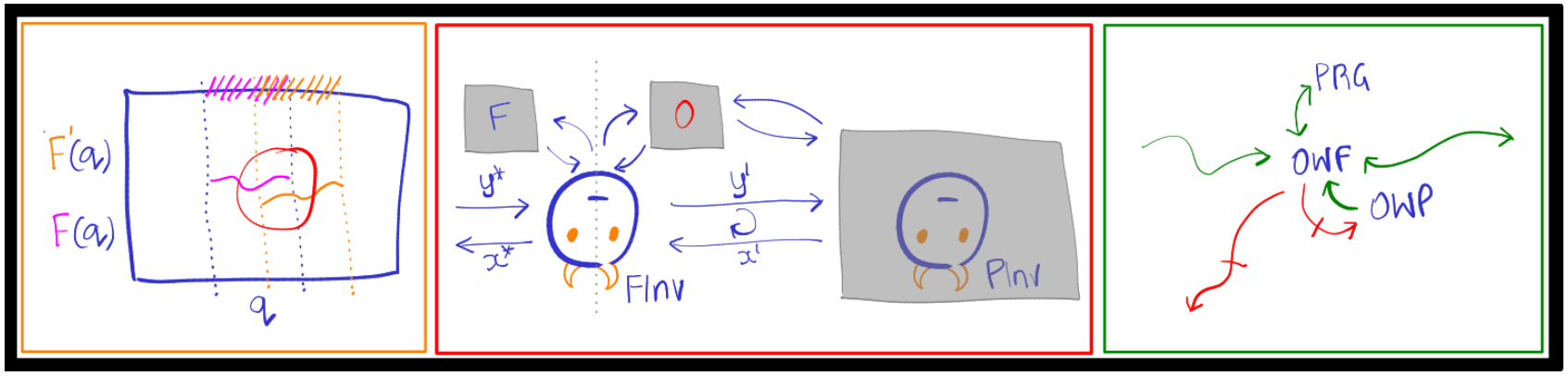


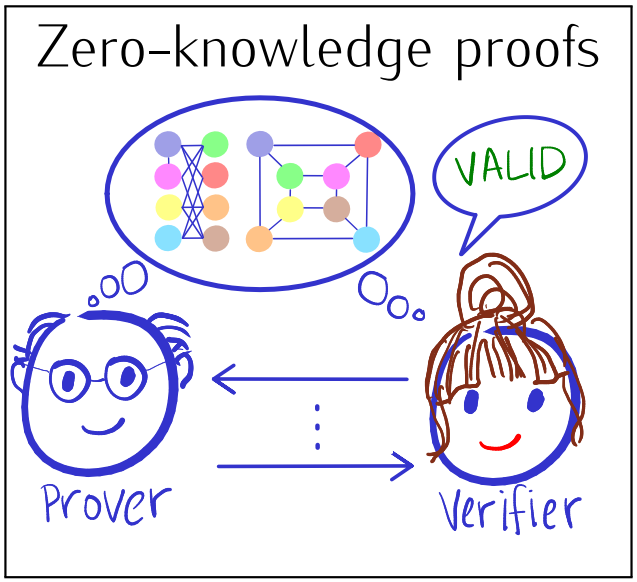
CS783: Theoretical Foundations of Cryptography

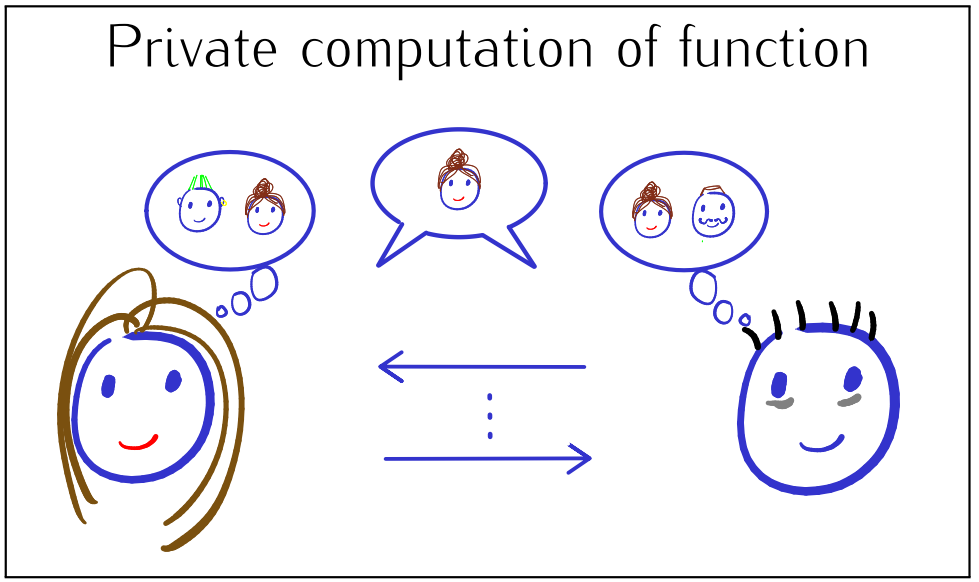
Lecture 21 (22/Oct/24)

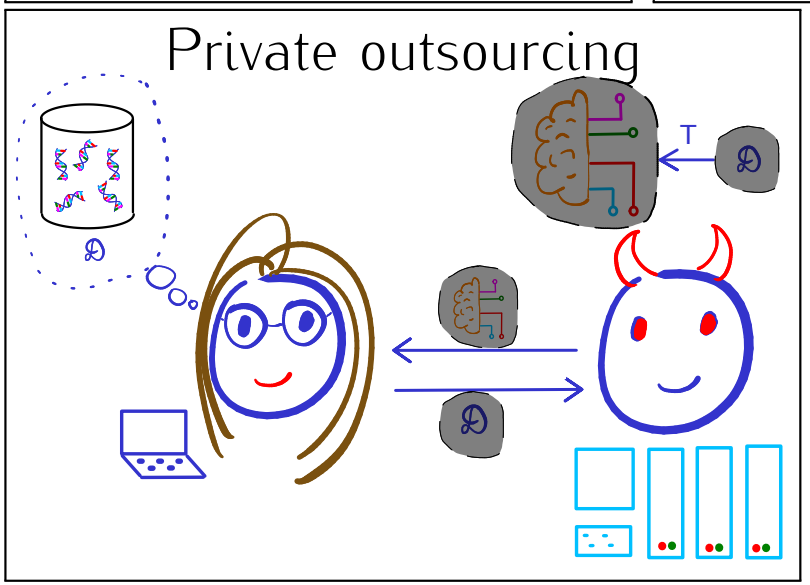
Instructor: Chethan Kamath

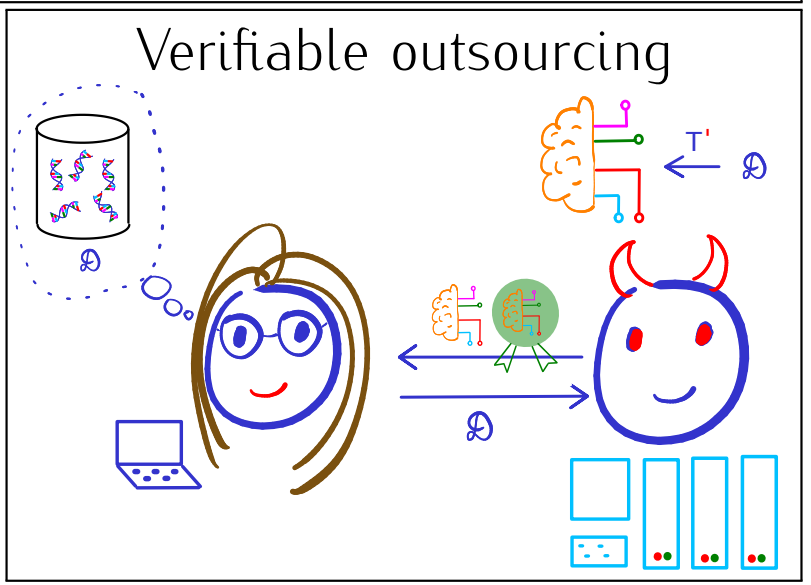
Recall from Last Module

- We saw several avatars of secure computation









Plan for Next Three Lectures...

MODULE 1
(Shared keys)

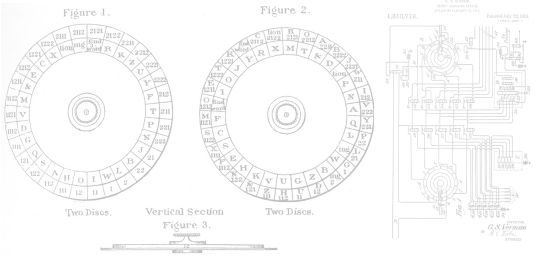
MODULE 2
(Public keys)

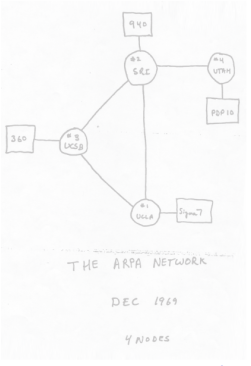
MODULE 3
(Secure comp.)

MODULE 4
(Adv. tasks)

For a large part of history

↷ Advent of internet ↷ Ubiquity of computing





*

**

```
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int main(int argc, char *argv[])
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Plan for Next Three Lectures...

- Focus today: limitations of *black-box reductions*

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  - Formalise black-box reduction of one primitive (e.g., PRF) to another (e.g., PRG) PRG → PRF
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  - Certain primitives (e.g., PKE) *cannot* be "black-box reduced" to others (e.g., OWF) OWF ↛ PKE

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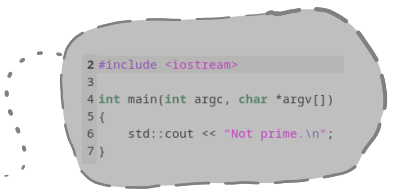

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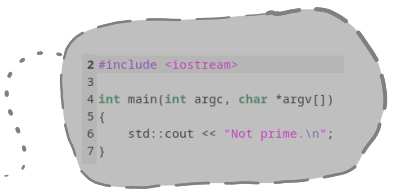

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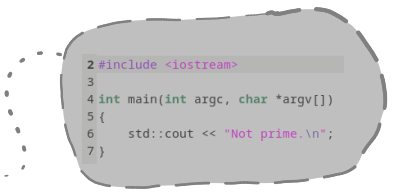

  - How to formalise security?
    - Virtual black-box (VBB) and indistinguishability obfuscator (IO)
  - Code obfuscation is powerful
    - Helps bypass certain black-box separations (e.g., OWF $\not\rightarrow$ PKE)
    - IO $\rightarrow$ most cryptographic primitives!

Plan for Today's Lecture

1. Black-Box Reduction $\rightarrow$
2. Black-Box Separation $\not\rightarrow$
3. Black-Box Separating OWF from OWP $\mathrm{OWF} \not\rightarrow \mathrm{OWP}$

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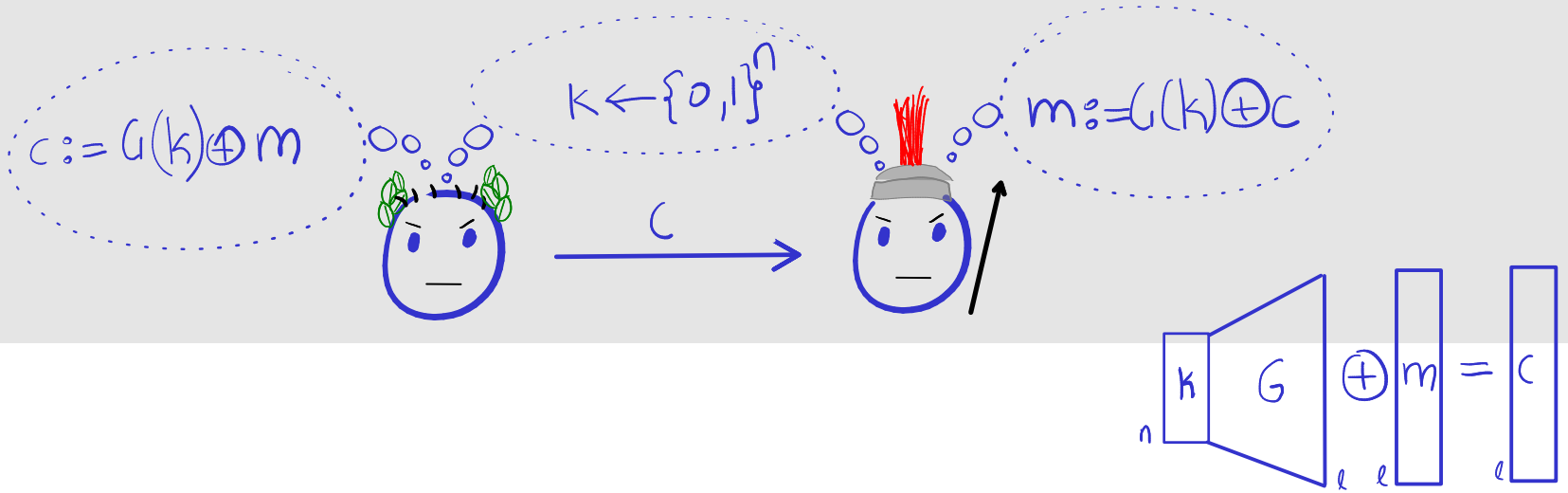
Recall Our First Cryptographic Reduction...

