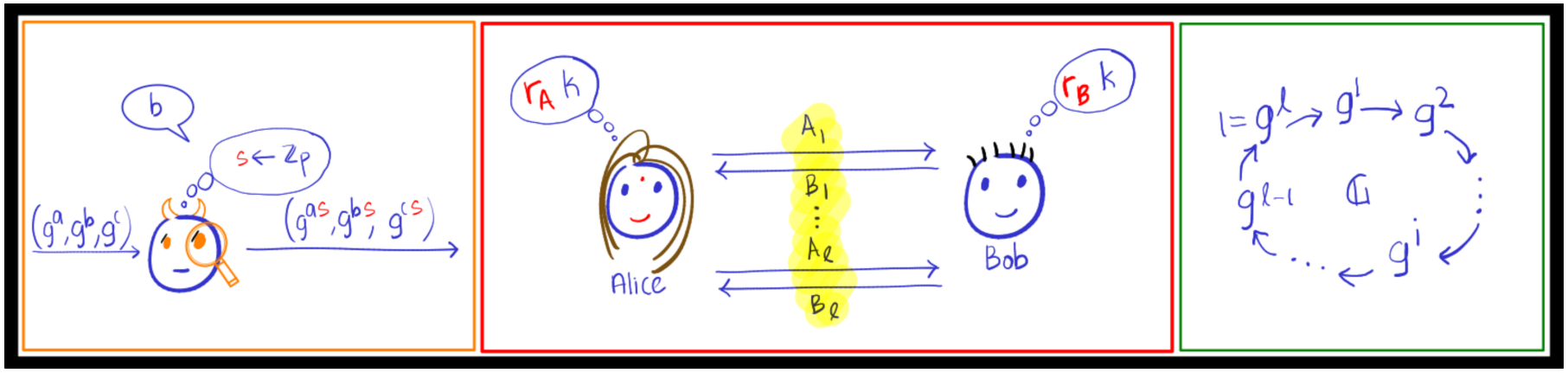
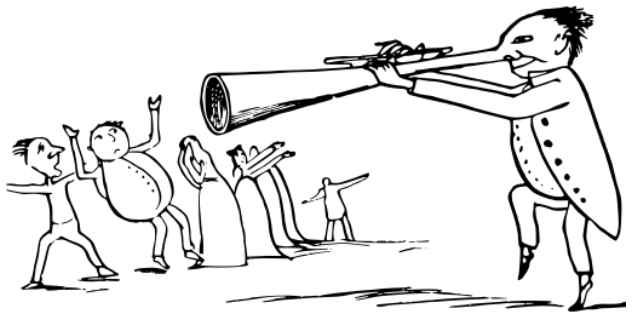


CS409m: Introduction to Cryptography

Lecture 12 (24/Sep/25)

Instructor: Chethan Kamath

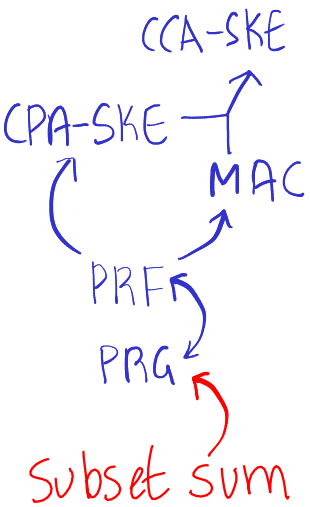
Announcements



- Mid-sem cribs session 12:30-14:30 Monday (29/Sep)
  - View your answer sheet ~~14:00-16:00~~ on ~~Friday~~ (~~26~~/Sep) in CC305
  - Submit cribs online by ~~Monday (29/Sep~~, 23:59) Wednesday 01/Oct
- Assignment 4 (ungraded) will be released on Friday (26/Sep)

Recall from Last Module

- We learnt: secure communication in the shared-key setting
- Primitives encountered: PRG, PRF, MAC
- Computational hardness assumptions: subset-sum problem



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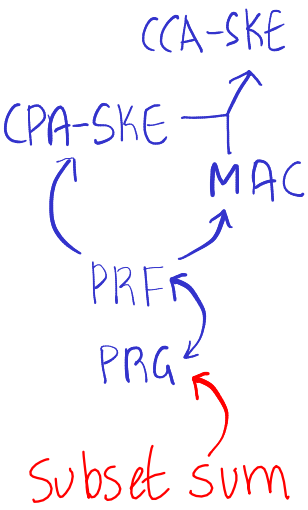
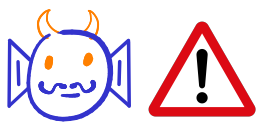


- Key conceptual takeaways:
  - Threat modelling
  - Computational security
  - What design choices lead to vulnerability?



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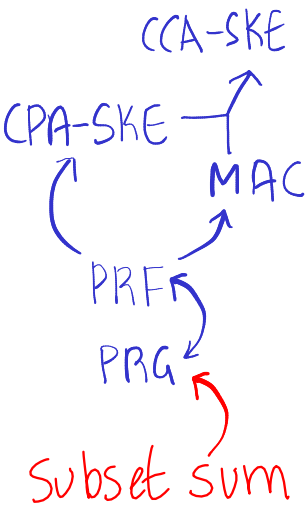

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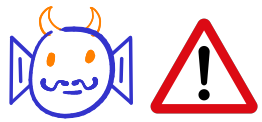
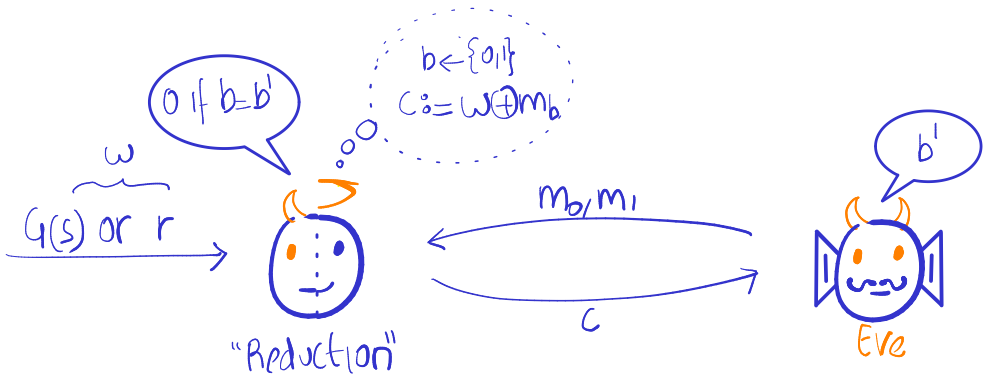
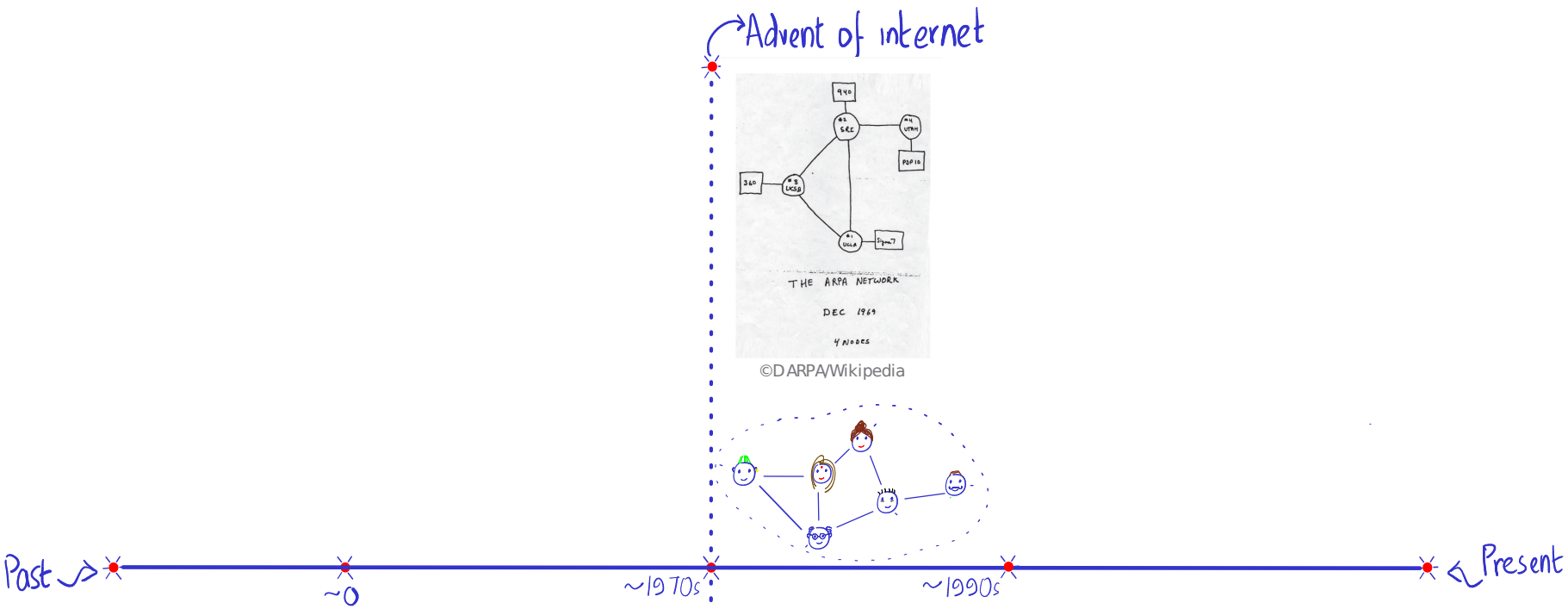


