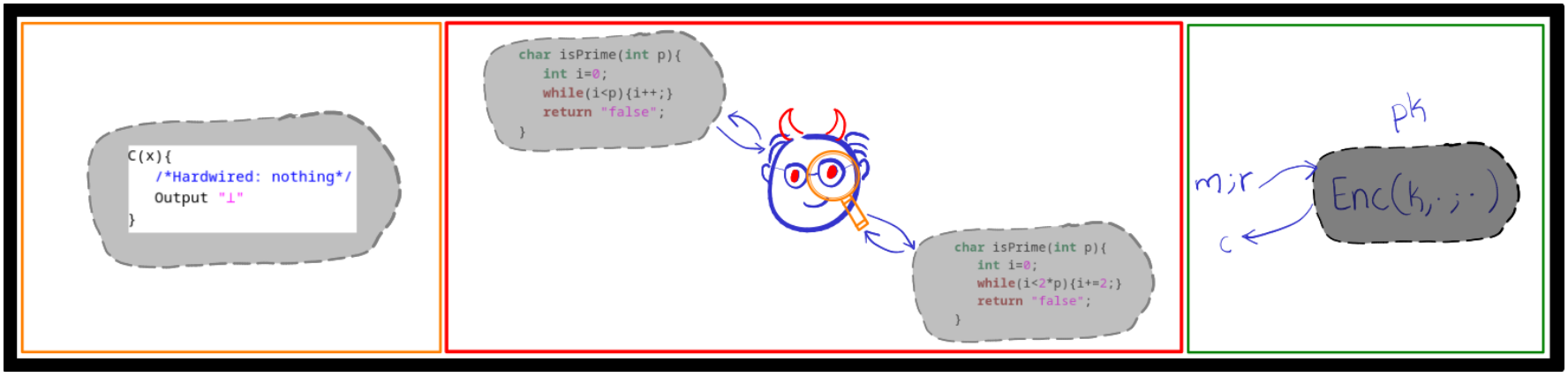


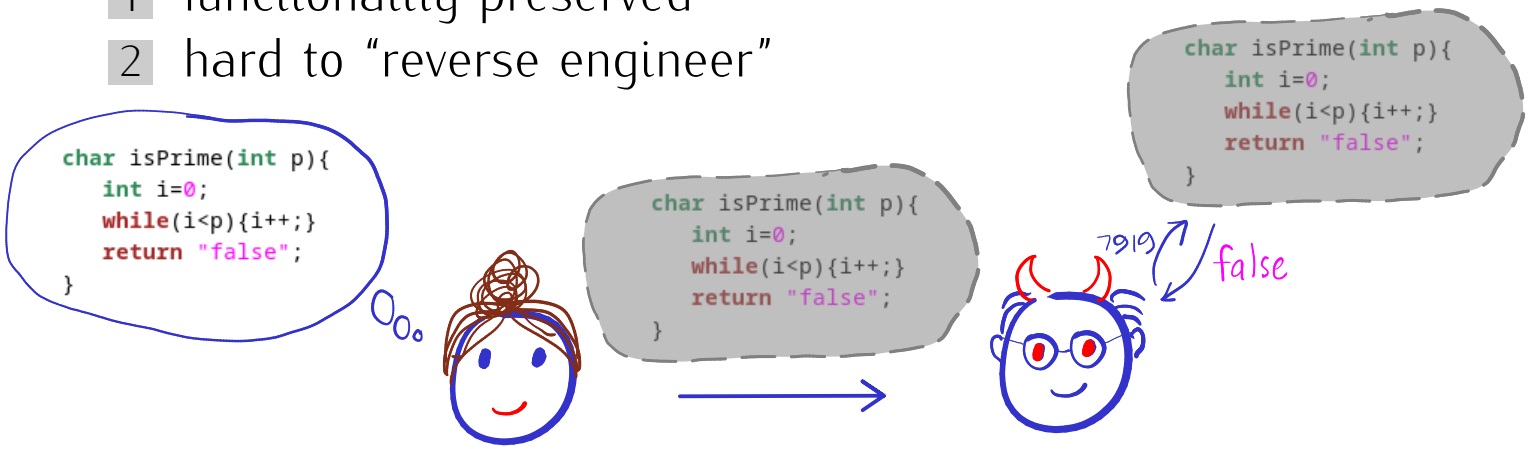
CS783: Theoretical Foundations of Cryptography

Lecture 23 (05/Nov/24)

Instructor: Chethan Kamath

Recall from Last Lecture...

- Program obfuscation: "scramble/encrypt" a program such that
  1. functionality preserved
  2. hard to "reverse engineer"



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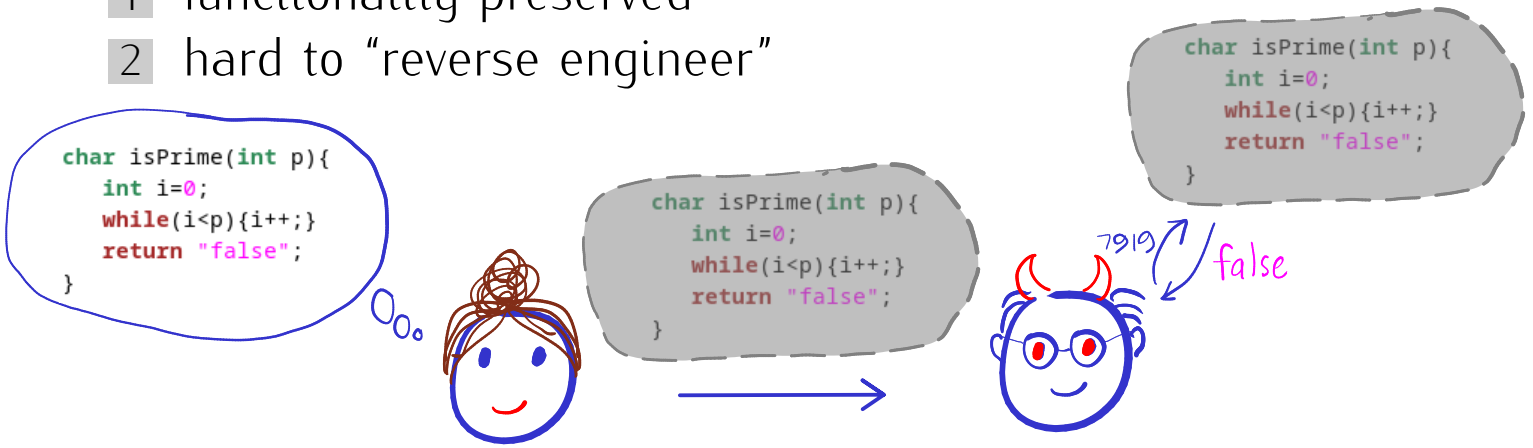

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  - Virtual black-box obfuscation (VBBO): anything learnable "white-box" given P is learnable 'black-box" given only oracle access to P

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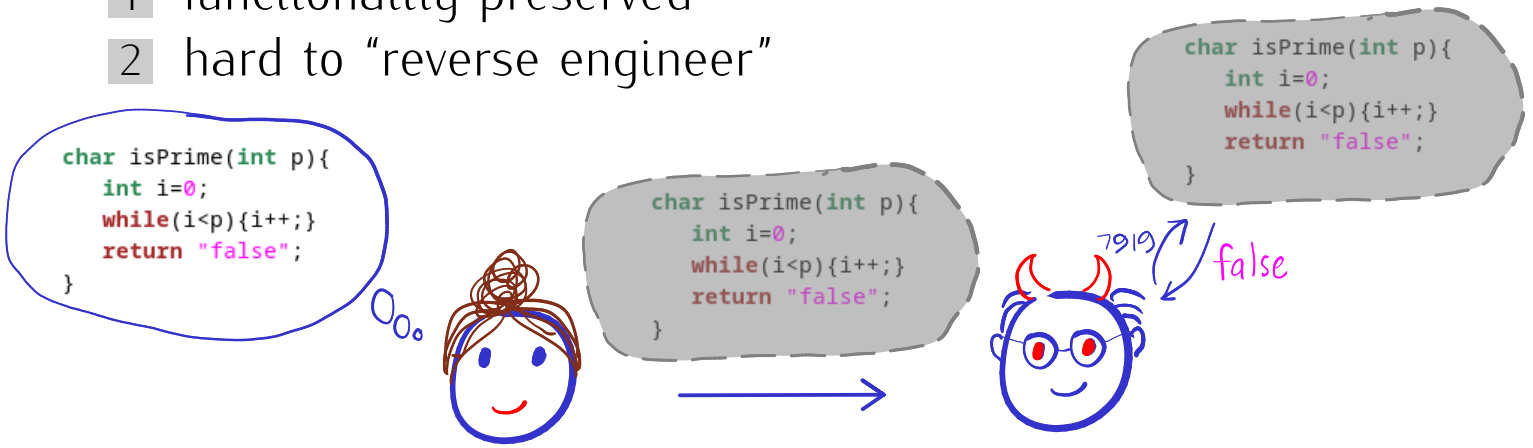