Construction 1 (Computational OTP Π from PRG G)



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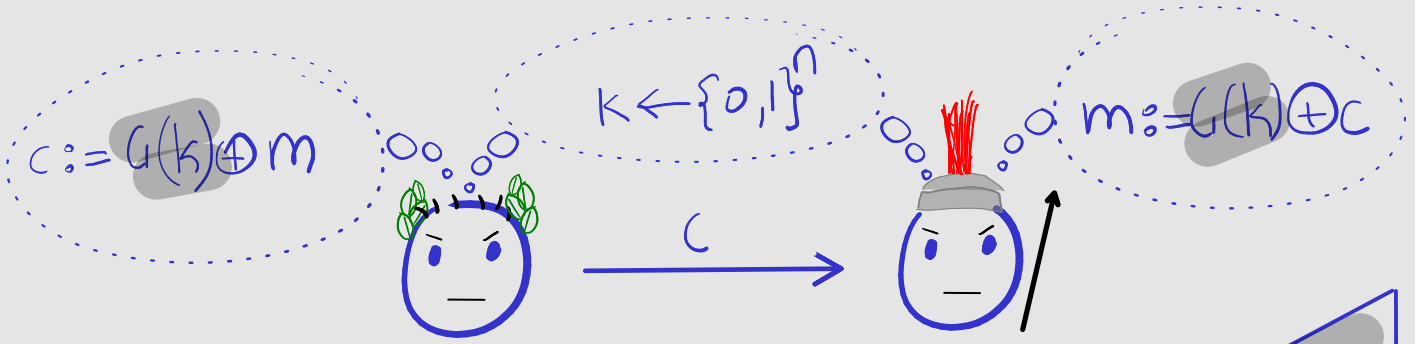

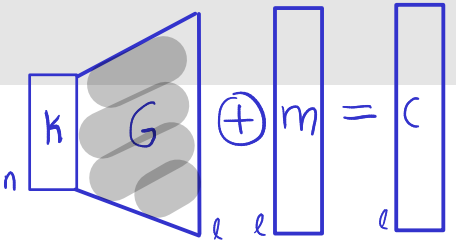

- Construction Π uses PRG G as a "black-box"
  - Only needs query access to G
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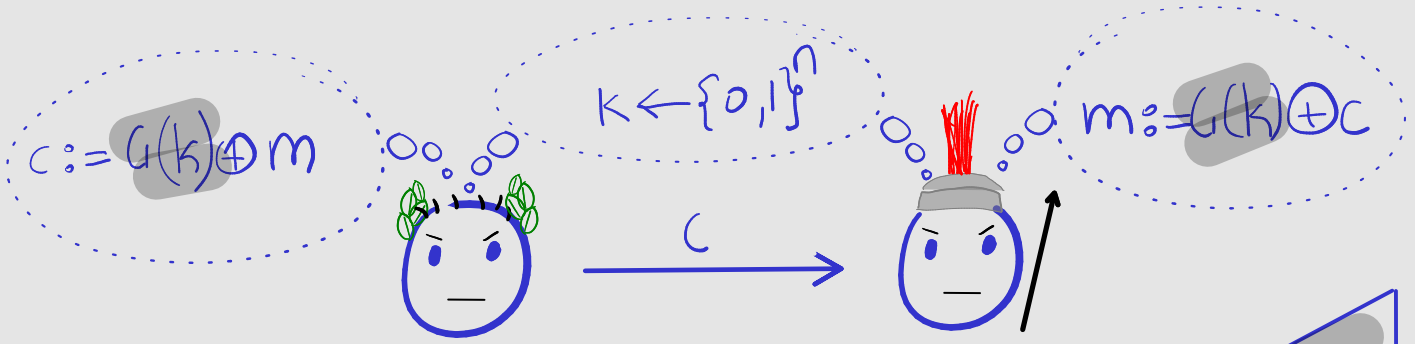

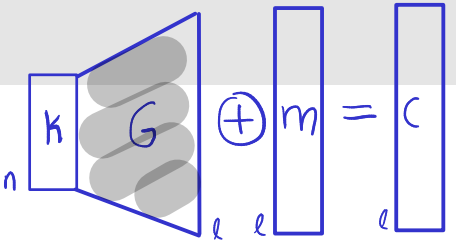

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Pseudocode 1 (of Π^G) ↪ denotes oracle/black-box access to G

- $\mathsf{Gen}(1^n)$: *output* $k \leftarrow \{0,1\}^n$
- $\mathsf{Enc}^G(k, m)$: *query* G *on* k *to obtain* y *and output* $c := y \oplus m$
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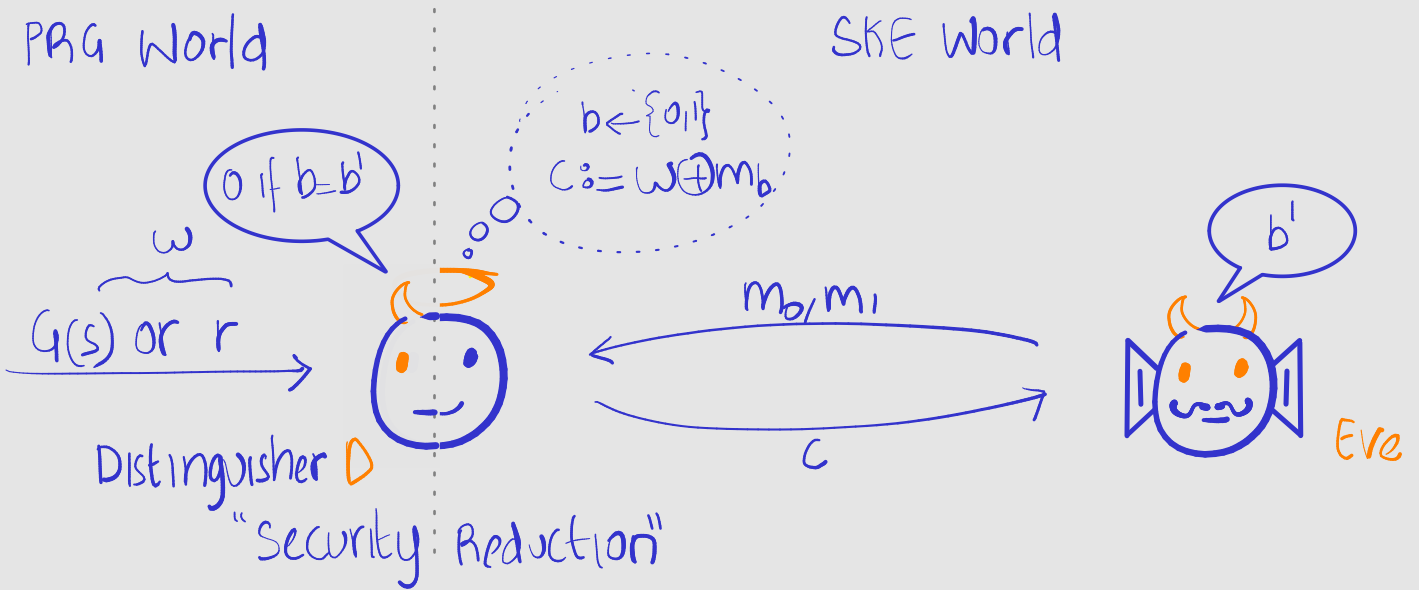
Theorem 1

Assuming G *is a PRG, Construction 1 is computationally secret.*

Proof by *security reduction*: ∃Eve breaking Π ⇒ ∃D for G.



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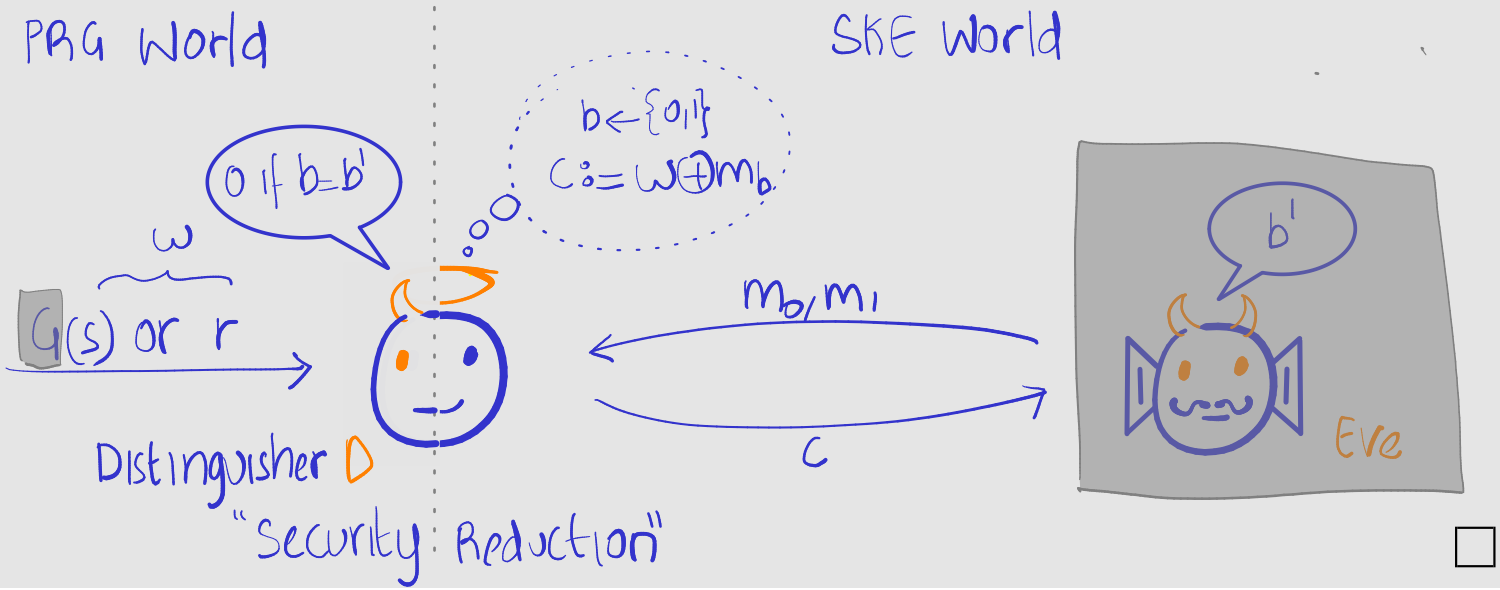

- Security reduction D (+analysis) uses G and Eve as black box
  - Only needs query access to G and Eve: $D^{G,Eve}$
  - Does not depend on exact implementation of G and Eve

Our Reductions So Far Have All Been "Black Box"...

1. PRG $\rightarrow$ PRF (GGM construction)
2. PRF $\rightarrow$ CPA-SKE
3. OWP $\rightarrow$ hardcore predicate (Goldreich-Levin construction)
4. PRF $\rightarrow$ MAC
5. TDP $\rightarrow$ PKE
6. Commitment $\rightarrow$ Computational ZKP for **NP** (Blum's protocol)
7. OT $\rightarrow$ 2PC (GMW protocol)

...