- Key tools: security reduction, hybrid argument



This Module...

MODULE 2
(Public keys)



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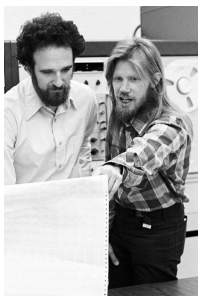

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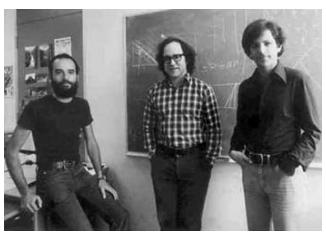

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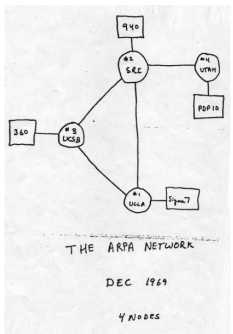
Advent of internet



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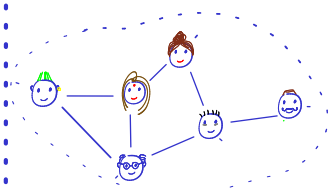

Past → ~0 ~1970s ~1990s ← Present

This Module...

MODULE 2
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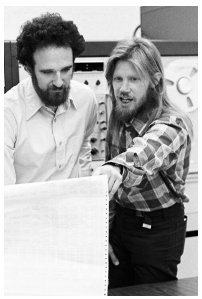
Advent of internet



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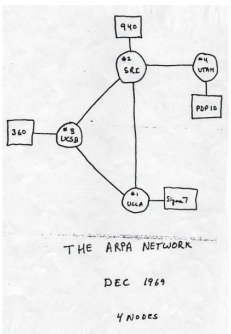

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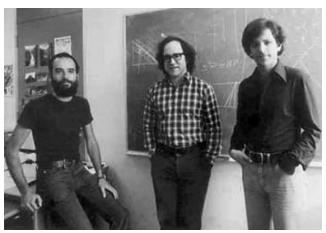

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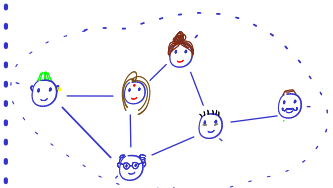


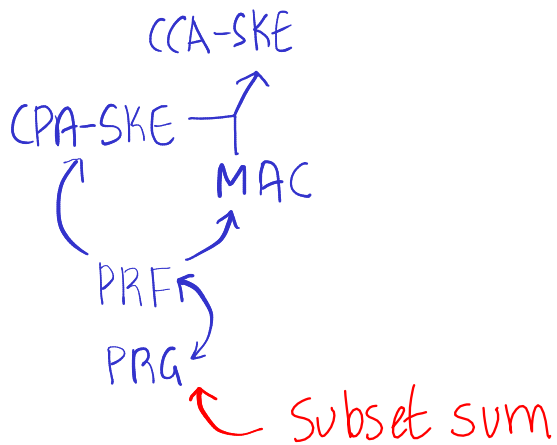
Past → ~0 ~1970s ~1980s ~1990s ← Present

Birth of "provable security"

©Oded Goldreich

This Module...

- Minicrypt to Cryptomania



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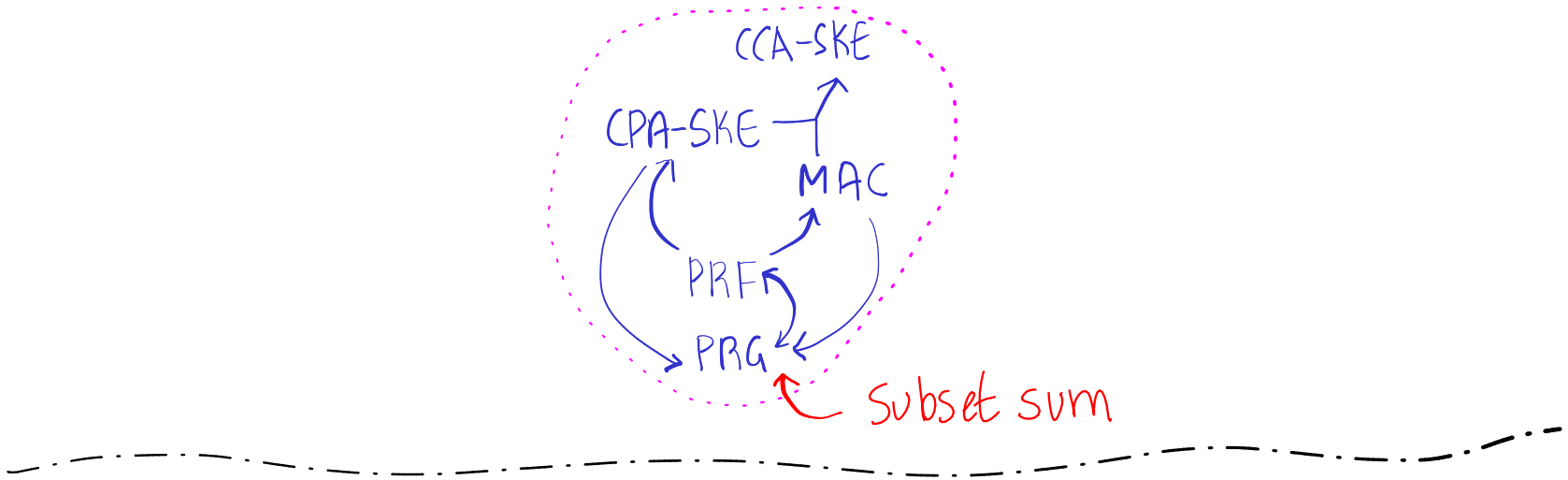

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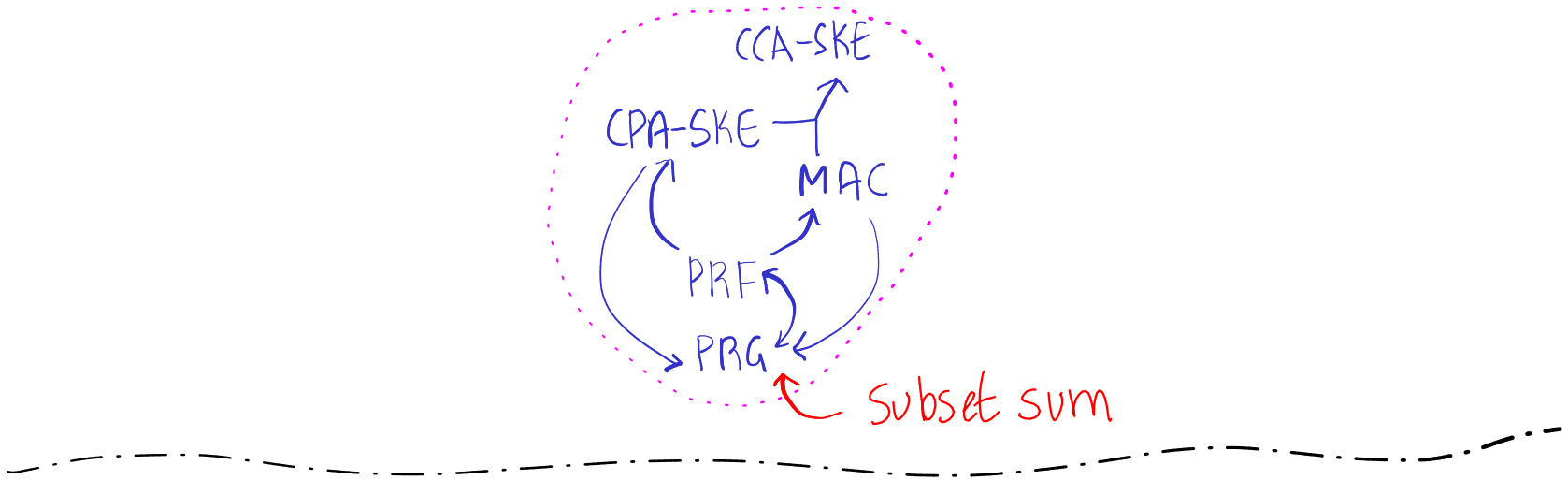