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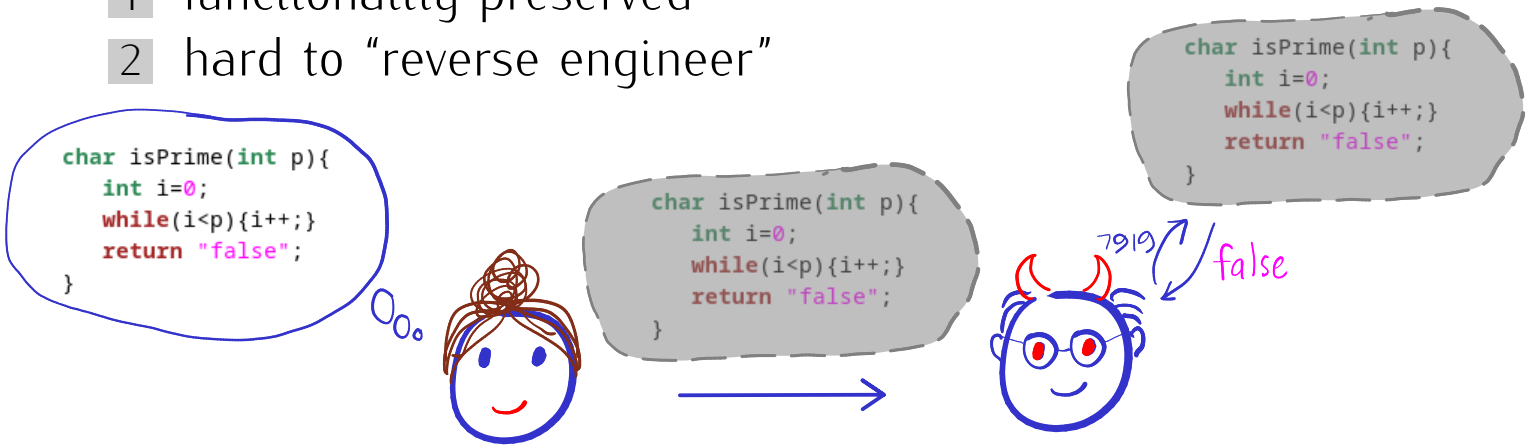

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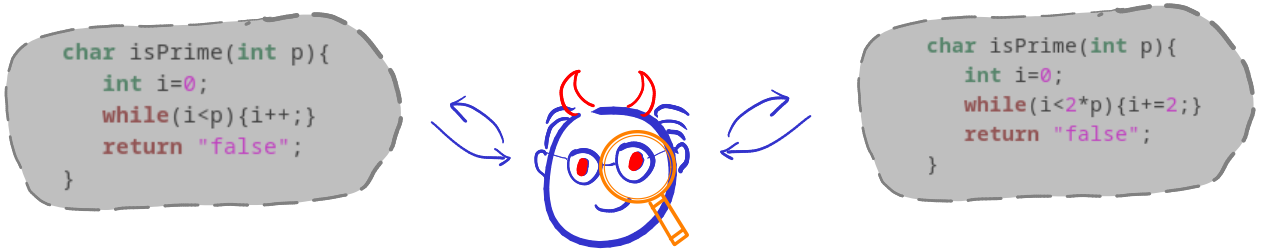
− Impossibility of VBBO for general programs

- Key idea: programs that spit out secret when run "on itself"

Plan for Today's Lecture...

- What do we do in the face of this impossibility?
  - Relax requirement to *indistinguishability* obfuscation (IO)



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- How to use IO: PRG $\xrightarrow{\text{IO}}$ PKE

- How to use IO: *punctured* programming
  - Puncturable PRF (PPRF)
  - PPRF $\xrightarrow{\text{IO}}$ PKE
  - PPRF $\xrightarrow{\text{IO}}$ digital signature

Plan for Today's Lecture...

General *template*:

1. Identify the task — program obfuscation
2. Come up with precise threat model M (a.k.a security model)
   - Adversary/Attack: What are the adversary's capabilities?
   - Security Goal: What does it mean to be secure?
3. Construct a scheme Π
4. Formally prove that Π in secure in model M

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Next lecture

Plan for Today's Lecture...

1. Indistinguishability Obfuscation (IO)
2. How to Use IO: PRG $\xrightarrow{\text{IO}}$ PKE
3. How to Use IO: Punctured Programming

Plan for Today's Lecture...

1. Indistinguishability Obfuscation (IO)
2. How to Use IO: PRG $\xrightarrow{\text{IO}}$ PKE
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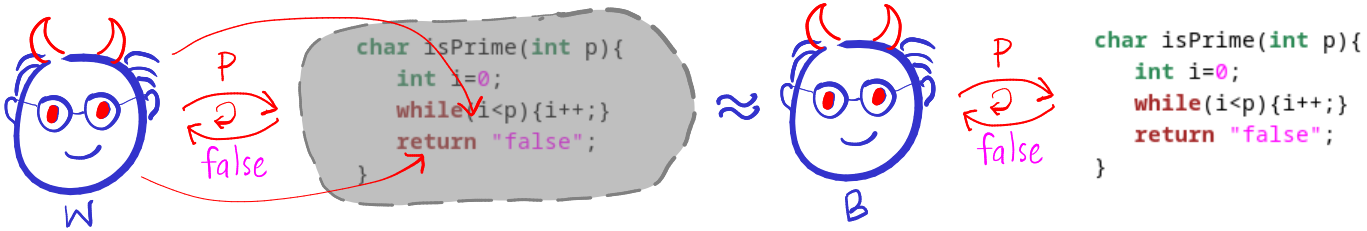
Recall VBBO

- Security via "simulation": anything learnable "white-box" given P is learnable "black-box" given only oracle access to P



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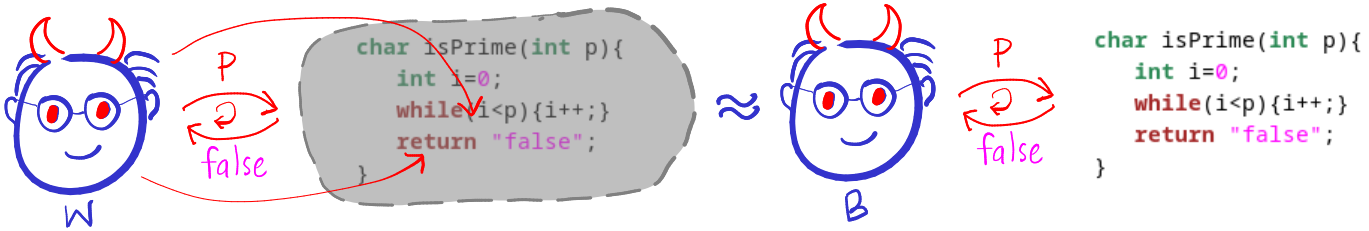

Defintion 1 (VBB obfuscator)

A PPT algorithm Obf *that takes as input any program* P *and a security parameter* n*, and outputs obfuscated program* $\tilde{\mathsf{P}}$ *such that:*

1. *Functionality preserved: for all inputs* x, $\tilde{\mathsf{P}}(x) = \mathsf{P}(x)$
2. *Small slowdown: run-time of* $\tilde{\mathsf{P}}$ *is poly. in* n *and run-time of* P

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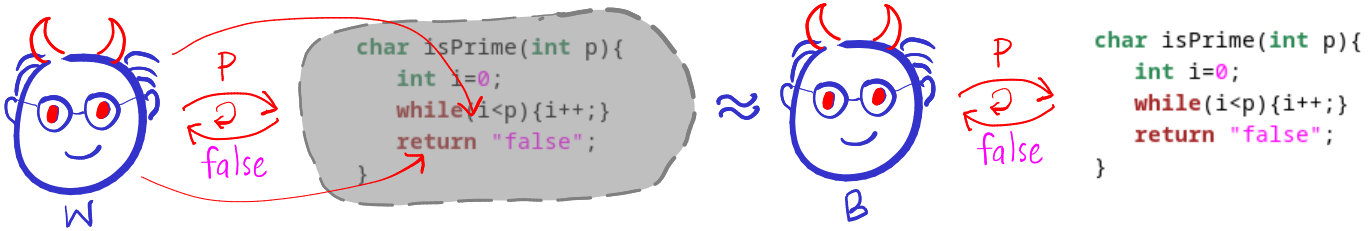

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3. *VBBO security: for every PPT* W*, there exists PPT* B *that can simulate* W*'s output on input* $\tilde{P}$ *using only oracle access to* P*. That is, the following is negligible:*

$$\left| \Pr[W(\tilde{P}) = 1] - \Pr[B^{P}(1^n, 1^{|P|}) = 1] \right|$$

$$\tilde{P} \leftarrow \mathrm{Obf}(1^n, P)$$

Let's Define Indistinguishability Obfuscation (IO)...

How to define IO?

Let's Define Indistinguishability Obfuscation (IO)...