Let's Formally Define (Fully) Black-Box Reduction

- *Both* construction and security reduction are black-box

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Defintion 1 ((Fully) black-box (BB) reduction of OWP to OWF)

A pair of efficient oracle-algorithms $(\Pi^{\cdot}, \mathsf{Finv}^{\cdot,\cdot})$ *such that*

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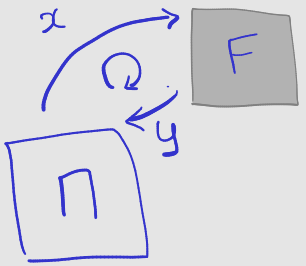
A pair of efficient oracle-algorithms $(\Pi^{\cdot}, \mathbf{Finv}^{\cdot,\cdot})$ *such that* (denotes oracle arguments)

1. *Correctness: for every function* F*, construction* Π^F *is permutation*



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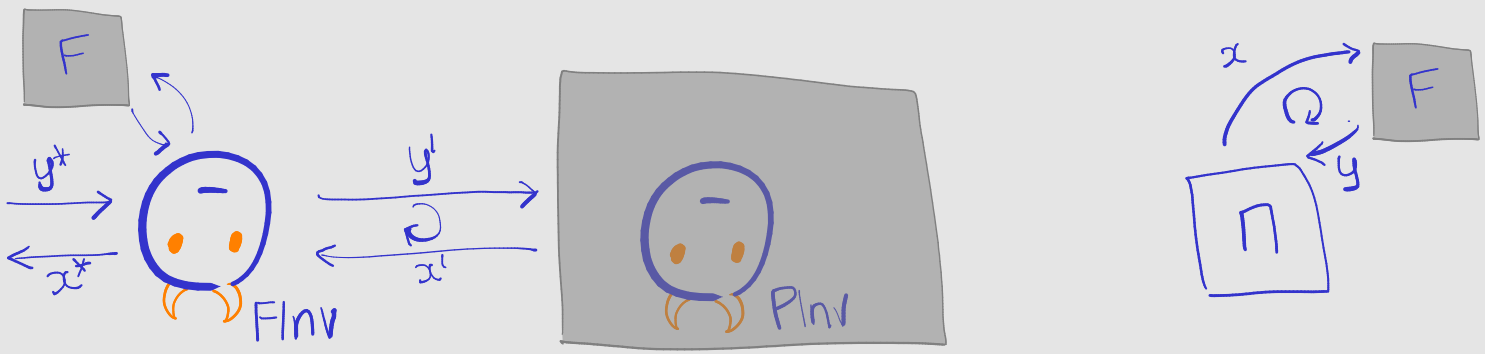

2. *Security: for every one-way* F *and for every OWP-inverter* $\mathbf{Pinv}^{\cdot}$ *that inverts* Π^F*, the security reduction* $\mathbf{Finv}^{\mathbf{Pinv},F}$ *inverts* F

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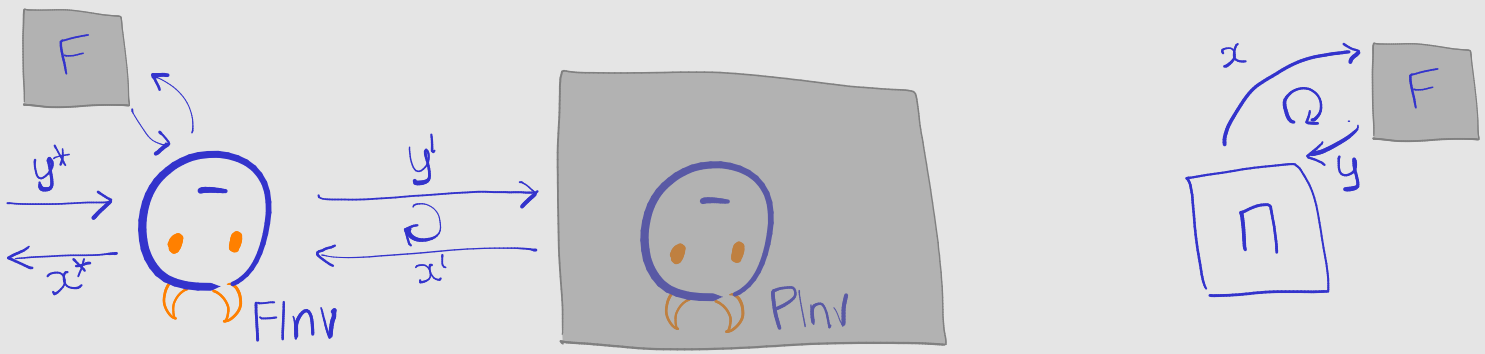