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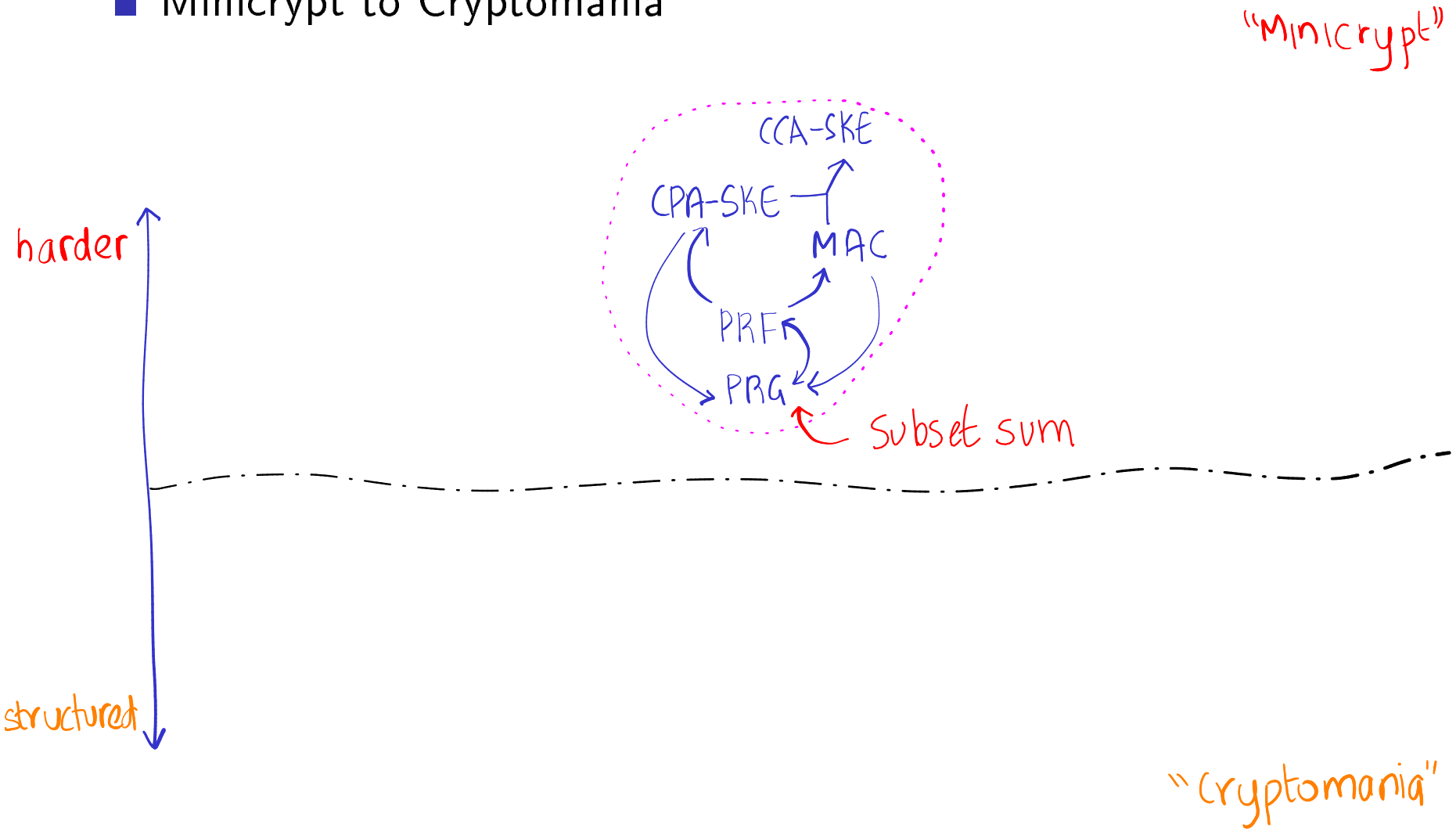

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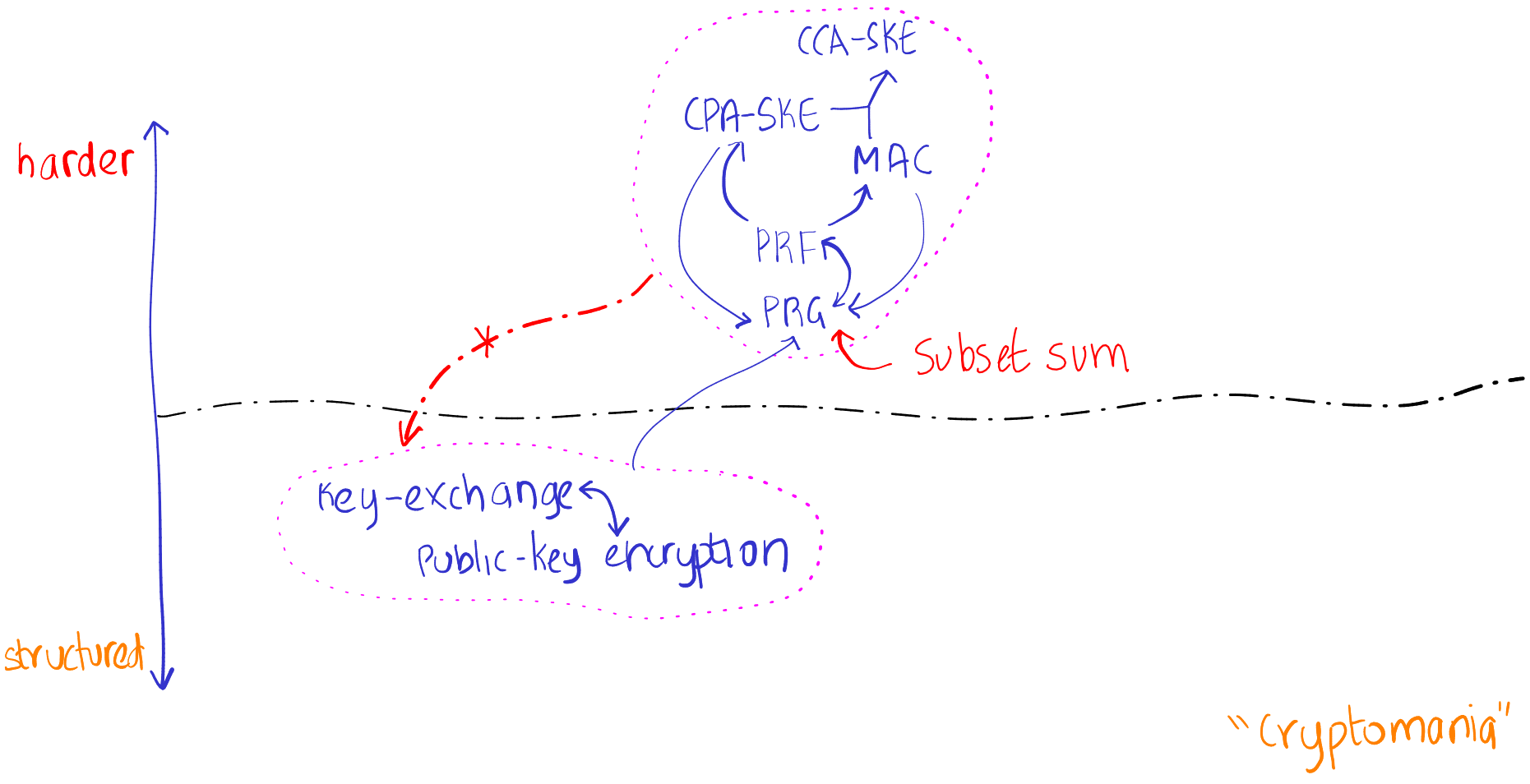