$\forall x : P_1(x) = P_2(x)$

How to define IO? Obfuscations of two *functionally-equivalent*, *same-sized* programs are computationally indistinguishable

runtime

```
char isPrime(int p){
   int i=0;
   while(i<p){i++;}
   return "false";
}
```

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char isPrime(int p){
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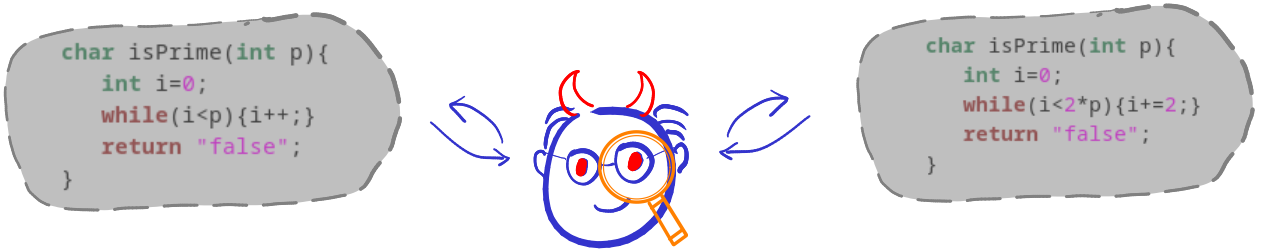

Defintion 2 (Indistinguishability obfuscator (IO))

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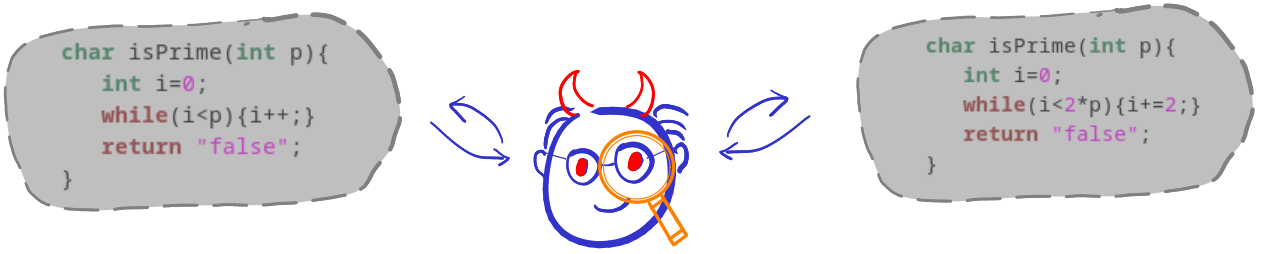

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$\tilde{P}_1 \leftarrow Obf(1^n, P_1)$ $\quad$ $\tilde{P}_2 \leftarrow Obf(1^n, P_2)$

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Assuming $\mathsf{P} = \mathsf{NP}$, can you construct an IO?

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1. *Show that VBBO $\rightarrow$ IO*
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Exercise 2 (IO is the "best-possible obfuscation")

If VBBO is possible for a program class $\mathcal{C}$, then an IO Obf for $\mathcal{C}$ is also a VBBO

Plan for Today's Lecture

1. Indistinguishability Obfuscation (IO)
2. How to Use IO: PRG $\xrightarrow{\text{IO}}$ PKE
3. How to Use IO: Punctured Programming

How to Use IO: PRG $\mathsf{G} \xrightarrow{\text{IO}}$ PKE...

How to construct PKE using PRG $\mathsf{G} : \{0,1\}^n \to \{0,1\}^{2n}$?

How to Use IO: PRG $\mathsf{G} \xrightarrow{\text{IO}}$ PKE...

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How to Use IO: PRG G $\xrightarrow{\text{IO}}$ PKE...

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Exercise 3

Prove that Construction 1 is secure if Obf *is VBBO*

How to Use IO: PRG $\mathsf{G} \xrightarrow{\text{IO}}$ PKE...

Theorem 1

If Obf *is an IO and* G *is a PRG then* Π *is a PKE*

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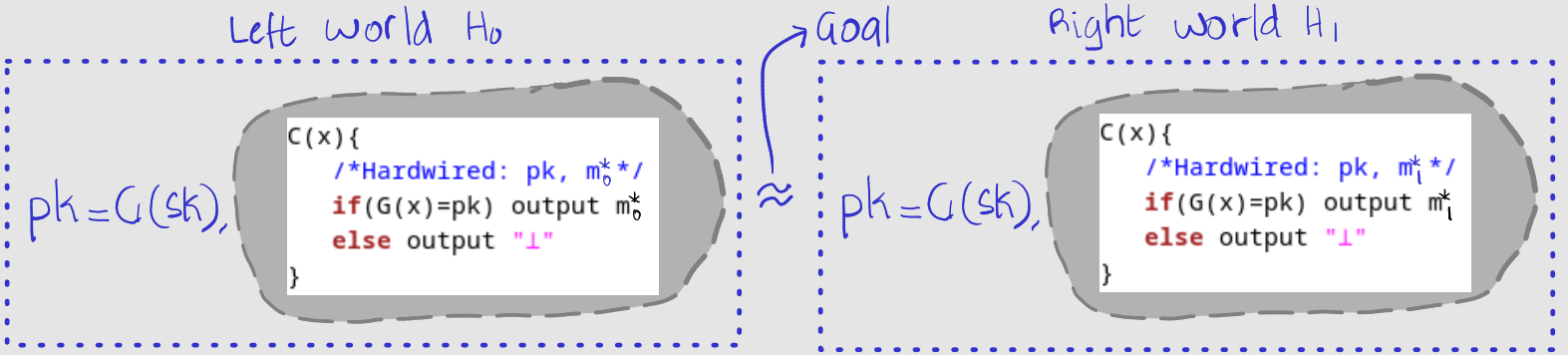

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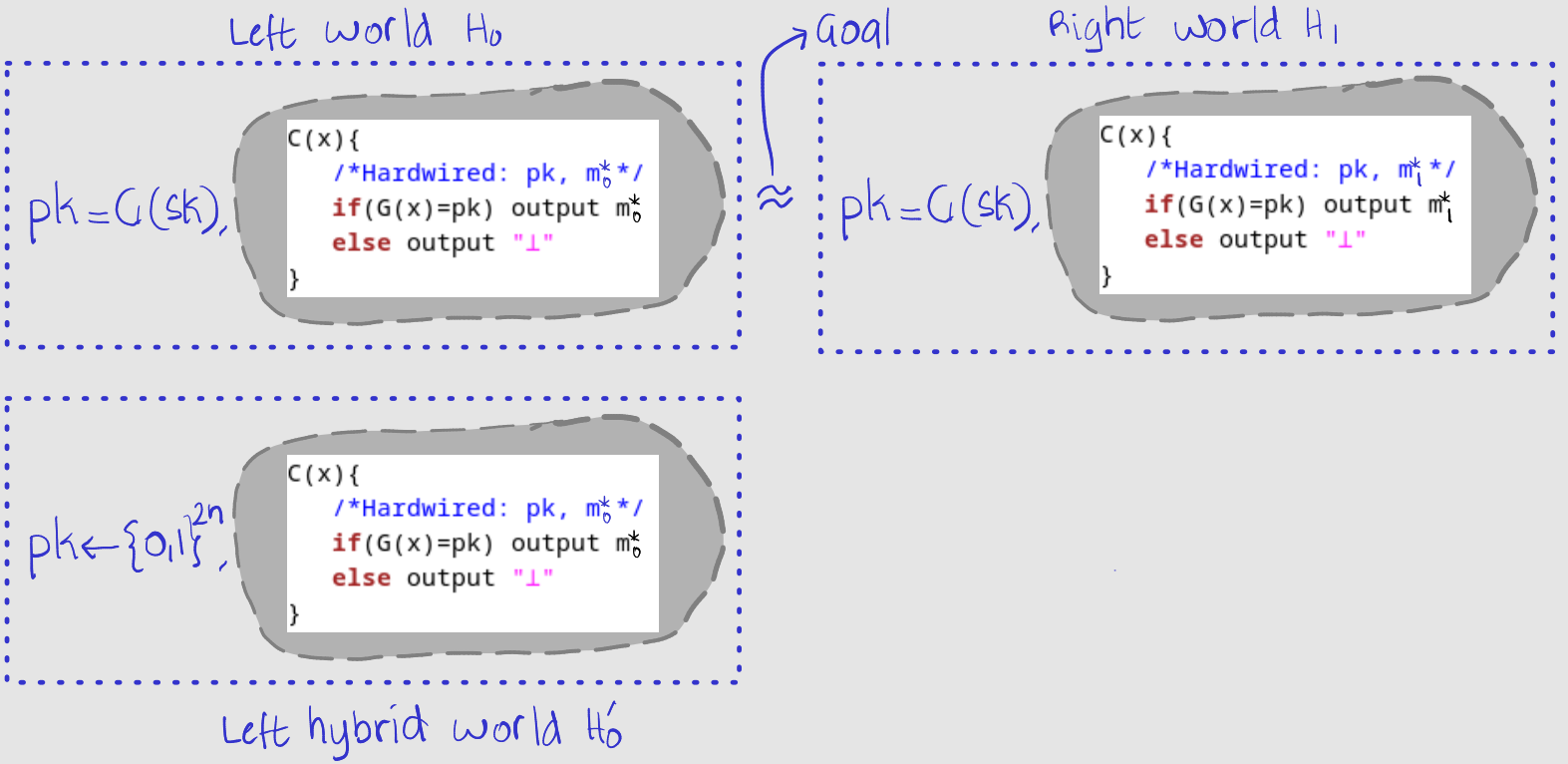