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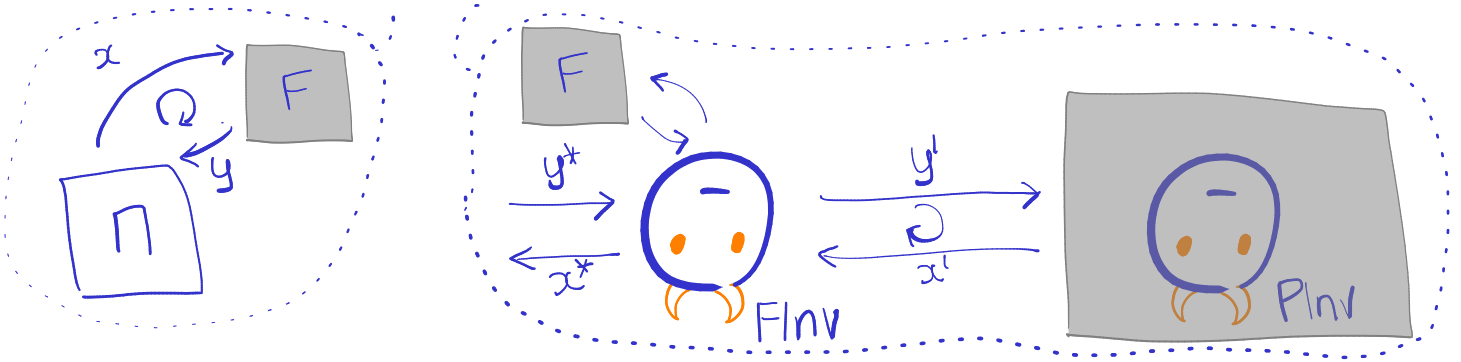
Exercise 1

Formulate the general definition for any two primitives $\mathcal{P}$ *and* $\mathcal{Q}$

BB Reduction "Relativises"...

- BB reduction $(\Pi^{\cdot}, \mathbf{FInv}^{\cdot,\cdot})$ of OWP to OWF:



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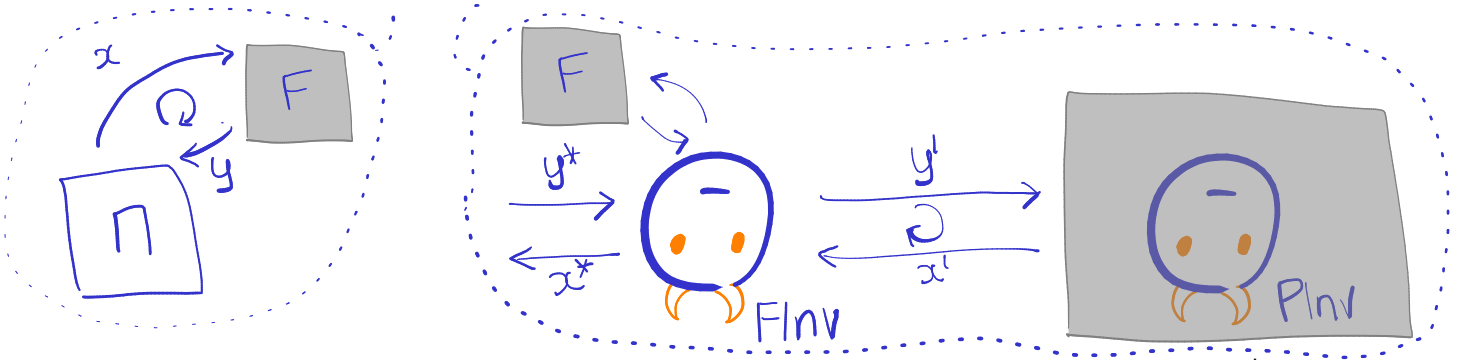

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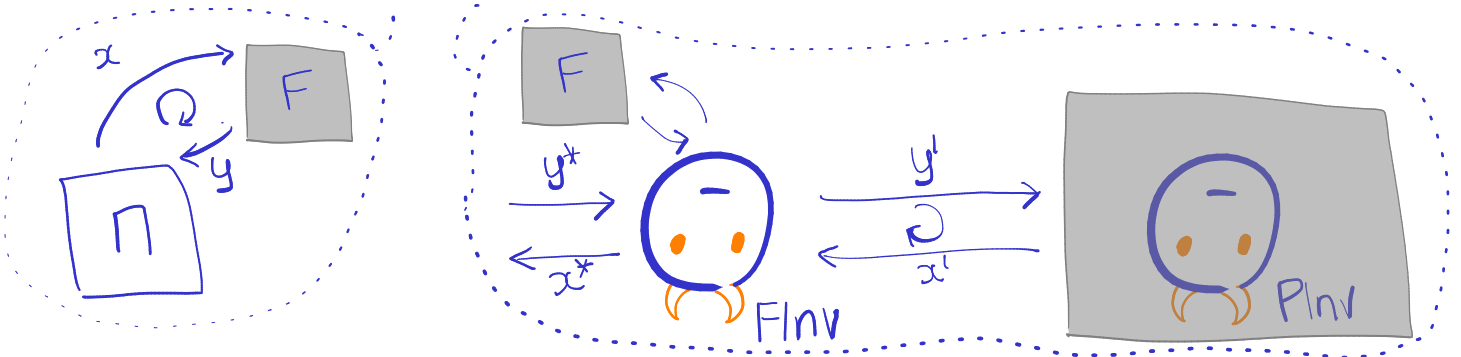

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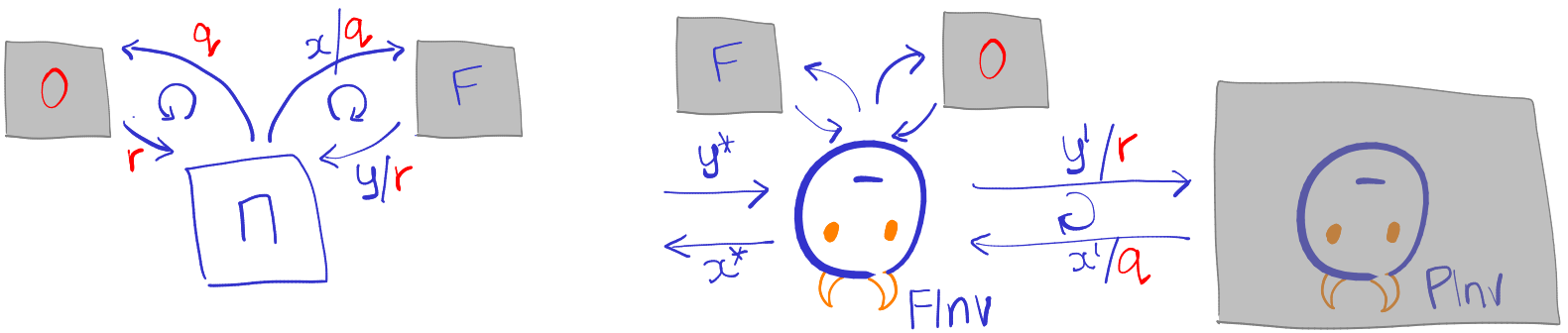

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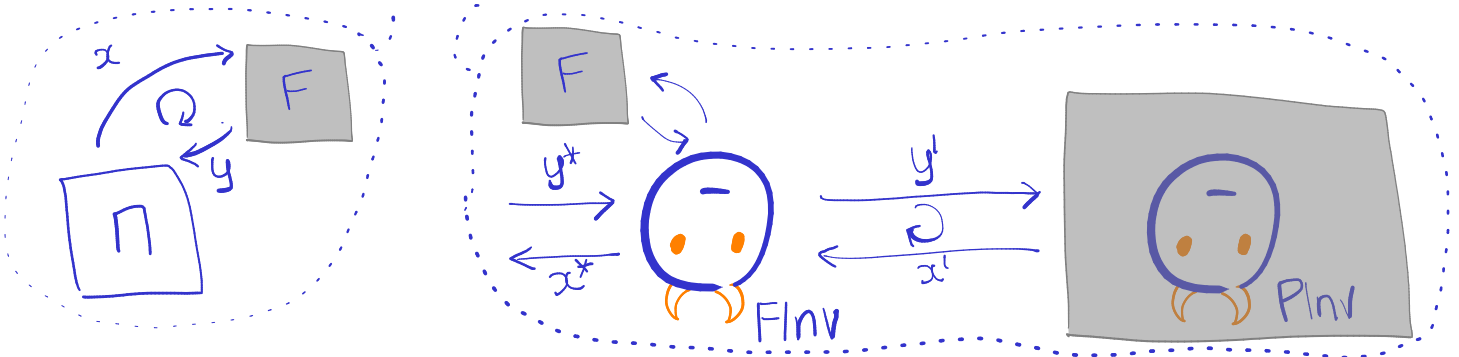

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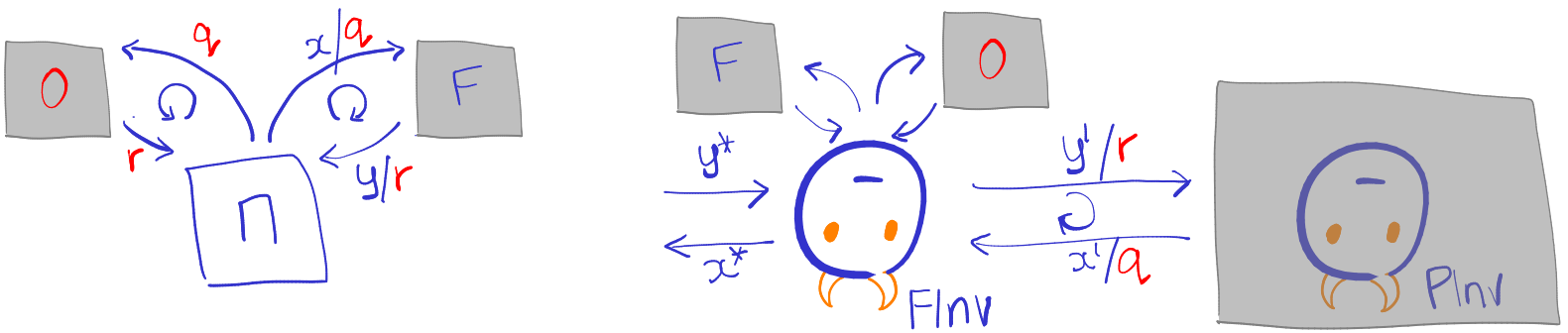

Claim 1 (Contrapositive)

$(\Pi^{\cdot,\cdot}, \mathsf{FInv}^{\cdot,\cdot,\cdot})$ *does not exist relative to some oracle* $\mathrm{O} \Rightarrow (\Pi^{\cdot}, \mathsf{FInv}^{\cdot,\cdot})$ *does not exist*

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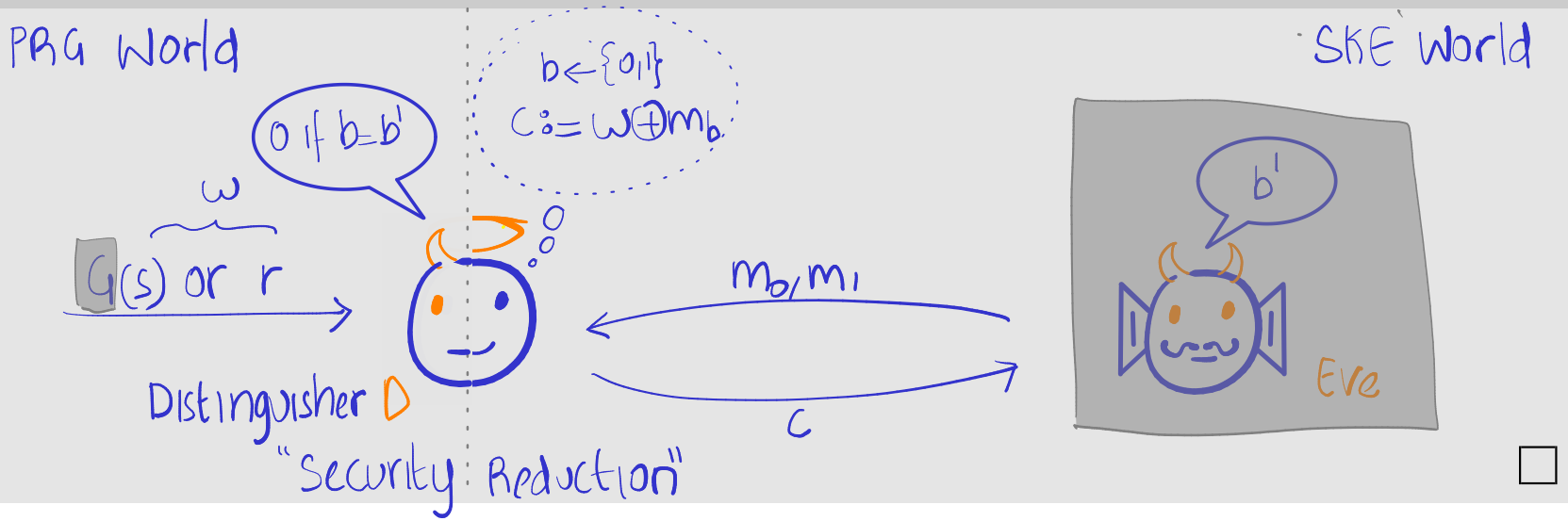