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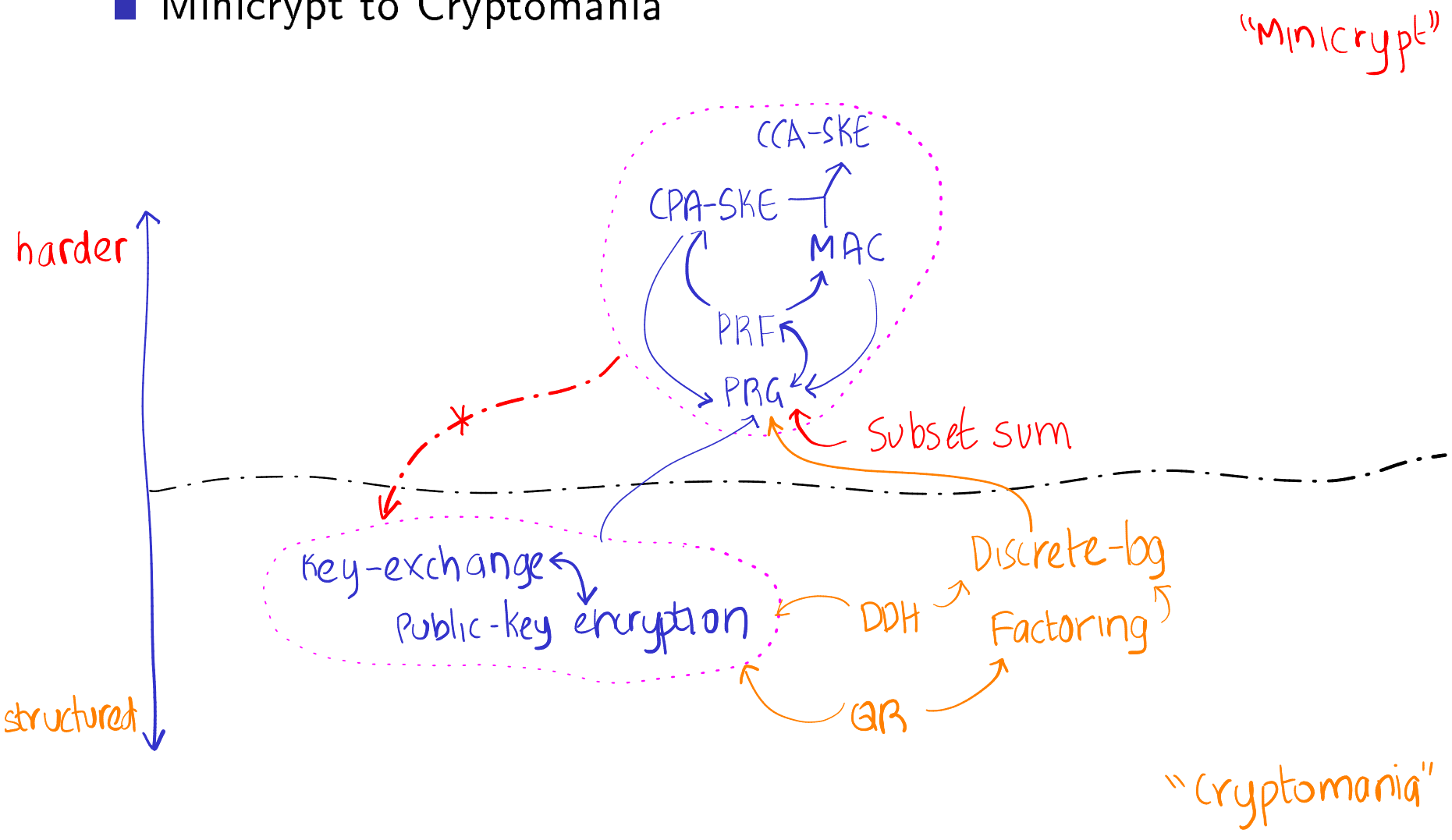

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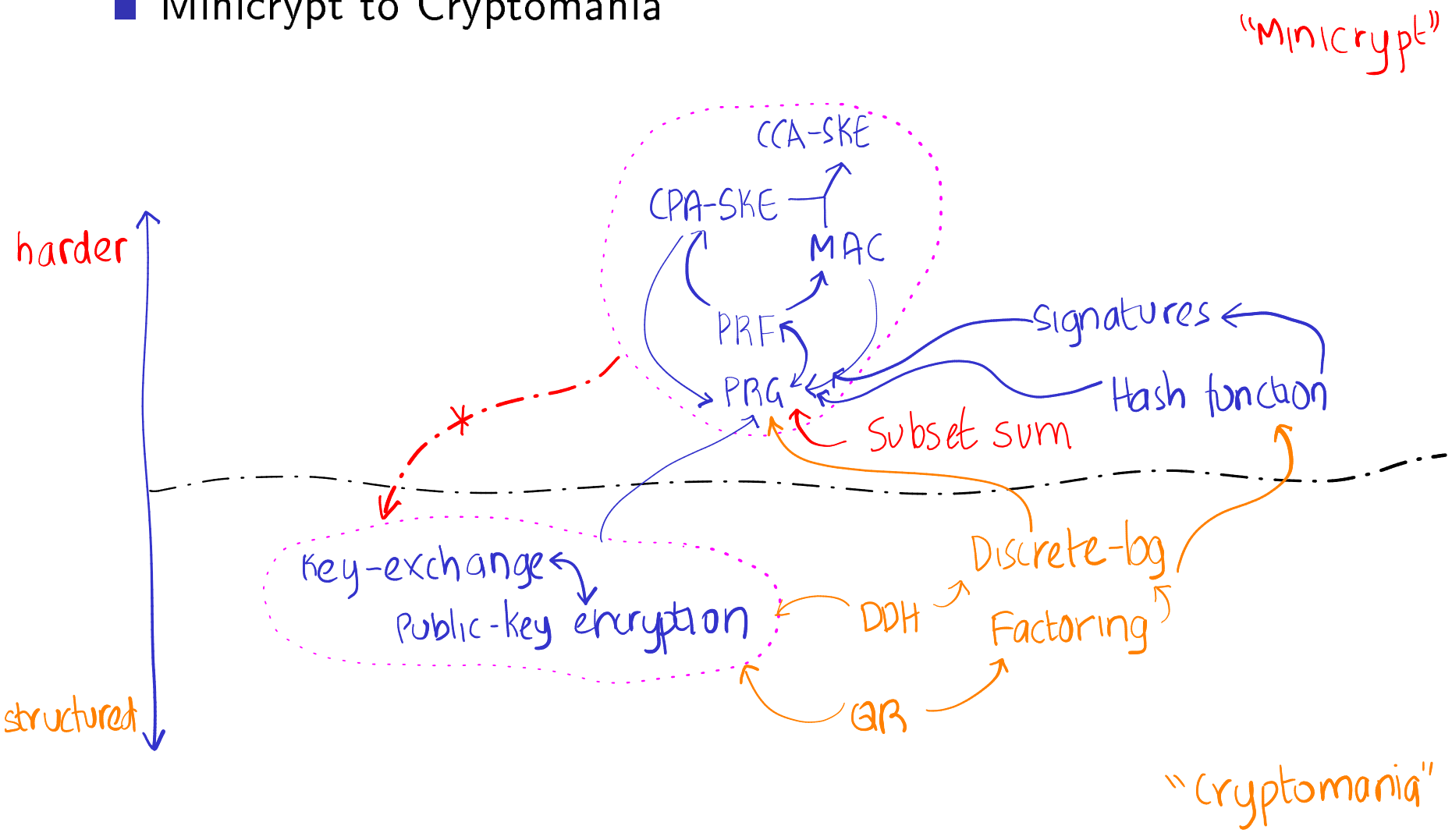