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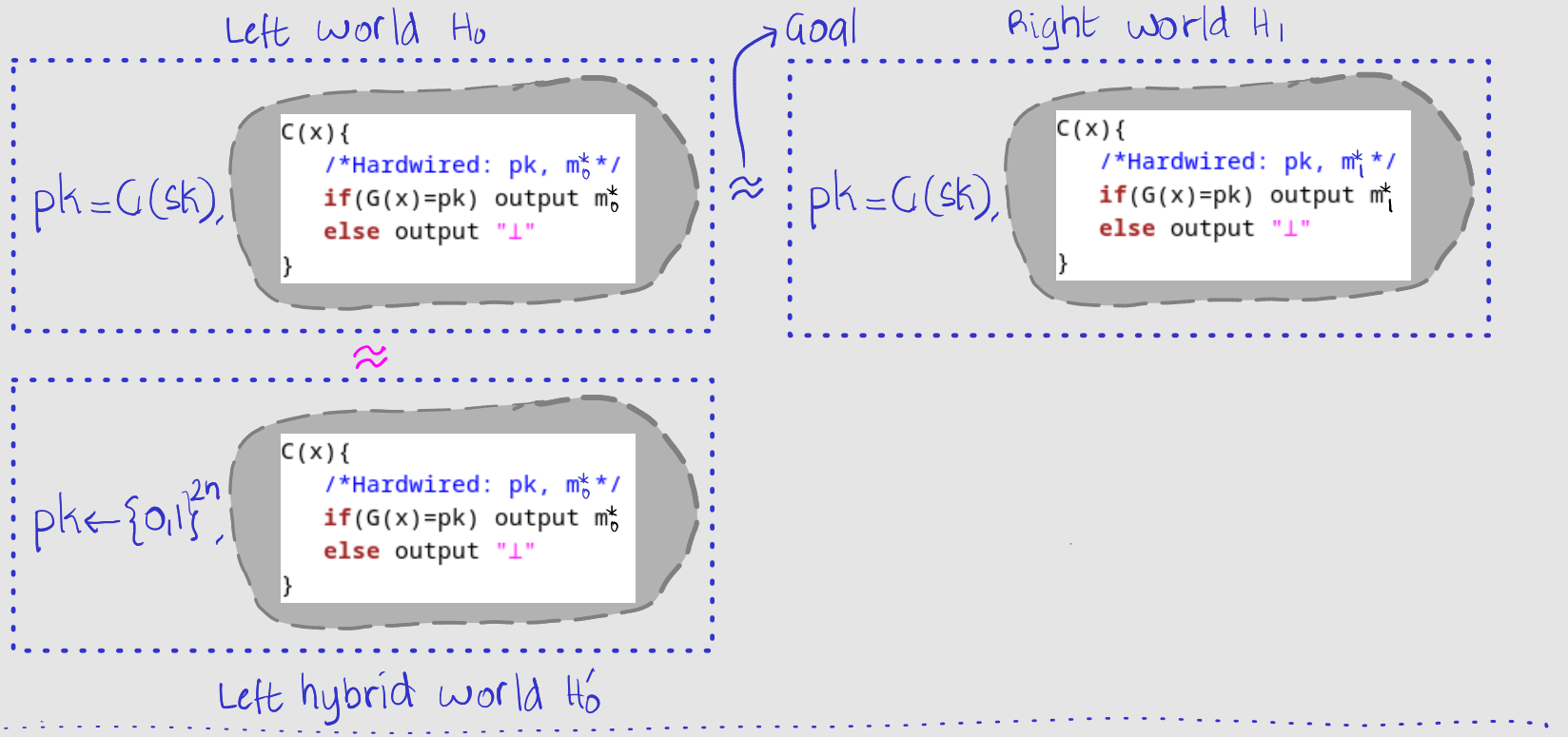

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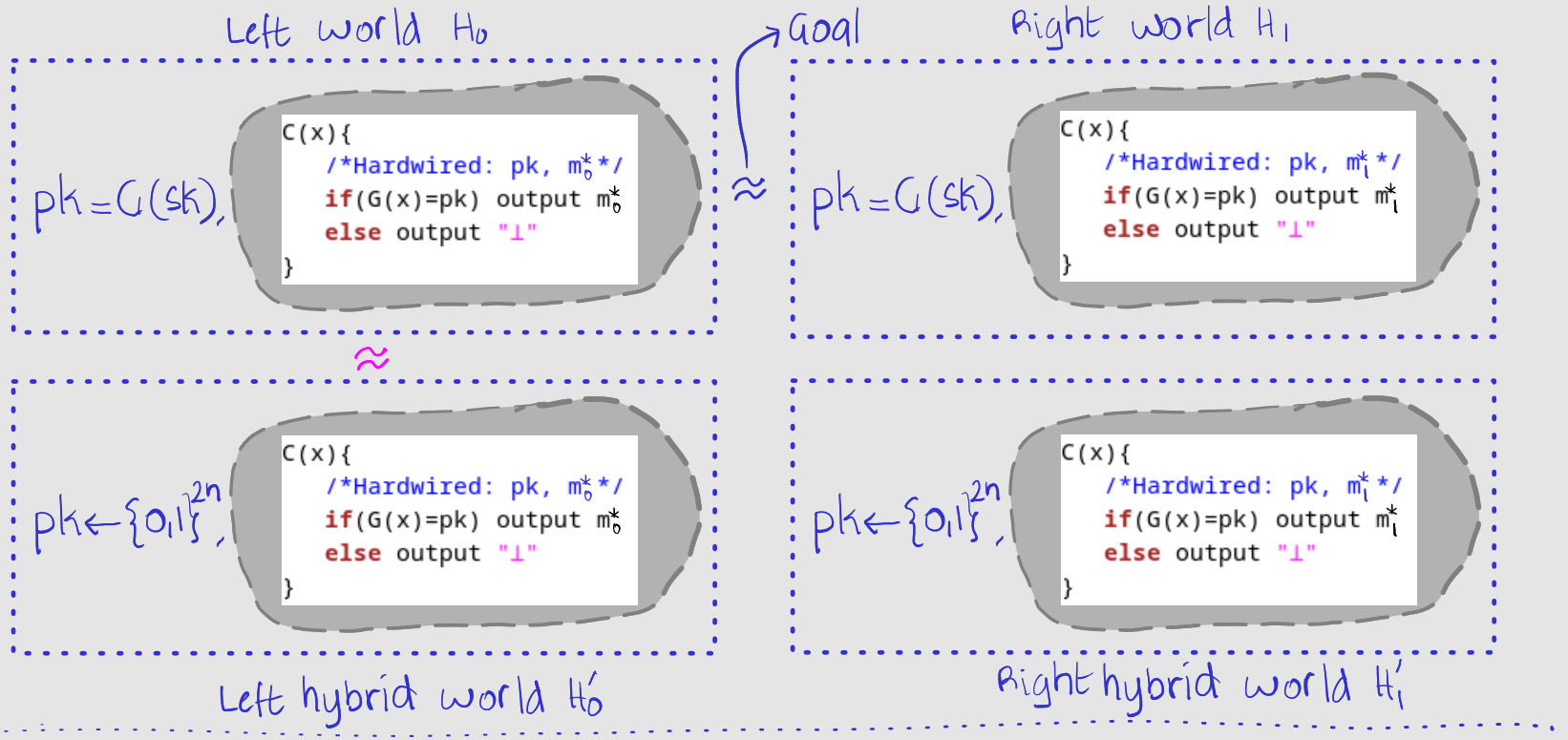

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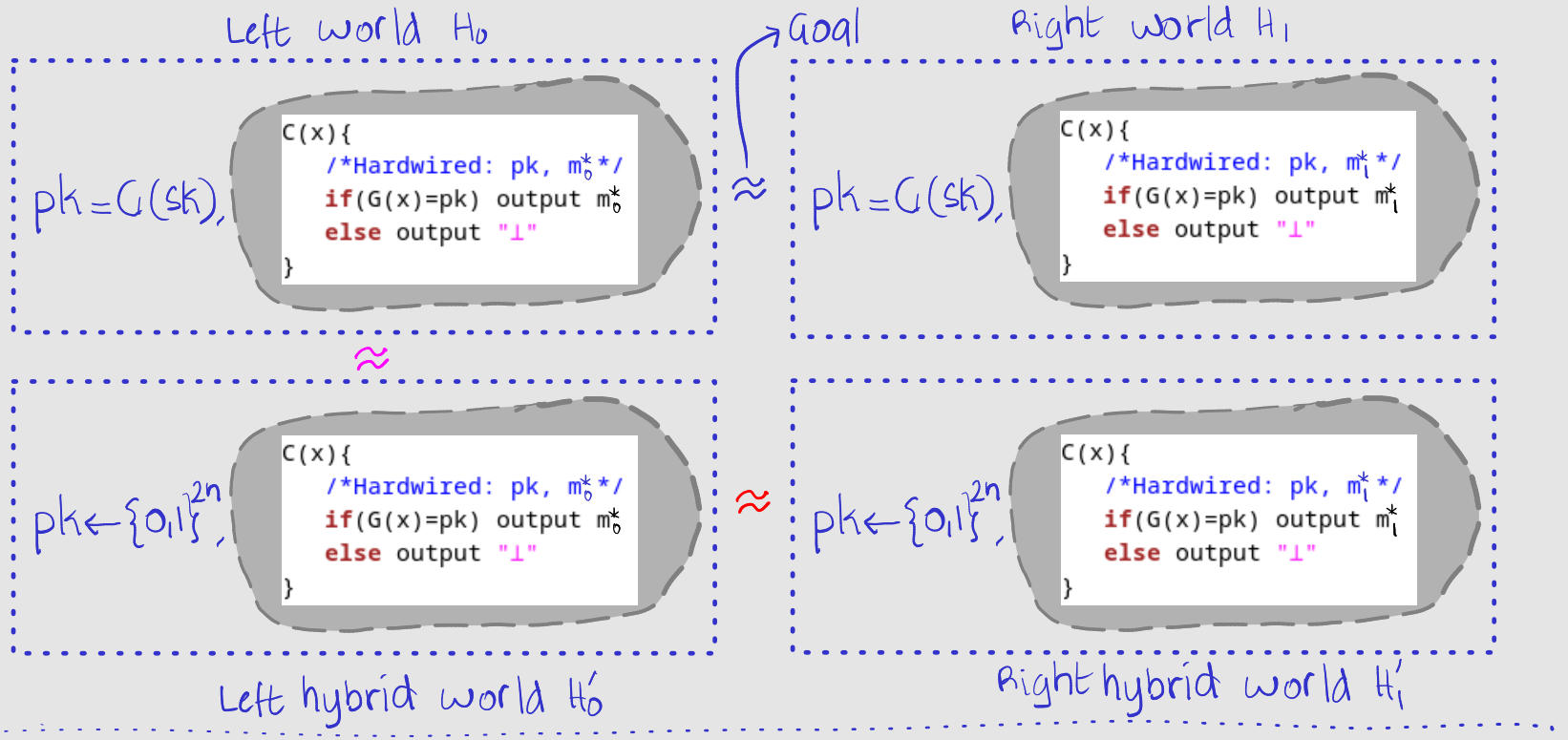