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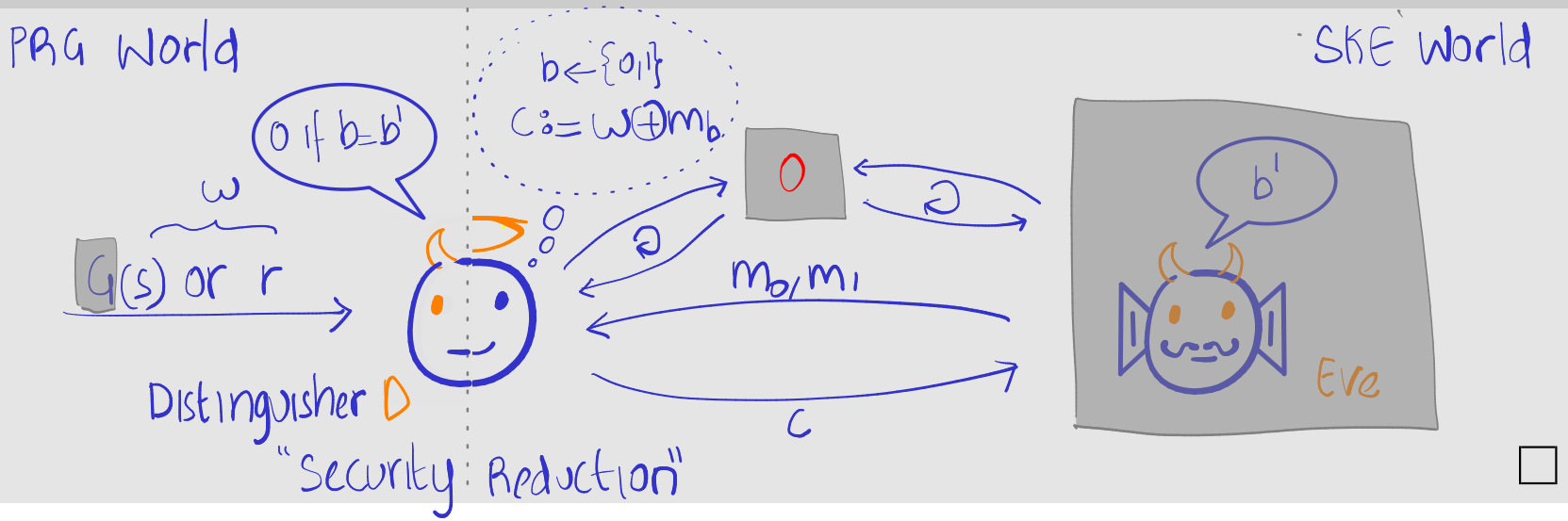

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Plan for this Session

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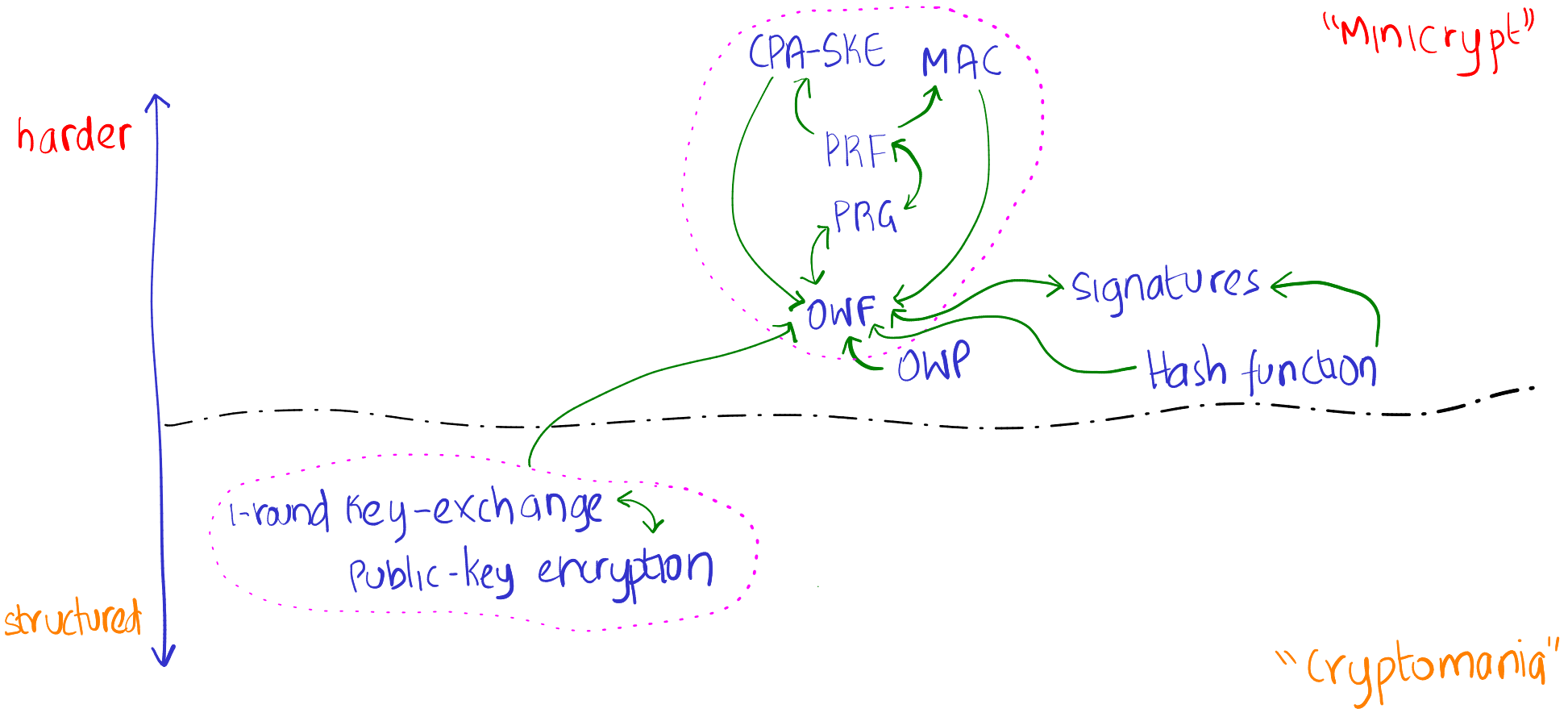
Recall Our Landscape



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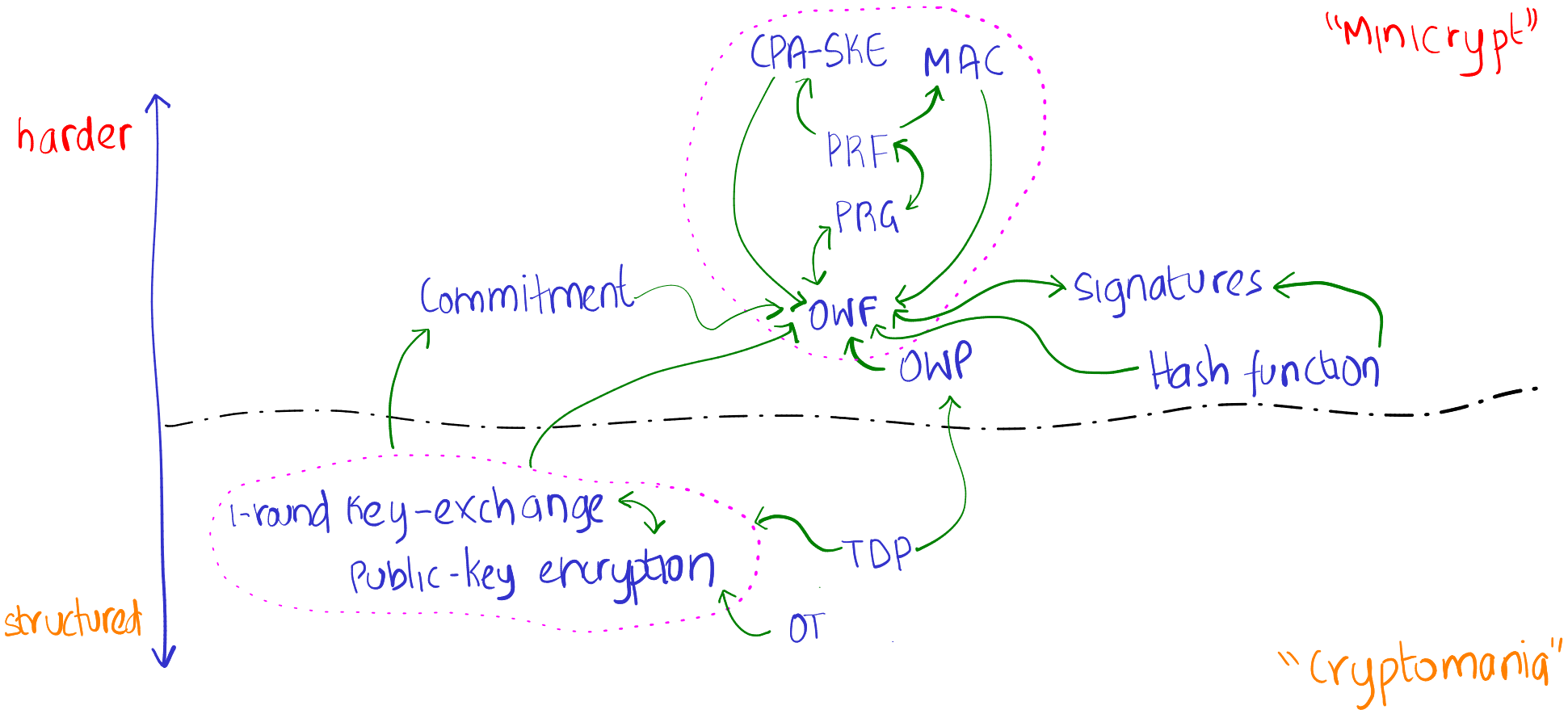

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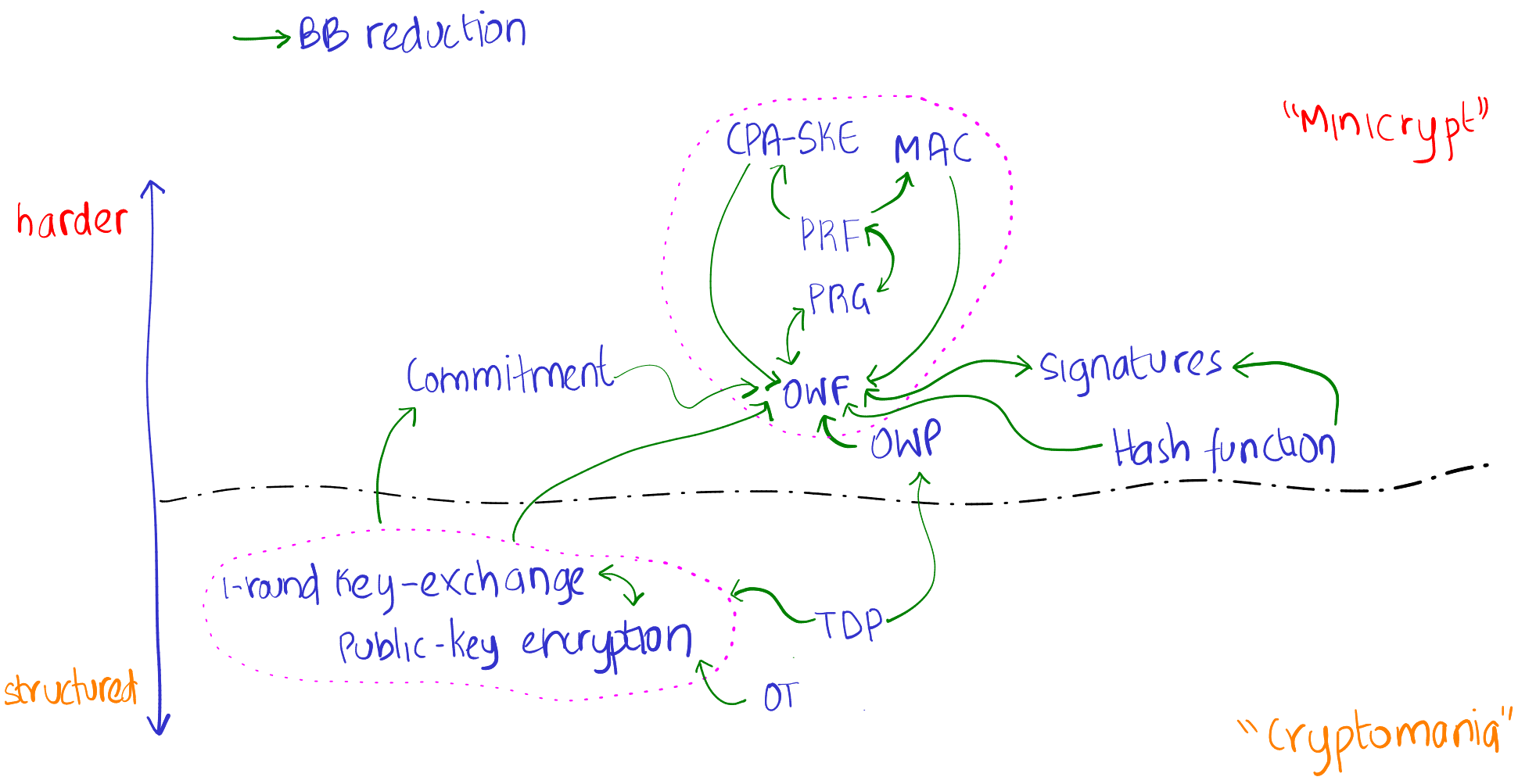

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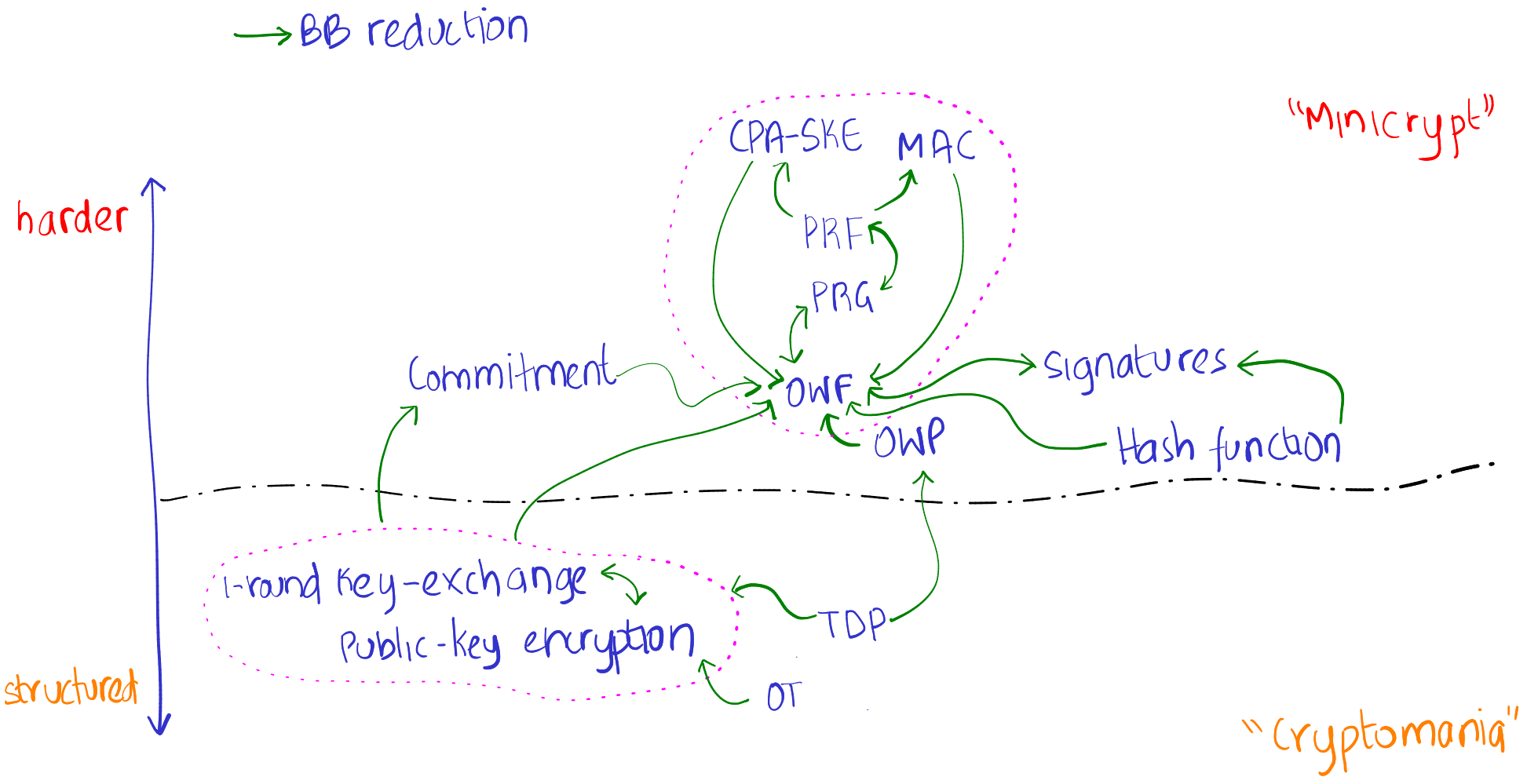

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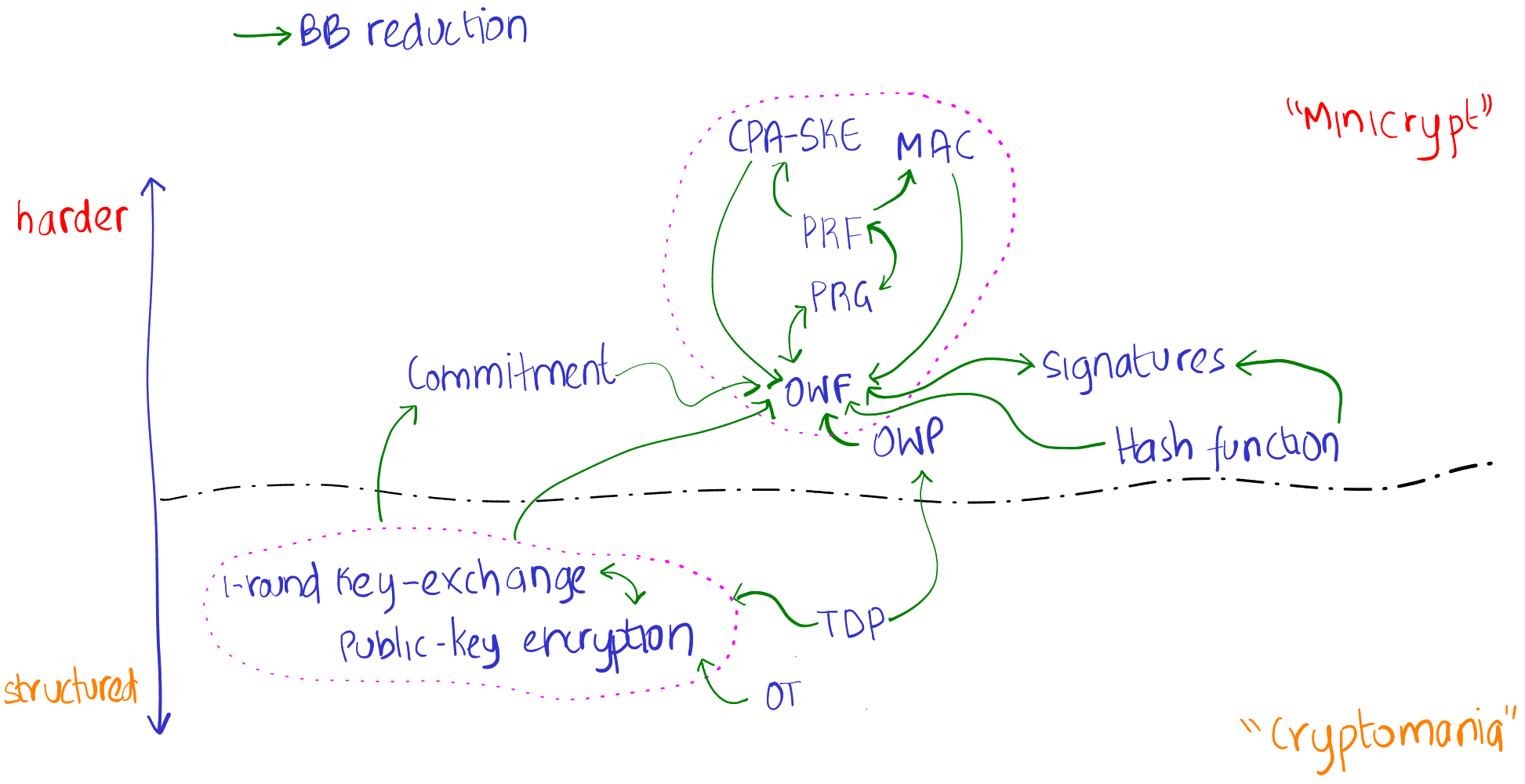

- What about OWF → OWP or OWF → PKE? We don't know

“Separating” OWF from OWP

Show that OWF exists but OWP doesn't?

“Separating” OWF from OWP

- Show that OWF exists but OWP doesn't?
  - Both OWF and OWP are believed to exist
  - Implies $\mathsf{P} \neq \mathsf{NP}$

“Separating” OWF from OWP

- Show that OWF exists but OWP doesn't?