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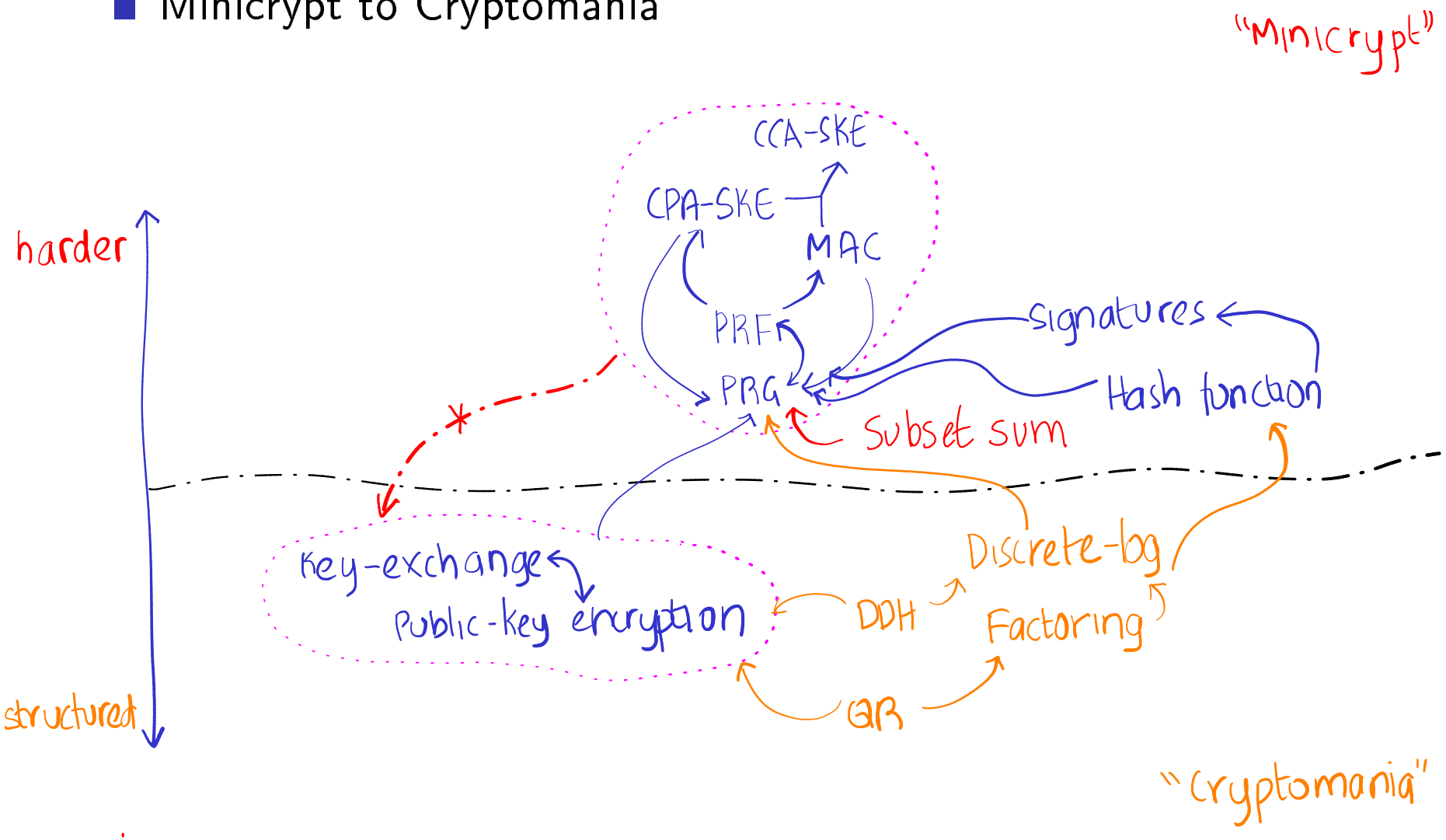

Today: how does one establish a shared key in the first place?

Plan for Today's Lecture

- Task: key exchange
- Threat model: computational secrecy against eavesdroppers

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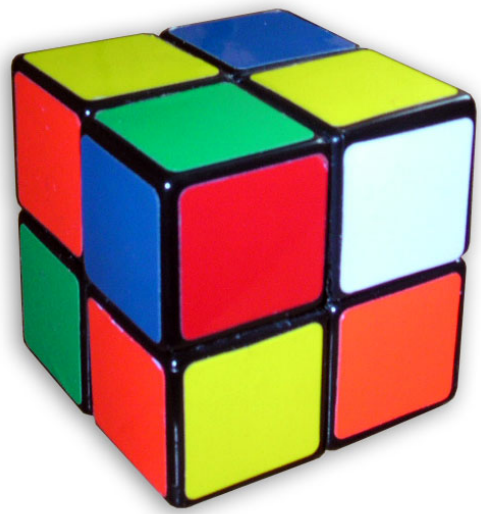
Key Exchange

©Wikipedia

Basic Group Theory



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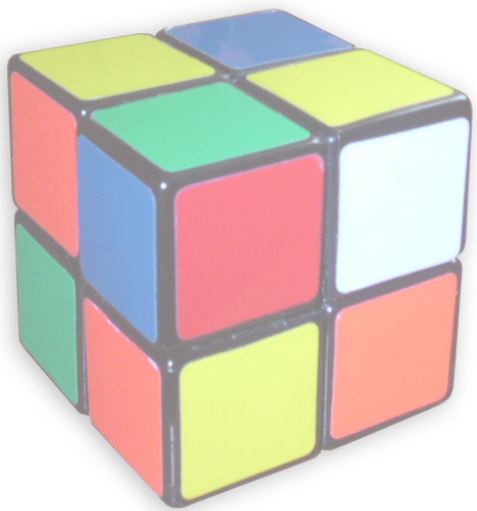

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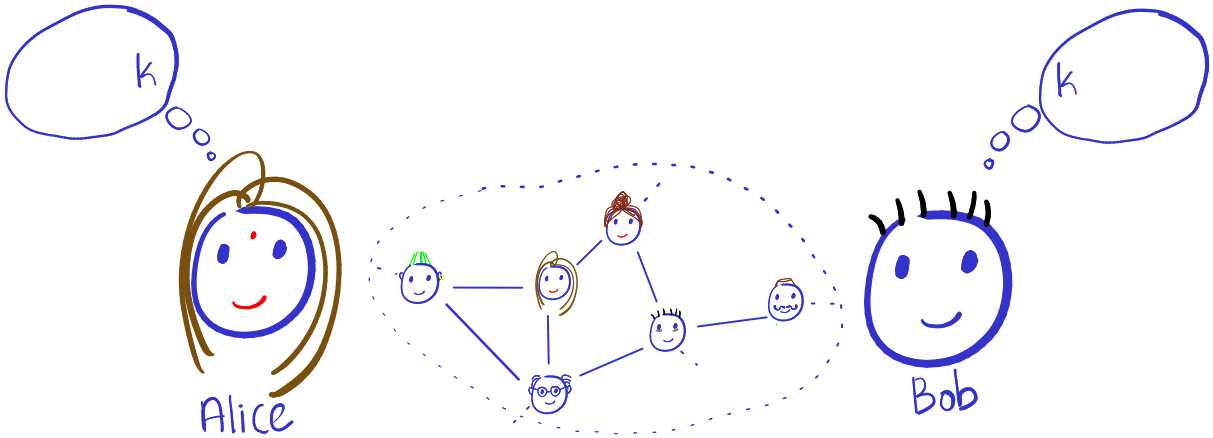
How To Establish a Shared Key in the First Place?