$\approx$ Claim 1 : $H_0 \approx H_0'$ by pseudorandomness of G

$\approx$ Claim 2 : When $pk \notin \text{Image}(G)$, $H_0' \approx H_1'$ by IO security.
↳ w/ prob. $1 - 1/2^n$

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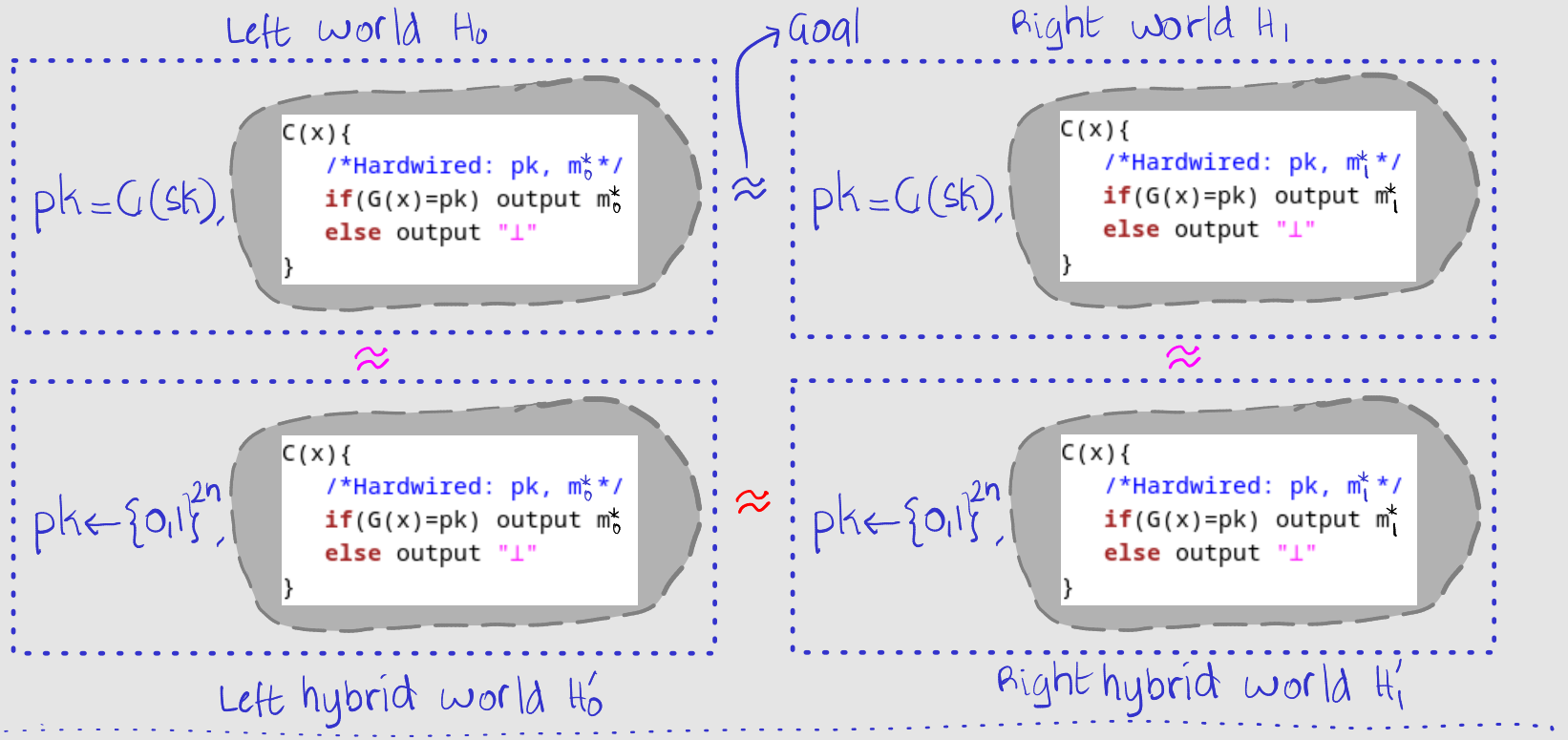

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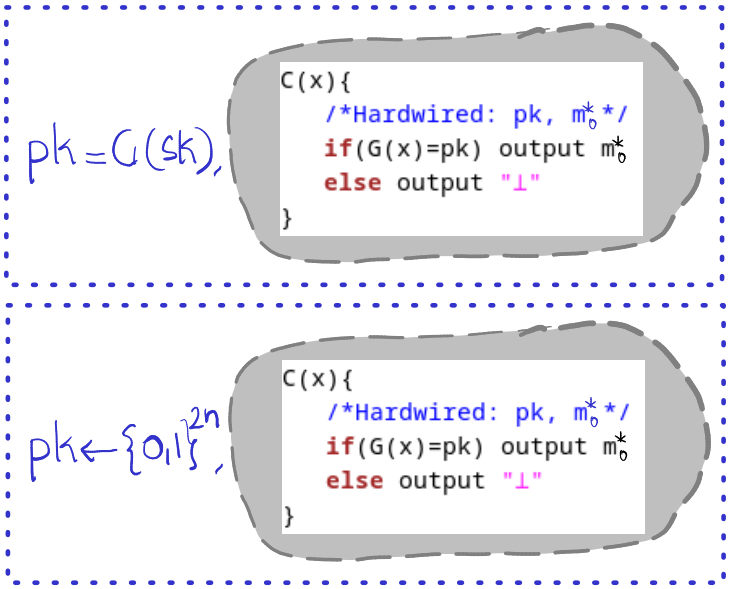

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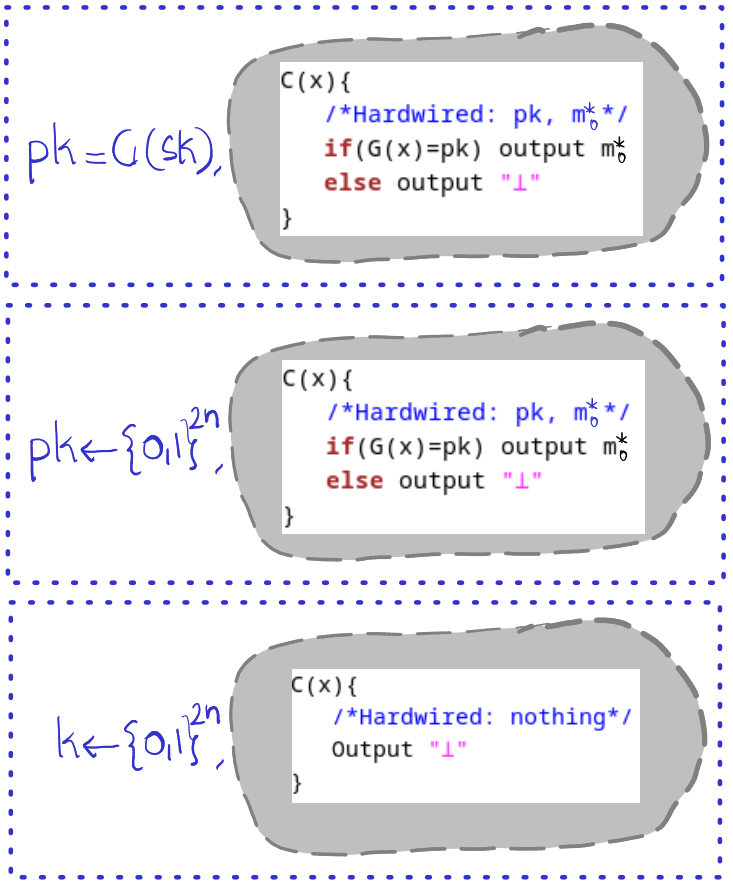
How to Use IO: PRG G $\xrightarrow{\text{IO}}$ PKE...