  - ⚠ Both OWF and OWP are believed to exist
  - ⚠ Implies $\mathsf{P} \neq \mathsf{NP}$

OWF $\not\to$ OWP

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  - ∀ 1 for every BB reduction $(\Pi^{\cdot}, \mathbf{FInv}^{\cdot,\cdot})$ of OWP to OWF
  - ∃ 2 there exists a OWF F and a OWP-inverter $\mathbf{PInv}^{\cdot}$ such that
  - — 3 $\mathbf{PInv}$ inverts Π^{F} but $\mathbf{FInv}^{\mathbf{PInv},F}$ does not invert F

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∃

- By Claim 1, suffices to show there exists oracle O such that:

∀ 1 for every BB reduction $(\Pi^{\cdot,O}, \mathbf{FInv}^{\cdot,\cdot,O})$ of OWP to OWF

∃ 2 there exists a OWF F^{O} and a OWP-inverter $\mathbf{PInv}^{\cdot,O}$ such that

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Plan for this Lecture

1. Black-Box Reduction $\rightarrow$
2. Black-Box Separation $\not\rightarrow$
3. Black-Box Separating OWF from OWP $\text{OWF} \not\rightarrow \text{OWP}$

High-Level Idea of the Separation

- We will come up with a "helper" oracle O such that

think of $O = \mathrm{PSPACE}$

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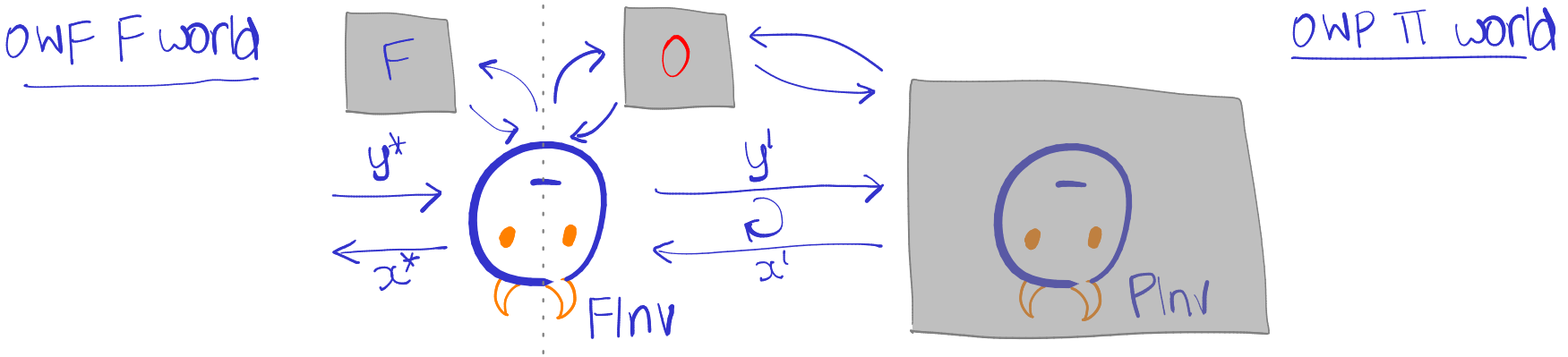

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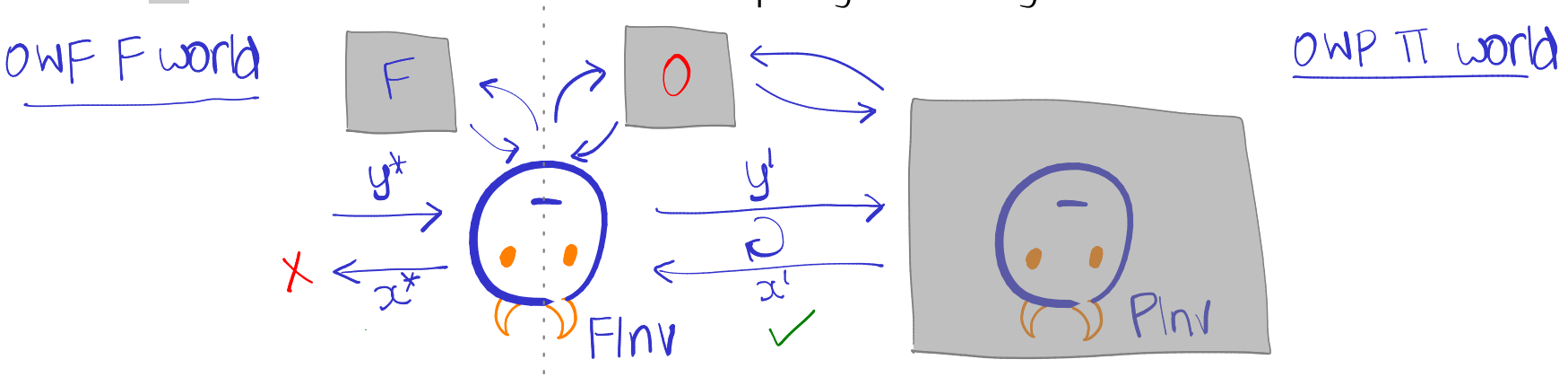

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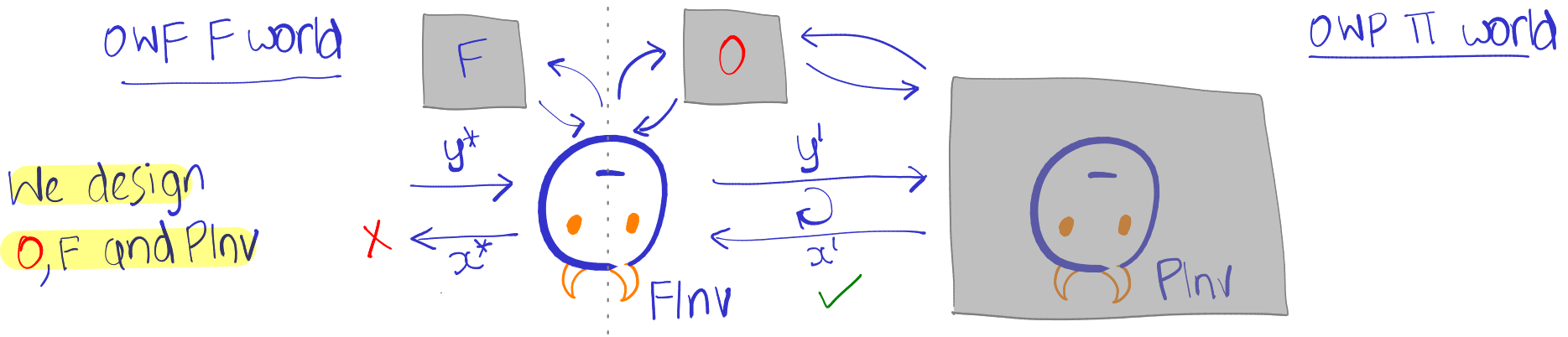

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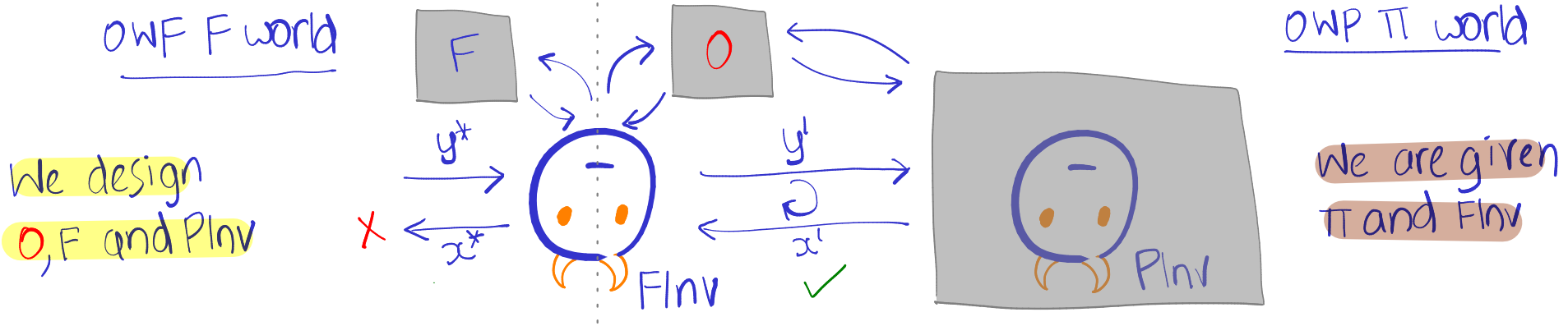

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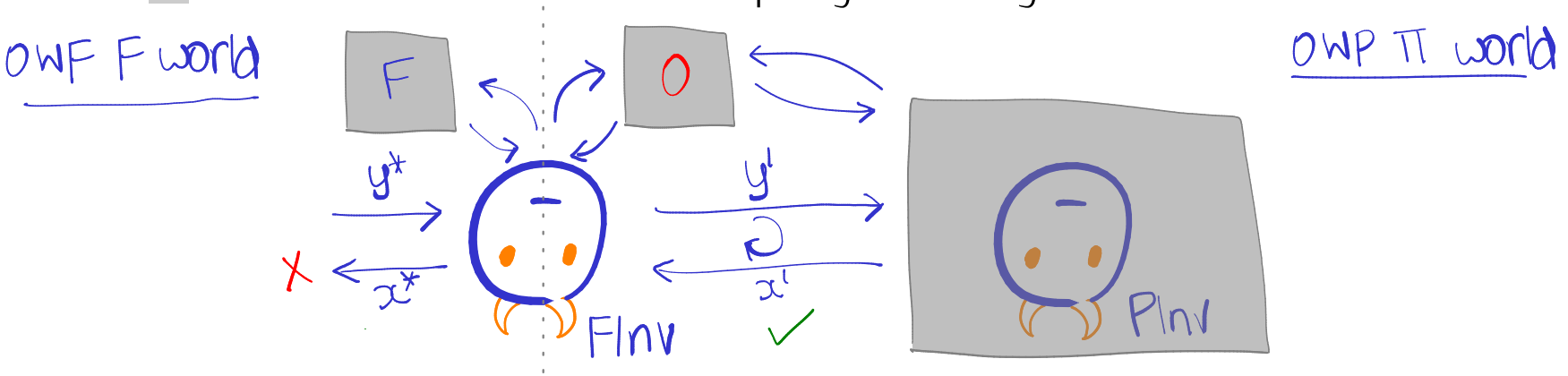

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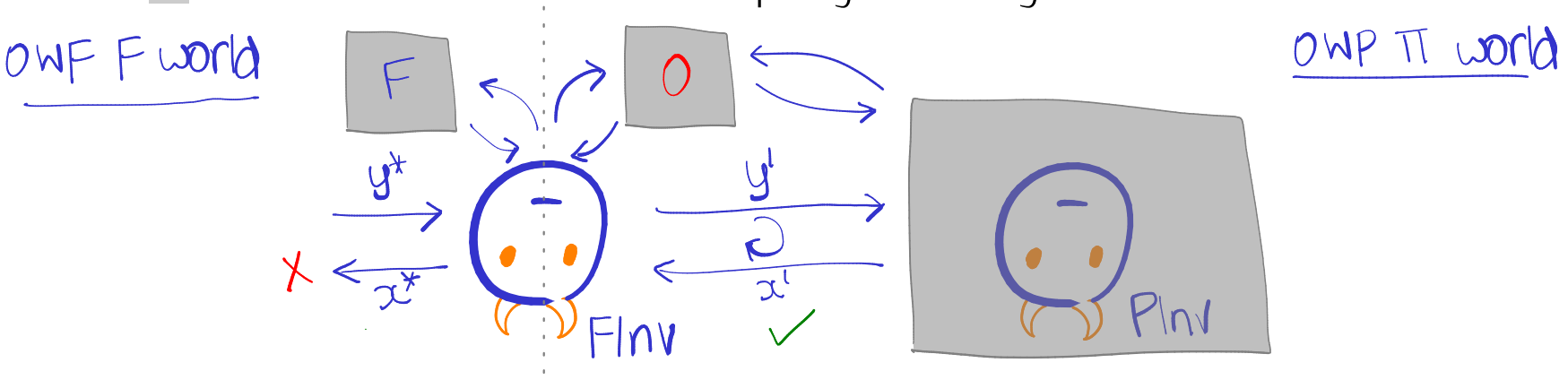

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- Step I: design query-learning **PInv** that *efficiently* breaks OWP Π^F given access to O
  - Idea: exploit the fact that Π^F is a permutation for *any* F
- Step II: show **FInv** can't break random oracle F even given O
  - Idea: random oracle output is unpredictable ⇒ one-wayness

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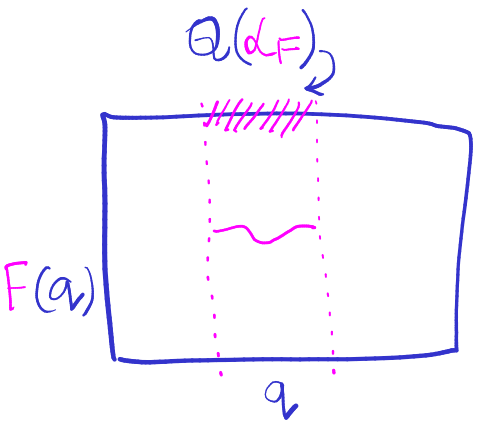

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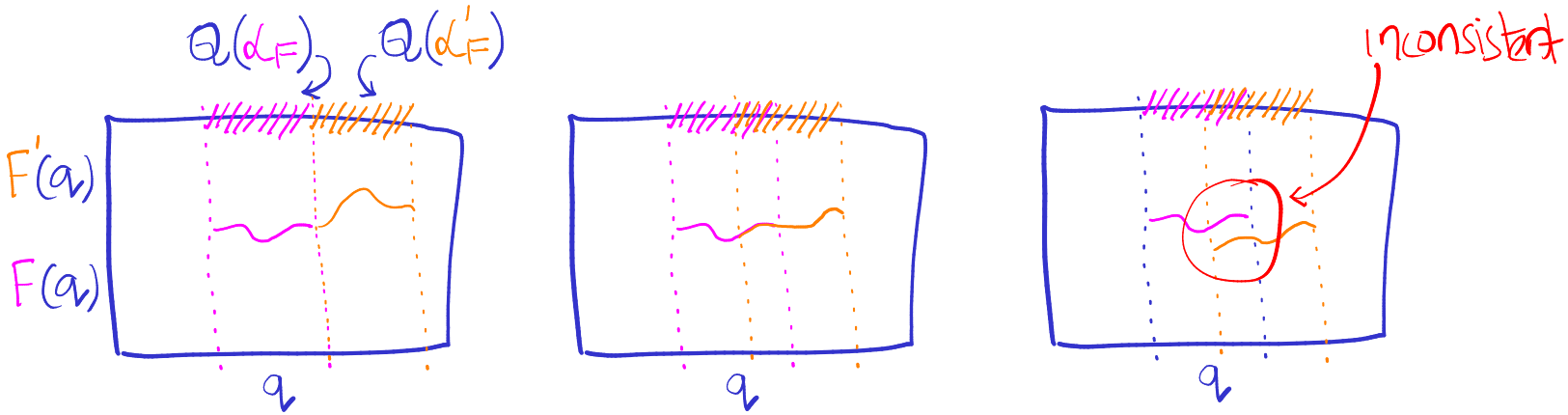

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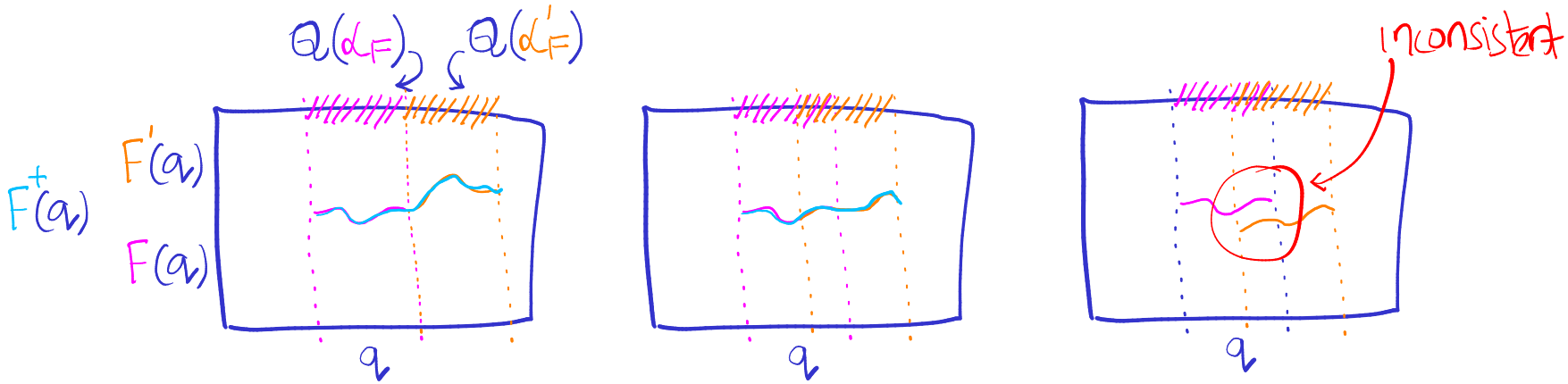

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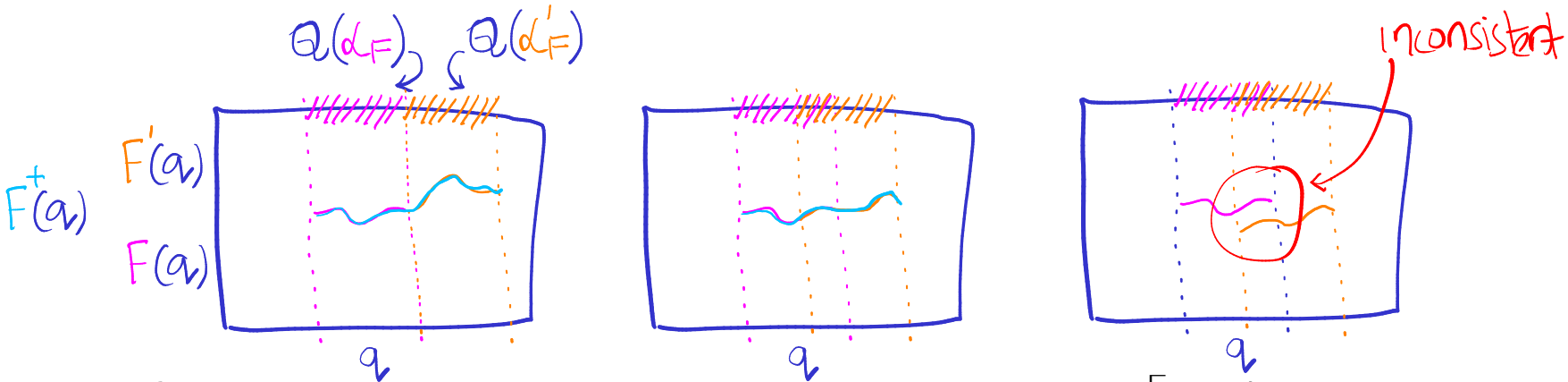