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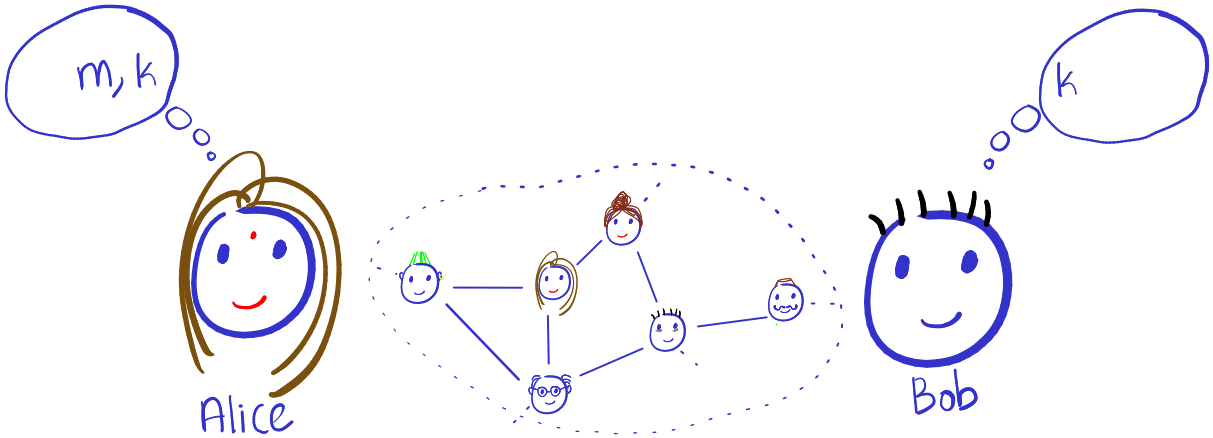

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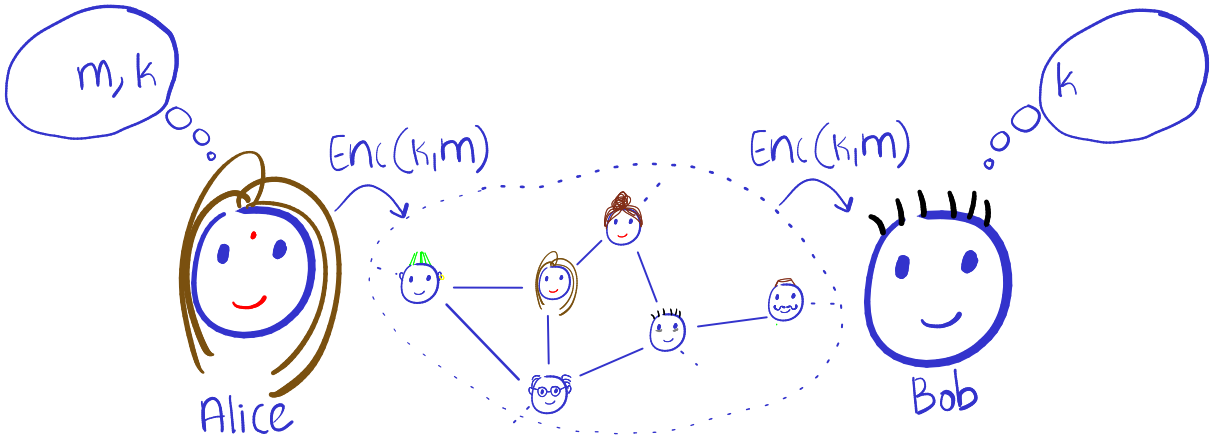

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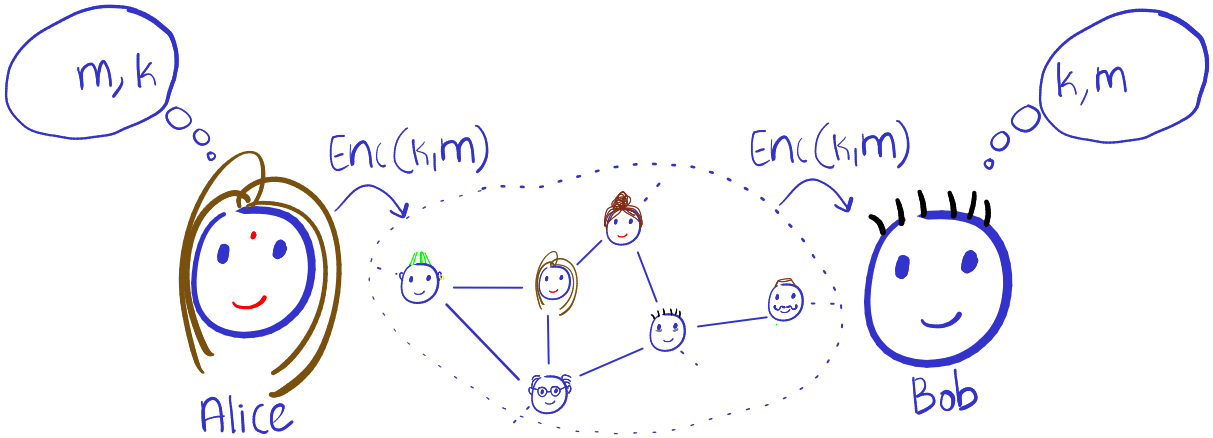

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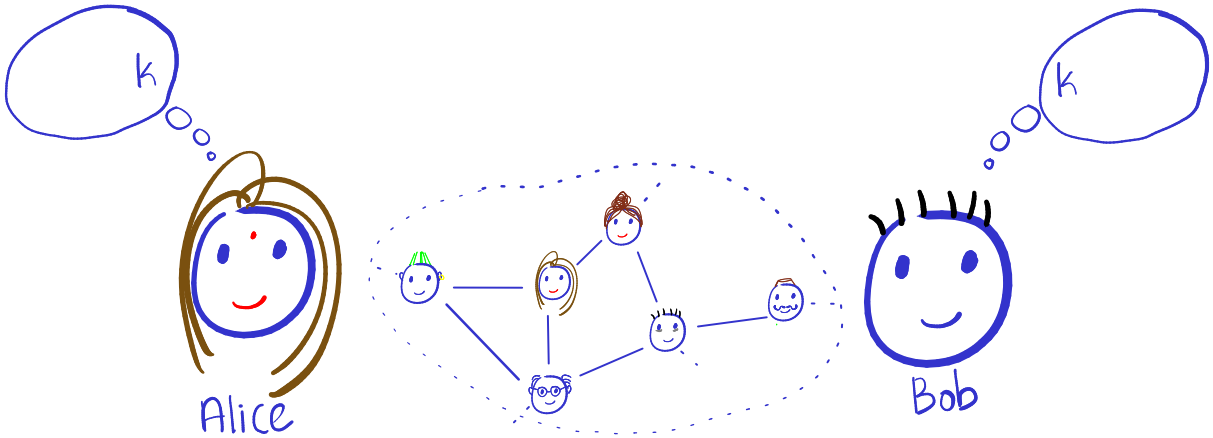

- The setting: Alice and Bob want to establish a shared key $k \in \{0,1\}^n$

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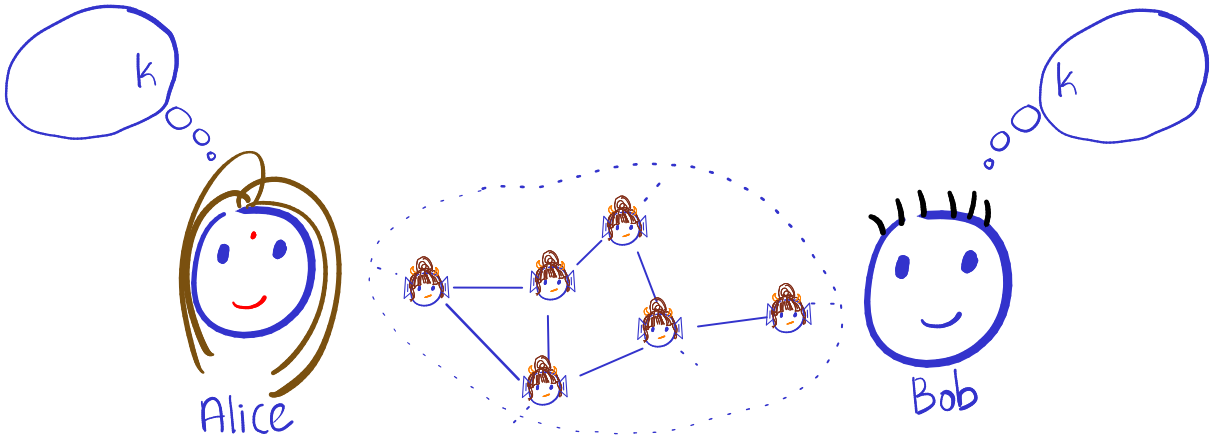