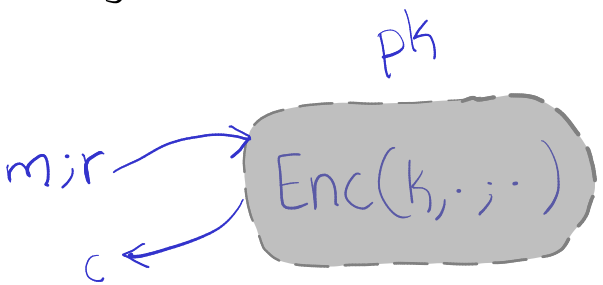
- What is going on?
  - IO *can* be used to hide secrets (in this case, messages)
  - Hiding exploits pseudorandomness of PRG:
    - When y is outside the image of G, message is never used
    - Switch from $pk := G(x)$ to $pk := y$ is indistinguishable

Plan for this Session

1. Indistinguishability Obfuscation (IO)
2. How to Use IO: PRG $\xrightarrow{\text{IO}}$ PKE
3. How to Use IO: Punctured Programming

Recall Our Attempt at SKE $\xrightarrow{\text{VBBO}}$ PKE...

- Public key is an obfuscation of SKE's encrypt algorithm **Enc** with secret key ***k*** *hardcoded*



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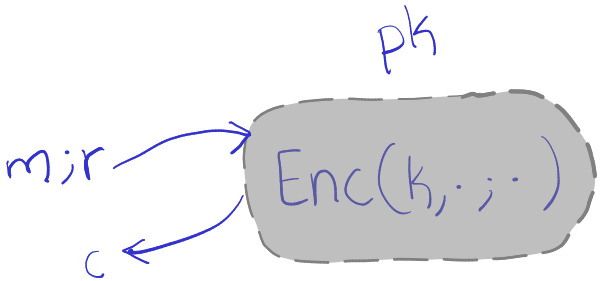

Construction 2 ($\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec}) \rightarrow \Pi' = (\mathsf{Gen}', \mathsf{Enc}', \mathsf{Dec}')$)

- $\mathsf{Gen}'(1^n)$:
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  - *Output* $\mathsf{Enc}(k, \cdot; \cdot)$ *as public key* $\mathbf{pk}$ *and* k *as secret key*
- $\mathsf{Enc}'(pk, m; r)$: *output* $c := pk(m; r)$
- $\mathsf{Dec}'(k, c)$: *output* $m := \mathsf{Dec}(k, c)$

Recall Our Attempt at SKE $\xrightarrow{\text{VBBO}}$ PKE...

- Using SKE based on PRF $\mathsf{F} : \{0,1\}^n \times \{0,1\}^{2n} \to \{0,1\}^{2n}$:

Construction 3 (PRF $\mathsf{F} \to \Pi' = (\mathsf{Gen}', \mathsf{Enc}', \mathsf{Dec}')$)

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$$\mathsf{Enc}(k, m; r) := (\mathsf{F}(k, r) \oplus m; r)$$

- $\mathsf{Enc}'(pk, m; r)$: *output* $(c, r) := pk(m; r)$
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Recall Our Attempt at SKE $\xrightarrow{\text{VBBO}}$ PKE...

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Why is Construction 3 insecure?

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  1. Run $pk(0^{2n}; r^*)$ to obtain $\mathsf{F}(k, r^*) \oplus 0^{2n} = \mathsf{F}(k, r^*)$
  2. Recover $m^* := \mathsf{F}(k, r^*) \oplus c^*$

Issue: adversary can control random coins used to encrypt

Let's Fix Construction 2

- How to "hide" random coins used to encrypt

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Why does the attack not work now? Need to invert PRG

- To formally prove IND-CPA security, we need additional properties from PRF F...

PRF F Needs to be "Puncturable"

Definition 2 (Puncturable PRF (PPRF))

A PRF $\mathsf{F} : \{0,1\}^n \times \{0,1\}^{2n} \to \{0,1\}^{2n}$ that additionally supports

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- PPRF can be obtained by modifying tree-based PRF from Lecture 5
  - PRG→PPRF

Construction 4 is IND-CPA PKE

Theorem 3

If F *is a puncturable PRF and* Obf *is an IO then* Π' *is a PKE*

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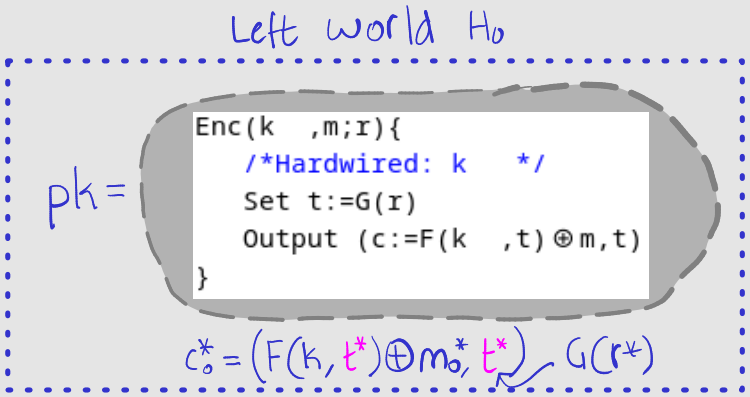
Proof Sketch (Hybrid argument).



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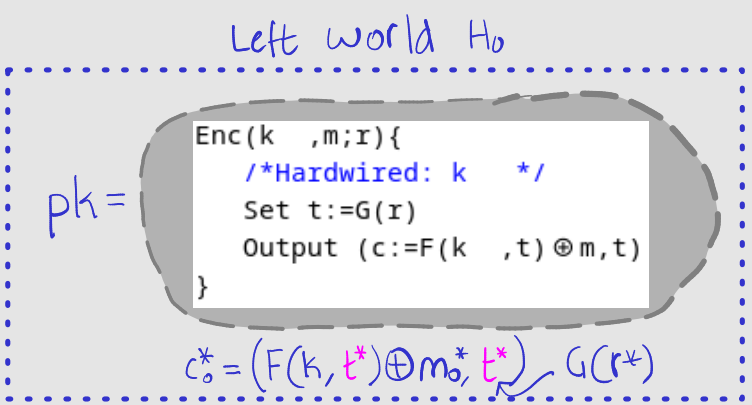

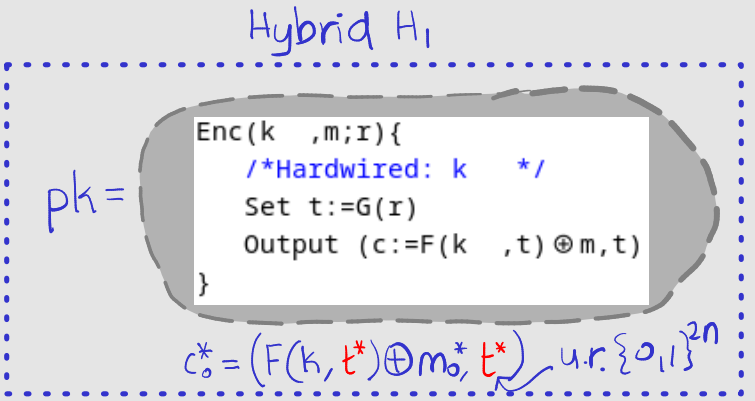

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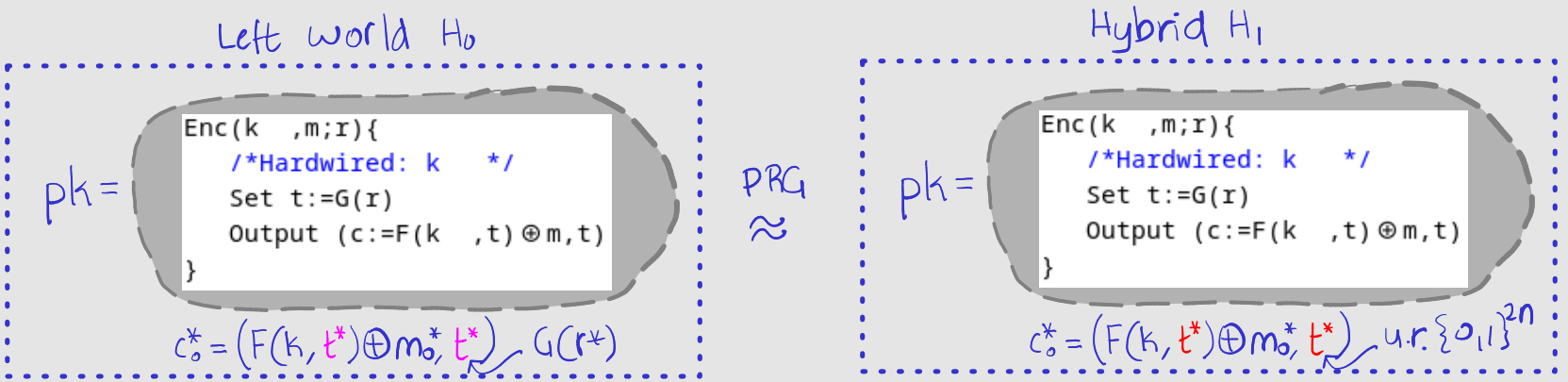