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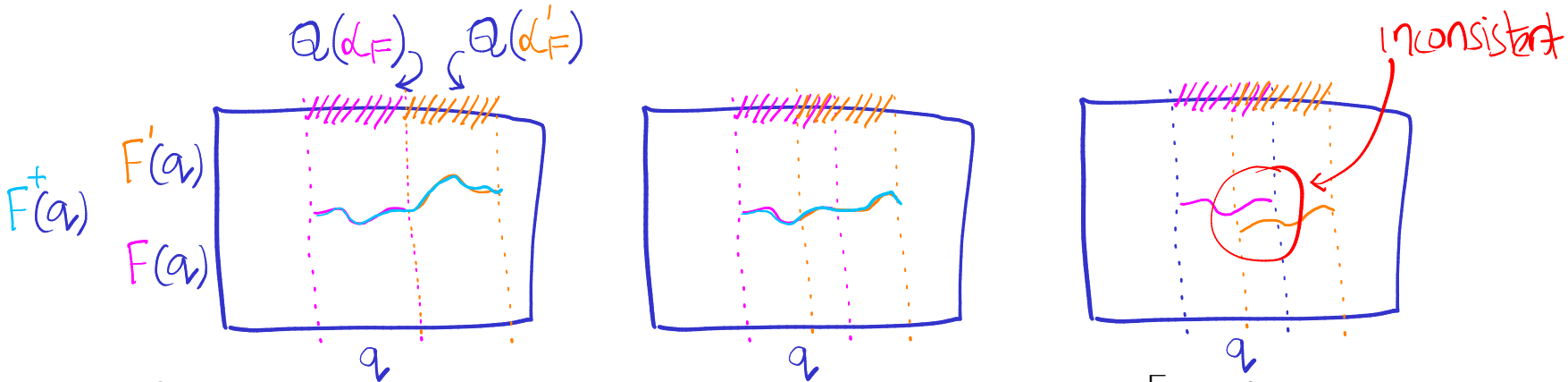

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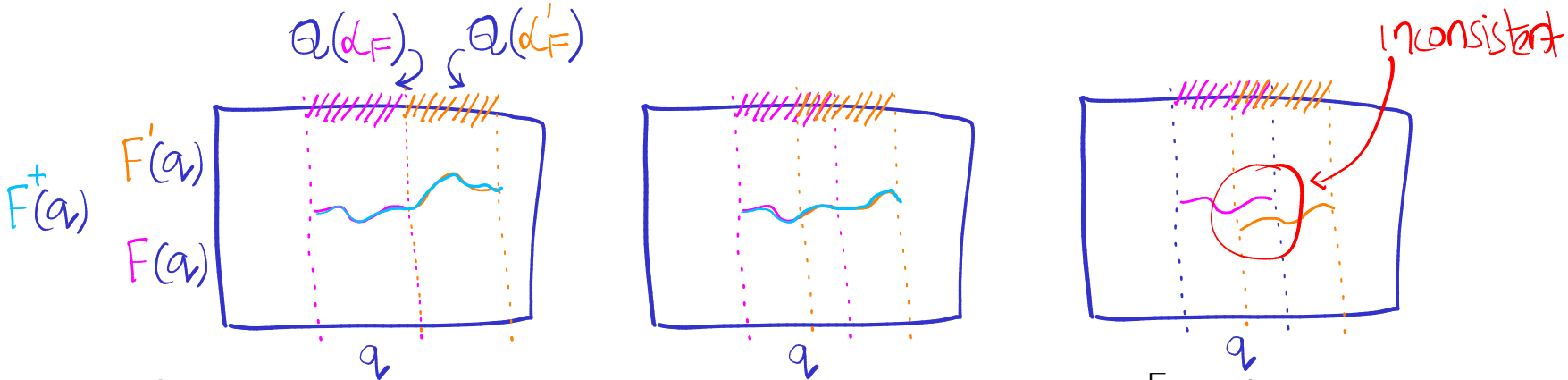

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  - Main observation: (partial F and F' consistent) and $(\Pi^F(x) = \Pi^{F'}(x') = y) \Rightarrow Q(\mathcal{C}_{F,x}) \cap Q(\mathcal{C}_{F',x'}) \neq \emptyset$. Why?

Step I: Design Query-Learning PInv...

strategy: learn at least one new query from $C_{F,x}$ per round

Construction 2 (Query-learning inverter $\mathbf{PInv}^{\Pi,F,O}(y)$) $y = \Pi^F(\alpha)$

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RO ↖ ↗ PSPACE

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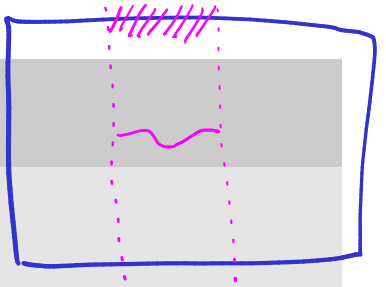

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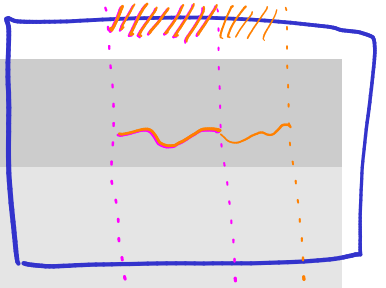

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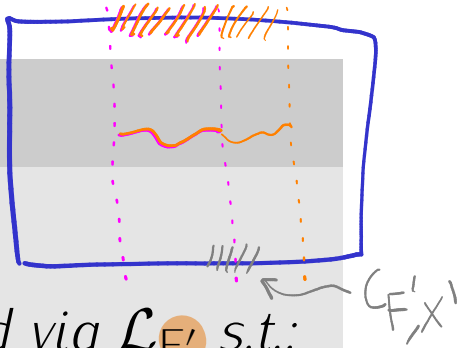

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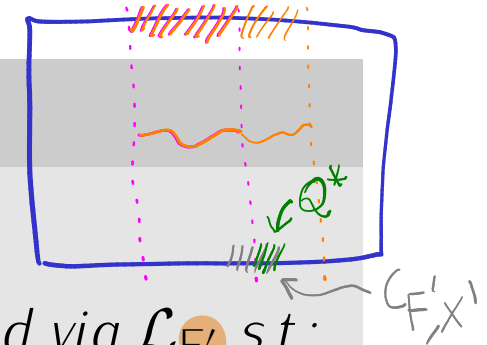

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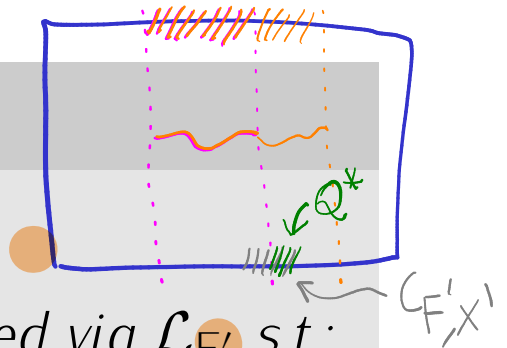

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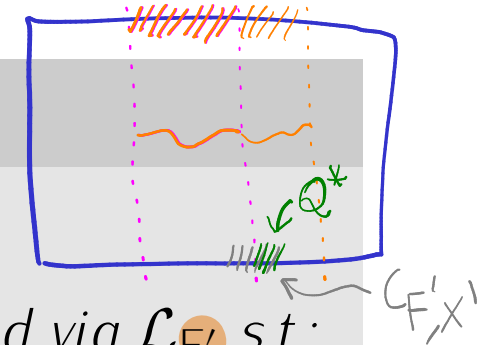

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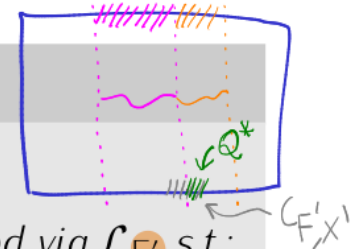
Lemma 2

If $O = \mathsf{PSPACE}$*,* PInv *outputs* $x : \Pi^F(x) = y$ *in polynomial time*

Why does PInv Work?



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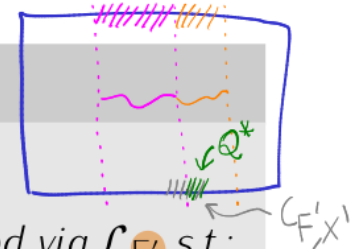
Proof of Lemma 2 (PInv outputs $x : \Pi^F(x) = y$ in polynomial time).

1. Claim 1: PInv outputs x such that $\Pi^F(x) = y$ and halts

Why does PInv Work?

Construction 2 (Query-learning inverter $\mathbf{PInv}^{\Pi,F,O}(y)$)



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Proof of Lemma 2 (PInv outputs $x : \Pi^F(x) = y$ in polynomial time).

1. Claim 1: PInv outputs x such that $\Pi^F(x) = y$ and halts
   - Sub-claim: $\mathcal{Q}^*$ contains at least one query $q^* \in Q(\mathcal{C}_{F,x} \setminus \mathcal{L}_F)$.
     - (?) Why?

Why does PInv Work?

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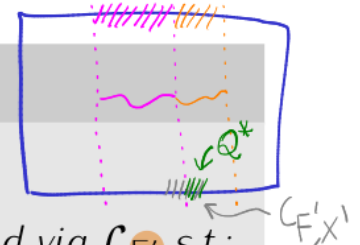

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     - (?) Why? Otherwise, possible to "merge" F and F' into F^+ such that collision! $\hookrightarrow$ $\Pi^{\mathsf{F}^+}(x') = \Pi^{\mathsf{F}^+}(x) = y$. How?