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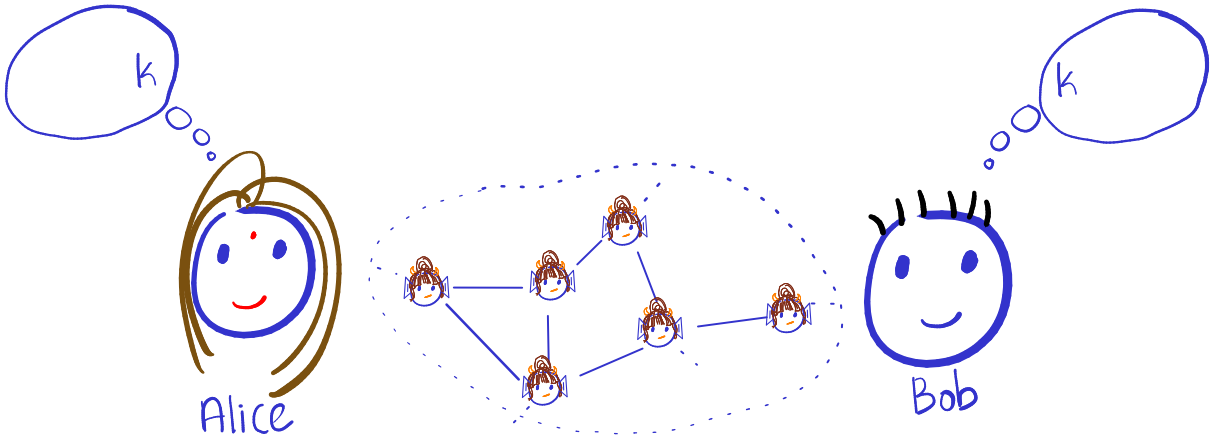

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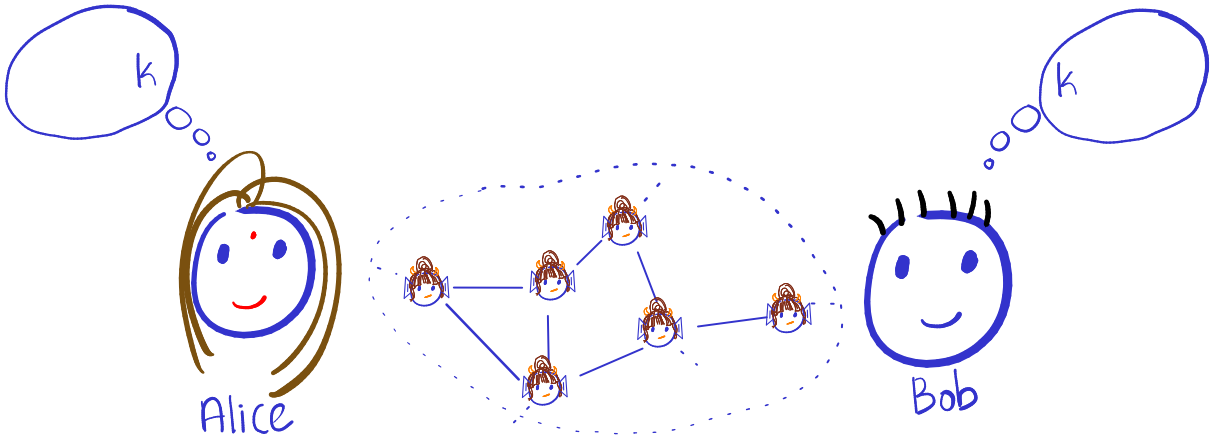

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- Alice and Bob execute a *protocol*, at the end of which they will have established a key
- Key Exchange IRL: HTTPs, TLS, SSH

Syntax of Key Exchange Protocol

Definition 1 (Key Exchange Protocol)

A (two-party) key-exchange protocol Π is a probabilistic protocol between two parties A and B at the end of which party A locally outputs $k_A \in \{0,1\}^n$ and party B locally outputs $k_B \in \{0,1\}^n$.

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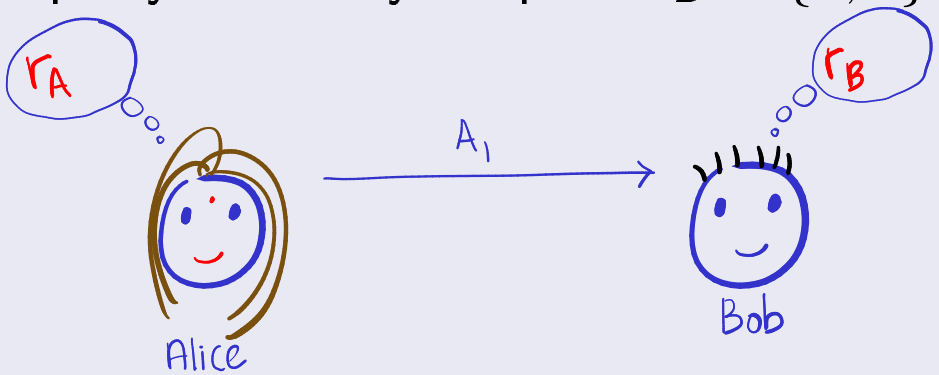

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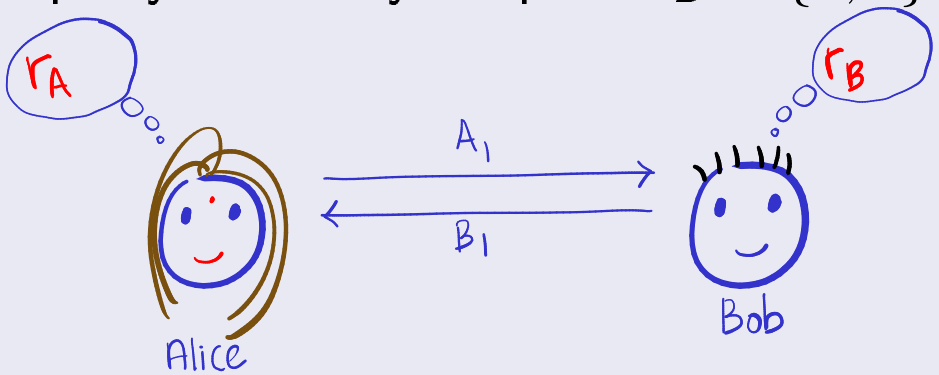

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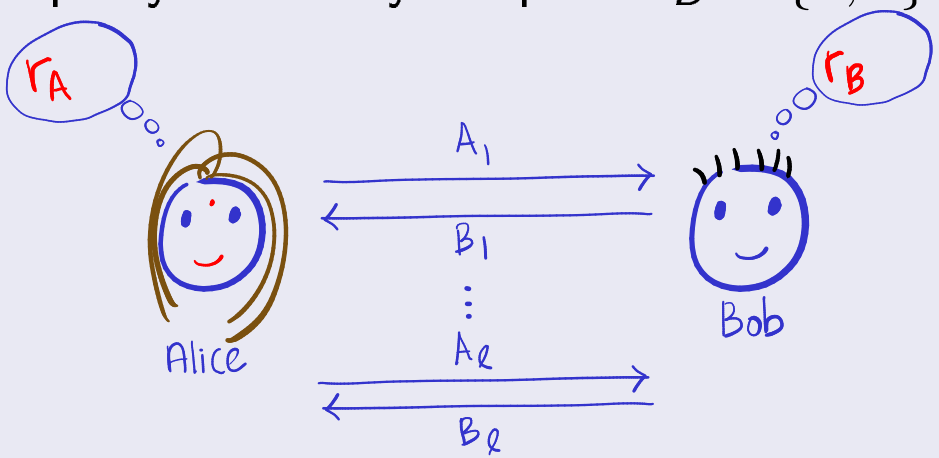

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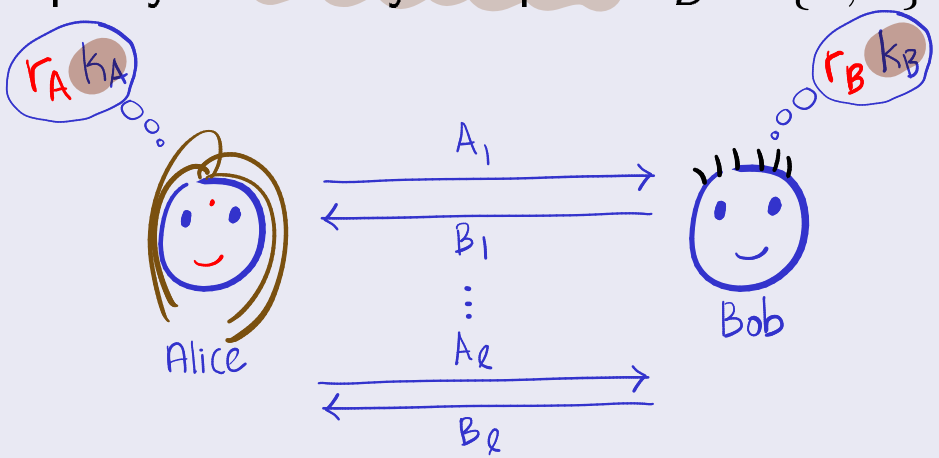