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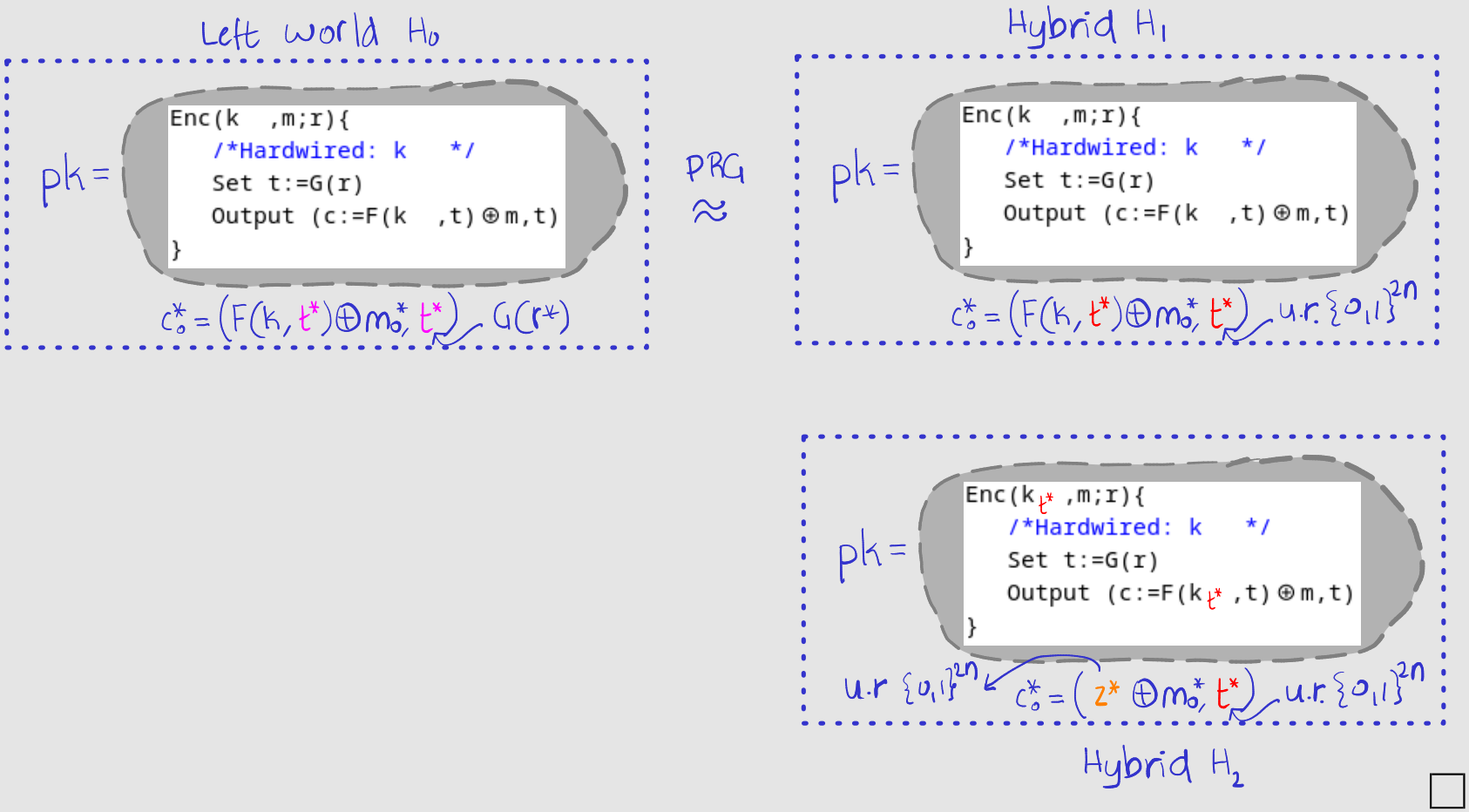

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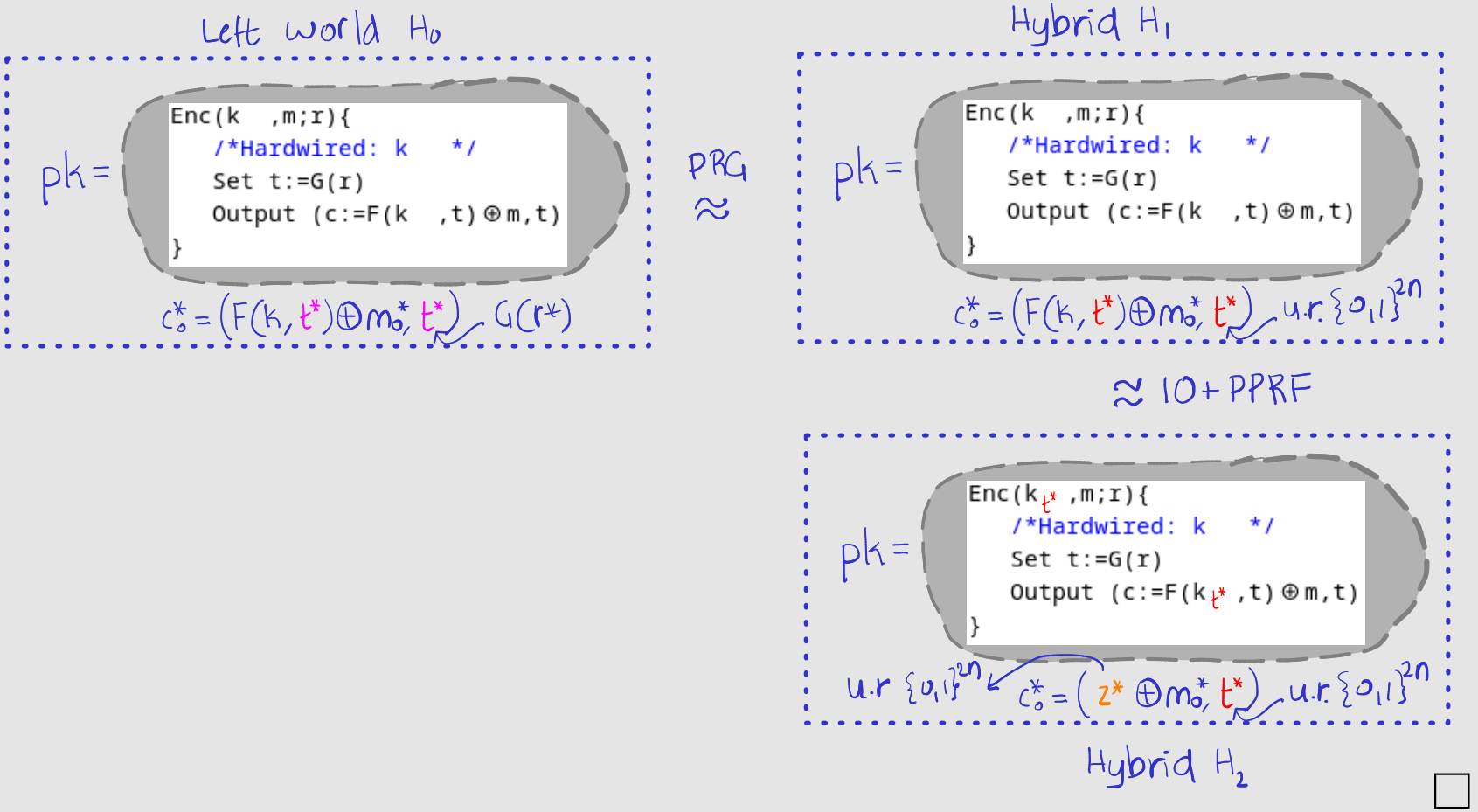