   - Sub-claim $\Rightarrow$ Claim 1 since $|\mathcal{C}_{\mathsf{F},x}|$ is polynomial $\square$

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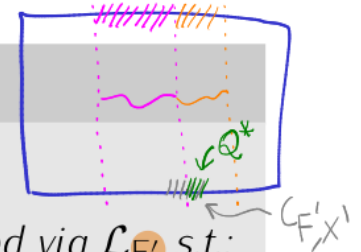

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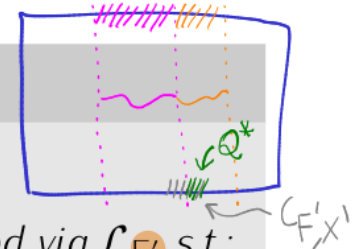

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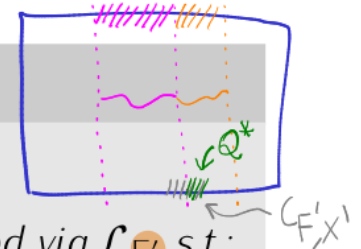

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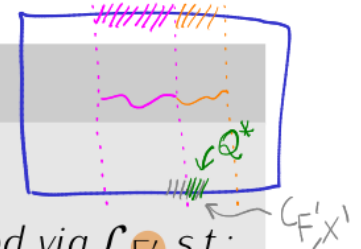

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   - Framed as NP language: $\{(y, \mathcal{L}_F), (\mathcal{L}'_F, x') : 2.1 \text{ and } 2.2 \text{ holds}\}$

What is PInv's run-time? □

Step II: Show F is One-Way Even Given O

Claim 2

For any fixed, efficient $\mathsf{FInv}^{\cdot}$*, the following is negligible*

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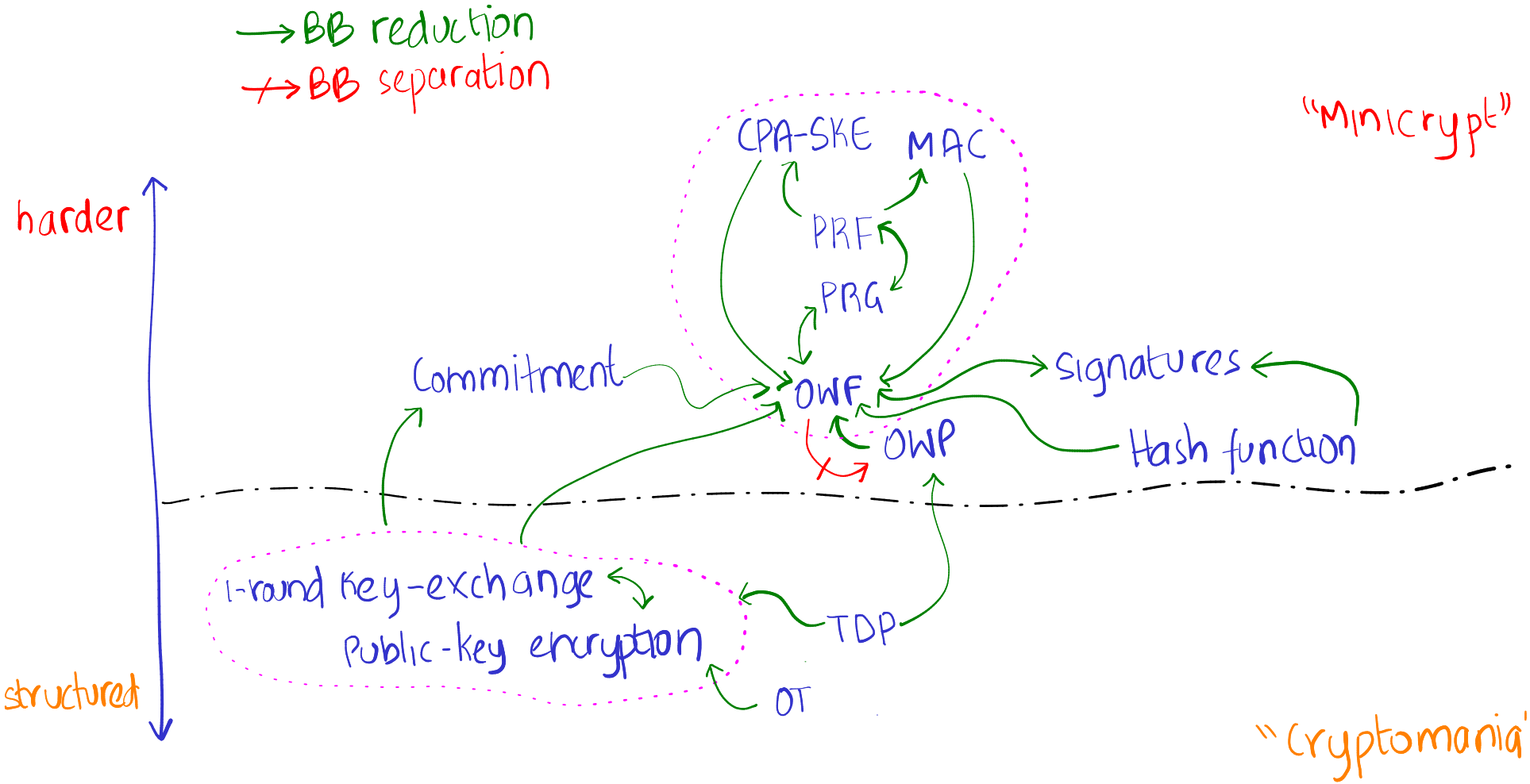
Exercise 2

Show that Claim 2 holds also with respect to the PSPACE *oracle* $\mathcal{O}$*.*

What Else Has Been BB Separated?



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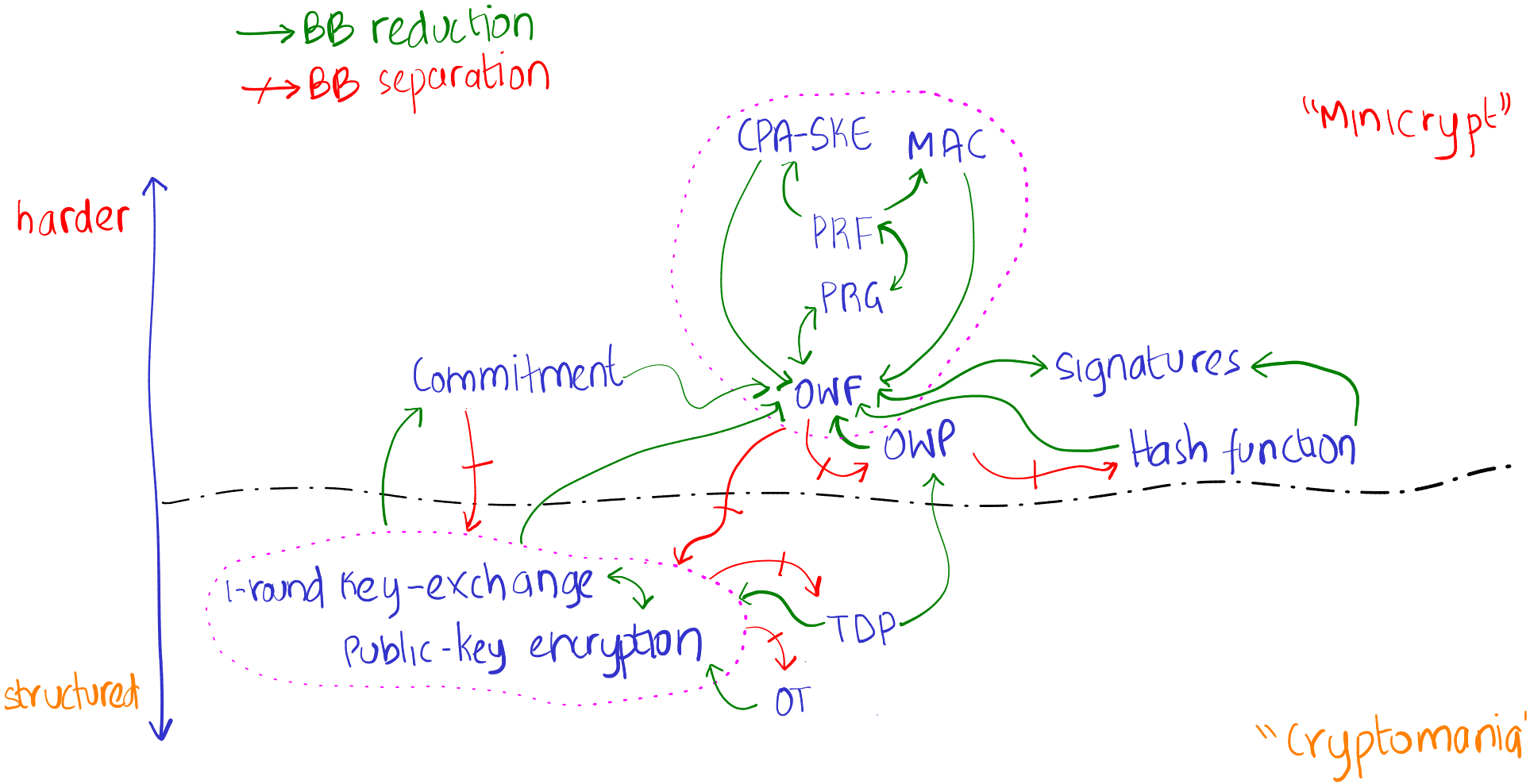

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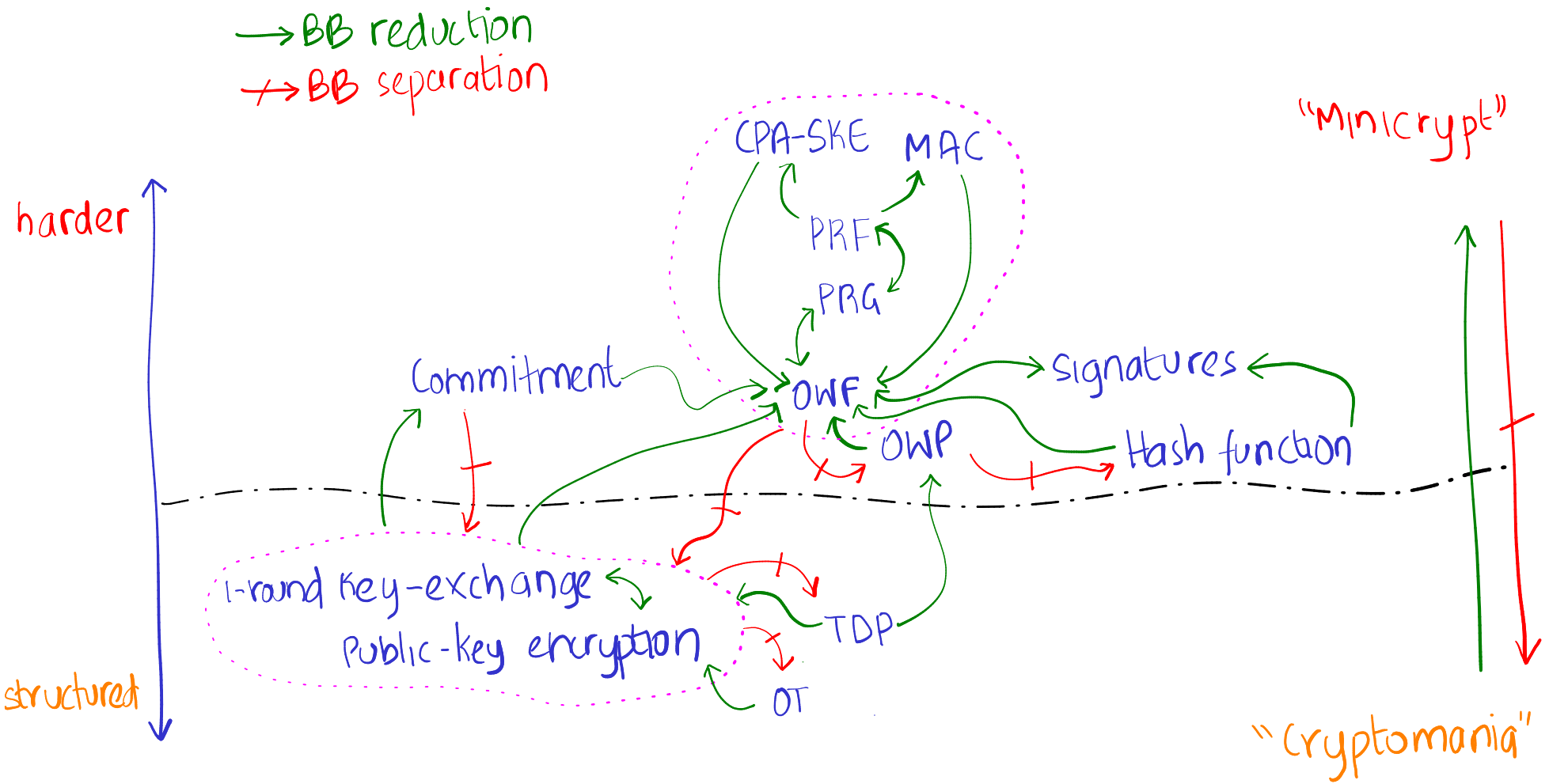

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- Separated OWF from OWP
- Key ideas:
  - Black-box reduction relativises: suffices to come up with an "oracle world" O where OWF exists but OWP doesn't
  - Efficient query-set learning algorithm that exploits perfect correctness of the construction
  - Random oracles are unpredictable, and hence one-way

Next Lecture

- Friday (25/Oct): crib session for Quiz 2

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- Tuesday (29/Oct): Obfuscation I
  - Virtual black-box (VBB) obfuscation
  - Bypassing separation OWF and OWP using code of the OWF:
    - OWF $\xrightarrow{\text{VBB obfuscation}}$ OWP
  - Impossibility of VBB obfuscation for general programs
  - Way around: relax to indistinguishability obfuscation (IO)

References

1. This lecture is mostly based on [Rud84, RTV04, Yer11]
2. Formal definition of fully BB reduction can be found in [RTV04, Yer11]. You can also find formal definitions of other notions of reductions (semi-BB, relativising etc.) there
3. The black-box separation of OWF from OWP is from Rudich's thesis [Rud84]
4. For more discussion on relativising reductions, refer to [AB09, §4.3]


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