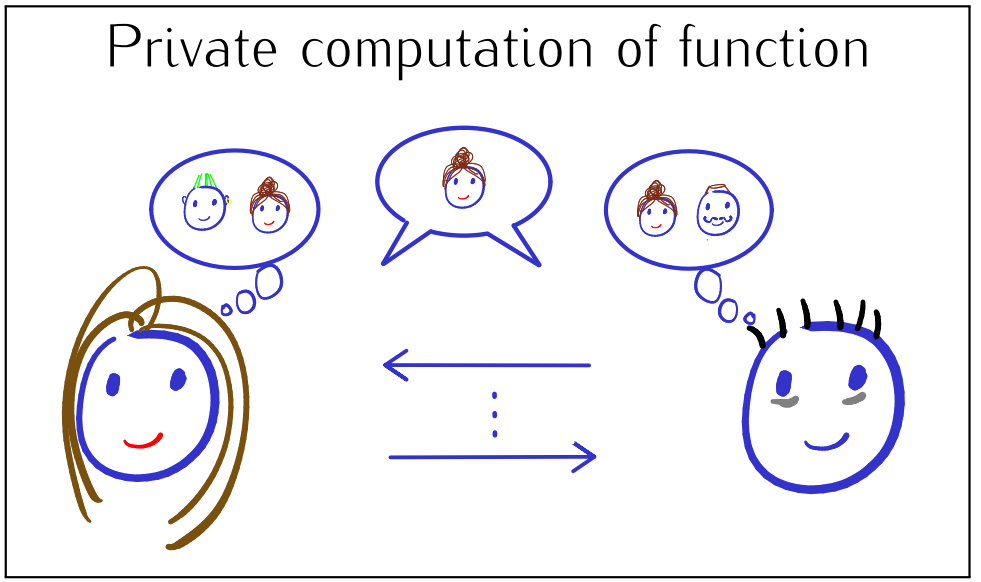

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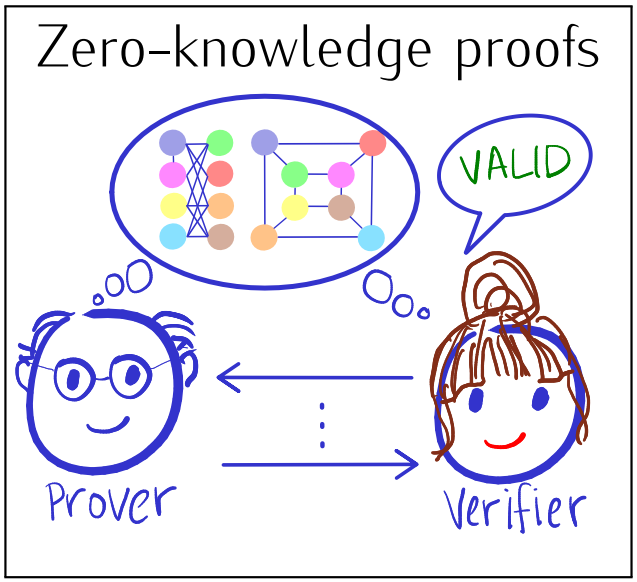
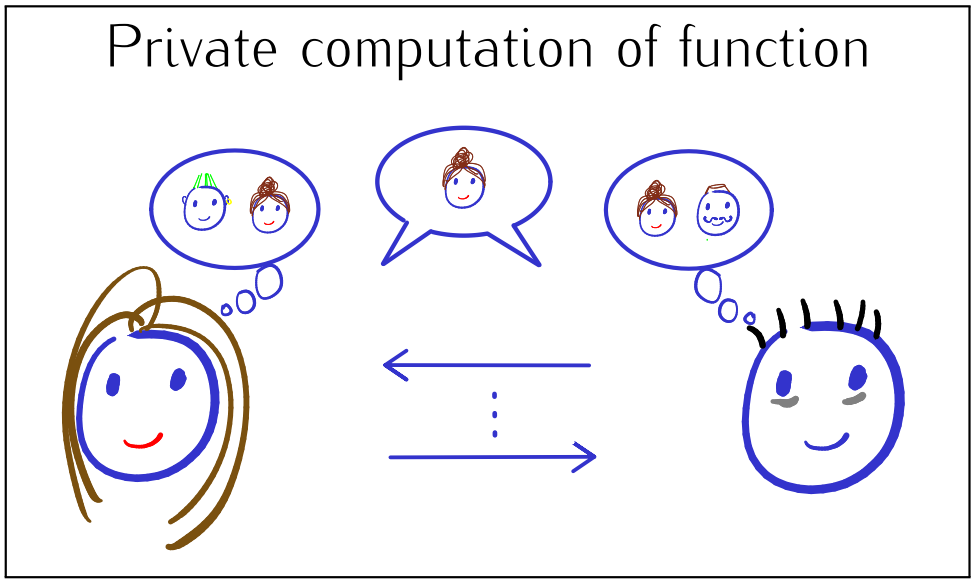
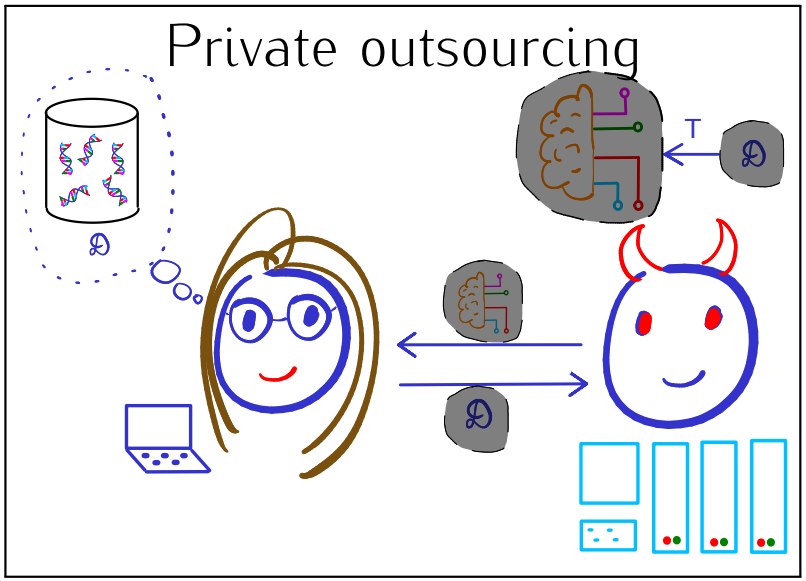
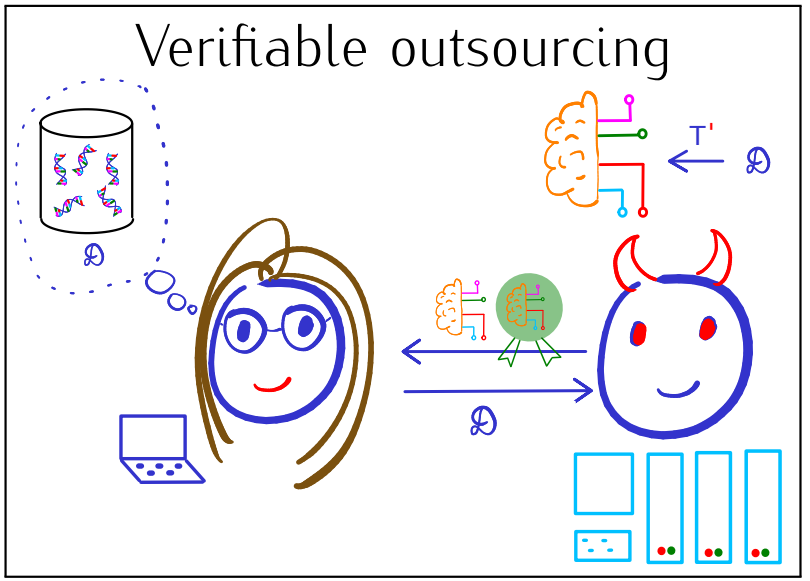
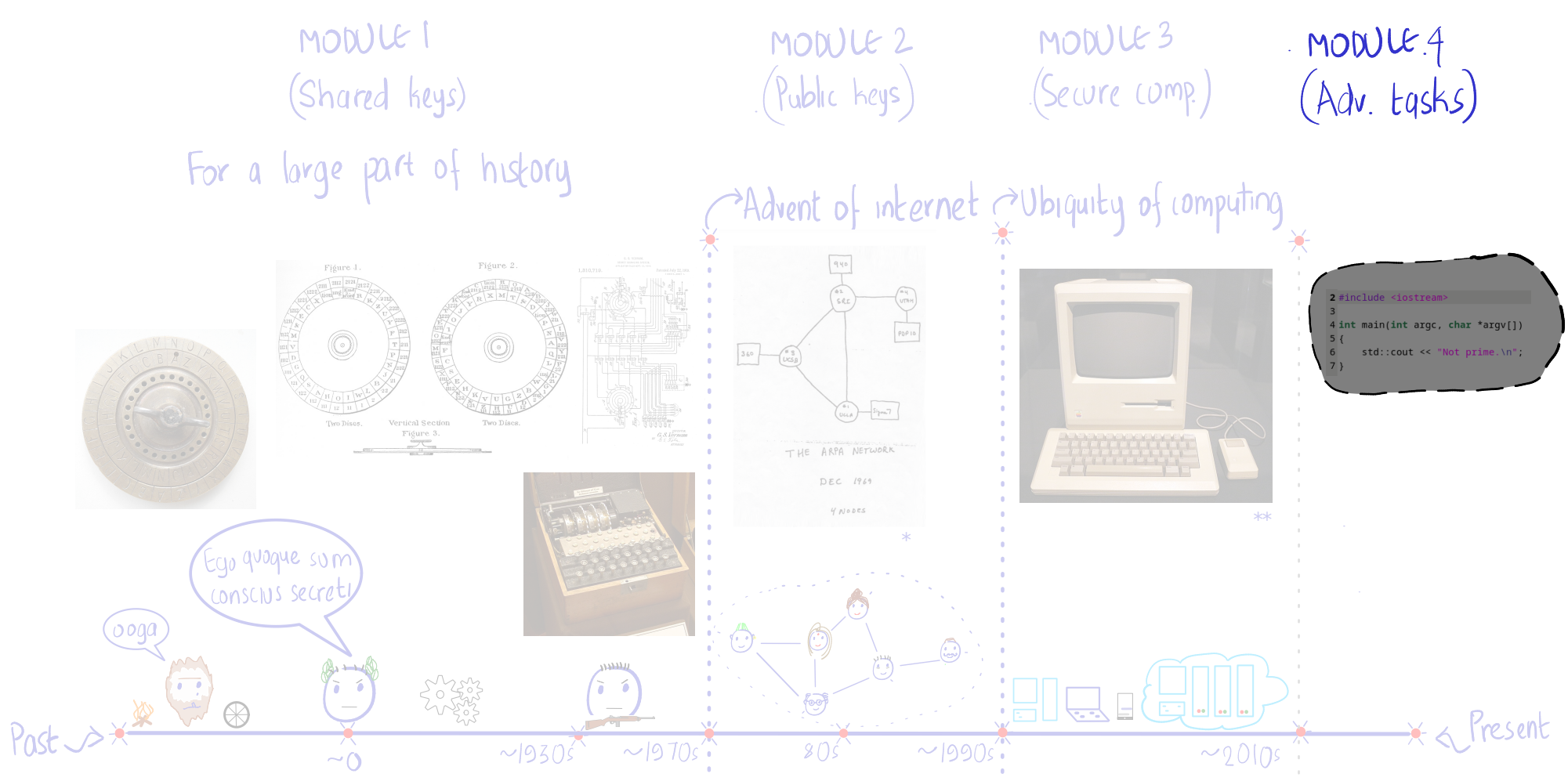








Module IV: Next Few Lectures



- Code obfuscation
  - What cryptography is possible if you can obfuscate code?

Credit for images: Wikipedia (*User:DARPA)(**User:Saliko)

References

1. SNARGs were introduced as "CS proofs" in [Mic94]. You can find a formal definition of SNARG in [BCI+13, §4].
2. The repeated squaring function was introduced in [CLSY93, RSW96] and is extensively used in timed cryptography. For example, the time-lock puzzle from [RSW96] are the verifiable delay functions from [Pie19, Wes19] are both based on *sequential* hardness of repeated squaring.
3. Formal description of Π and Π^{H} can be found in [Pie19]


Nir Bitansky, Alessandro Chiesa, Yuval Ishai, Rafail Ostrovsky, and Omer Paneth.

Succinct non-interactive arguments via linear interactive proofs.

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