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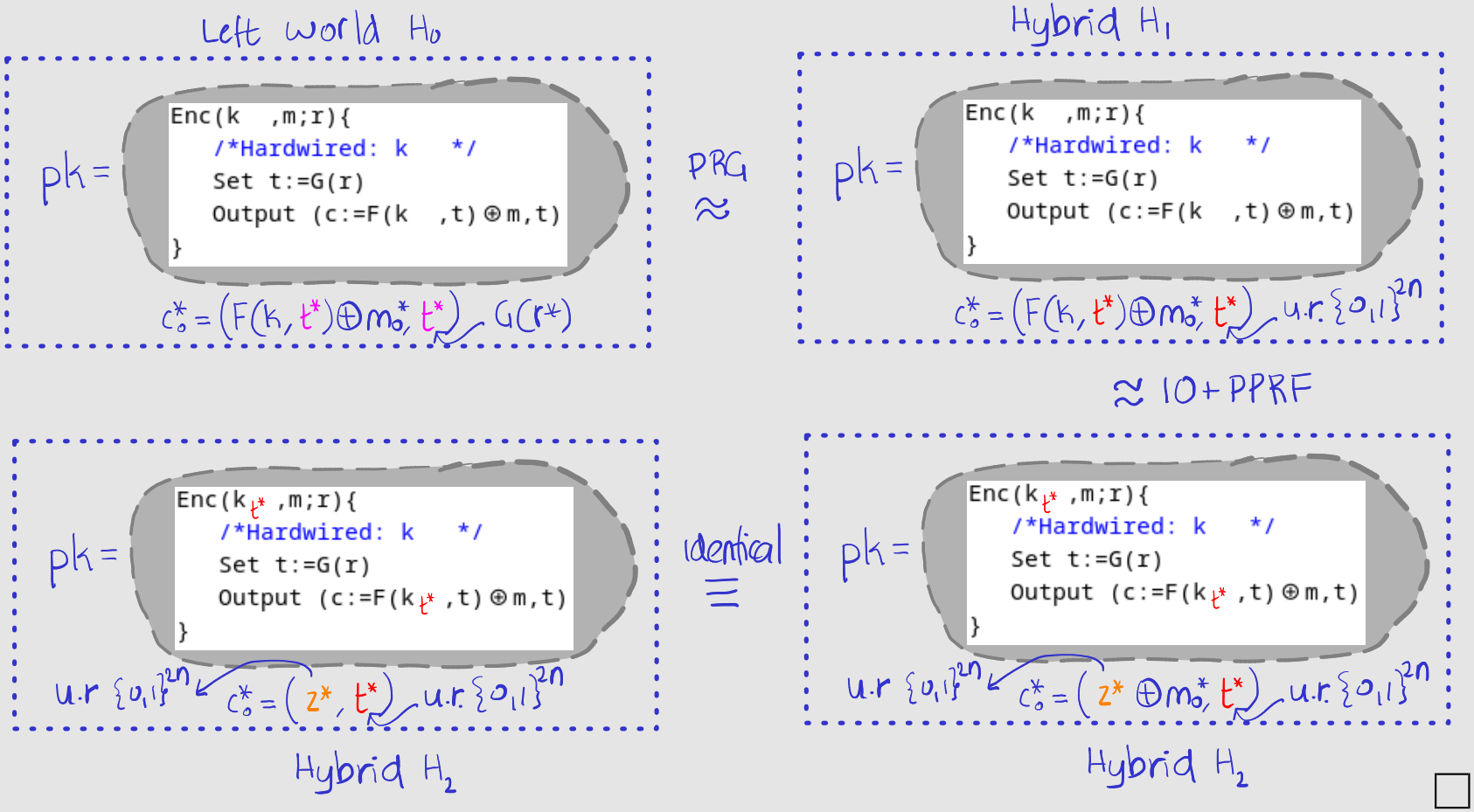

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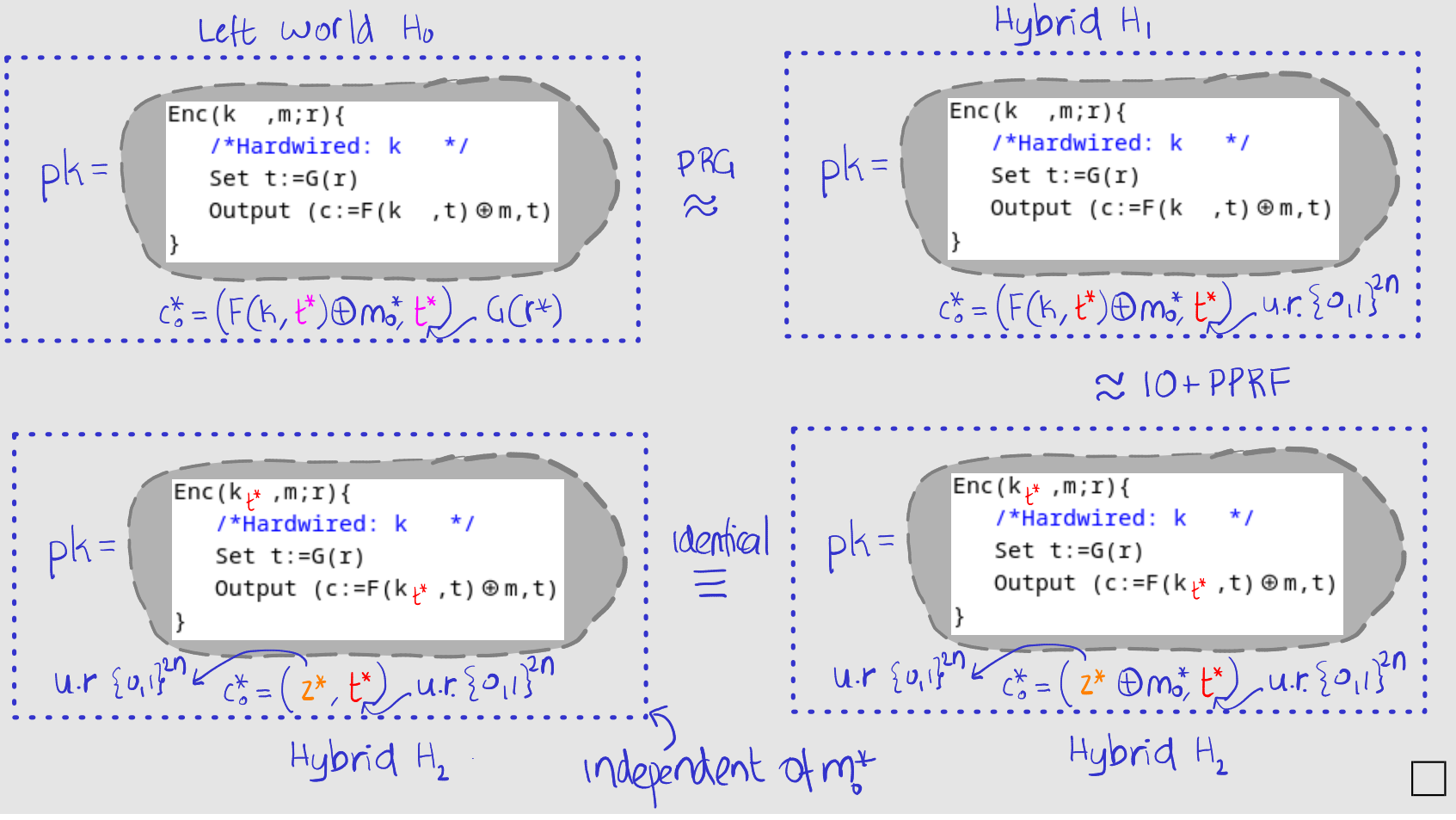

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(Short) Digital Signatures via Punctured Programming

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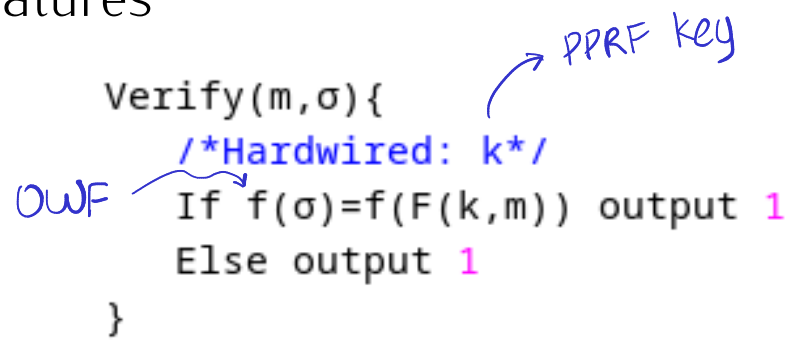
- Signature on m is evaluation of PPRF F on m
- Public key consists of obfuscation of the following program that verifies signatures



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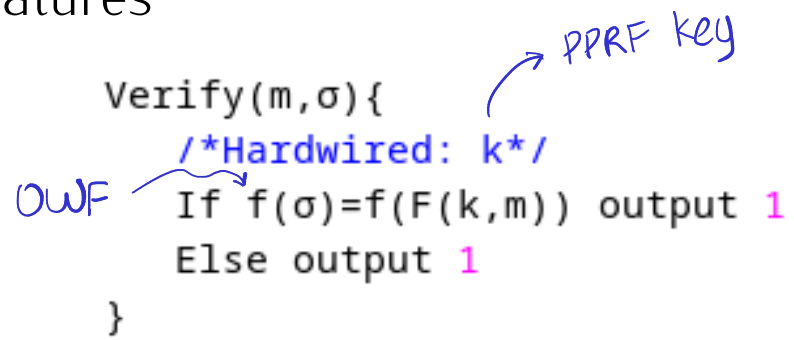

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Exercise 4

Prove Theorem 4

To Recap Today's Lecture

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  Key idea: how to use IO to hide secrets

- New tool: *punctured* programming
  - Puncturable PRF (PPRF)
  - PPRF $\xrightarrow{\text{IO}}$ PKE
  - PPRF $\xrightarrow{\text{IO}}$ digital signature

Next Lecture

- How to construct indistinguishability obfuscator (IO)
  - Bootstrapping theorem for IO
  - State of affairs for IO for NC^1

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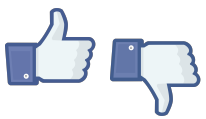
- Course feedback 

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1. The problem of constructing cryptographic primitives using IO was studied in [SW14]. That paper also introduces the "punctured programming" approach, and uses it to construct PKE, signatures, NIZK and several other primitives from IO.
2. Construction 1 is taken from Lecture 25 of Vinod Vaikuntanathan's MIT6875.
3. Mark Zhandry's COS 597C course (Autumn 2016) is an excellent source to learn further about program obfuscation.
4. Puncturable PRF was introduced in [BW13, BGI14, KPTZ13]. A formal definition can be found in [SW14].


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