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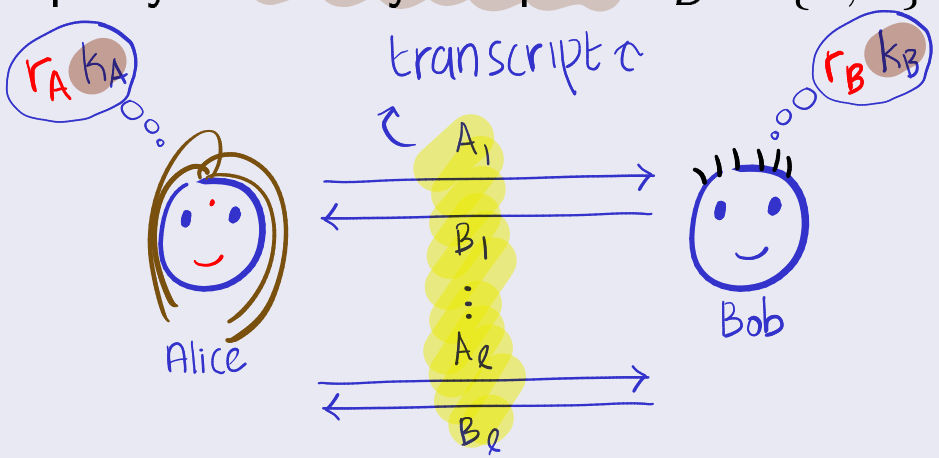

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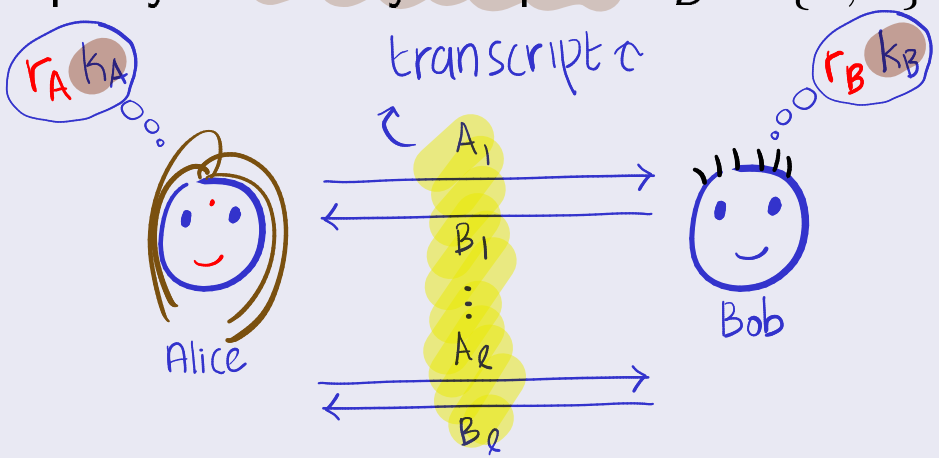
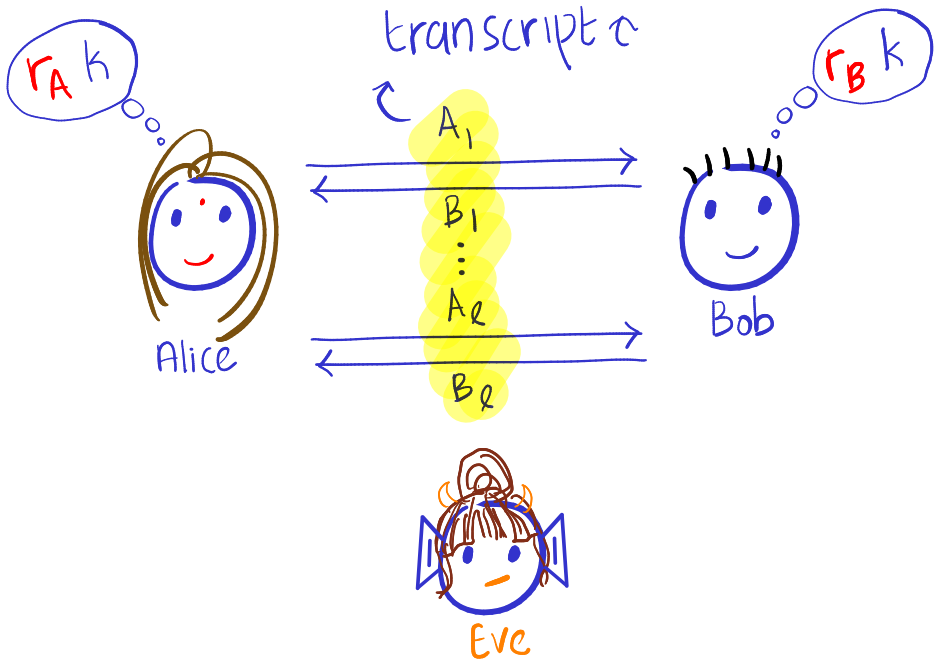


- Correctness of key exchange: for every $n \in \mathbb{N}$

$$\Pr_{(k_A,k_B,\tau)\leftarrow\Pi(1^n)}[k_A = k_B] = 1$$

How to Define Security?...

- Intuitively, what is the security requirement?



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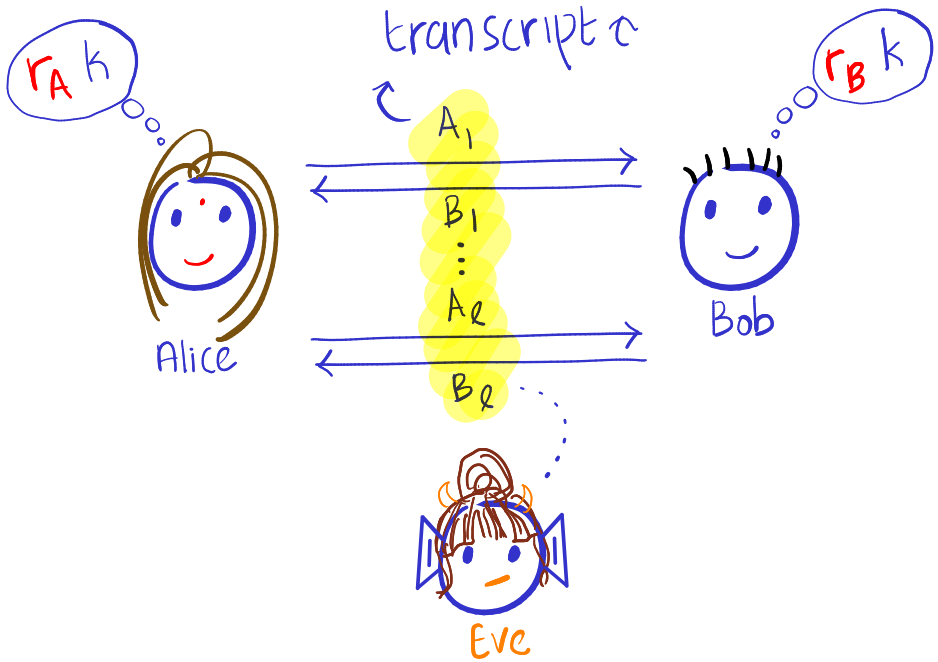

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Definition 2 (Secrecy Against Eavesdroppers)

A key-exchange protocol Π is computationally secret against eavesdroppers if for every PPT eavesdropper Eve the following is negligible.

$$\delta(n) := \left| \Pr_{(k,\tau)\leftarrow\Pi(1^n)}[\mathsf{Eve}(\tau, k) = 0] - \Pr_{\substack{(k,\tau)\leftarrow\Pi(1^n) \\ r\leftarrow\{0,1\}^n}}[\mathsf{Eve}(\tau, r) = 0] \right|$$

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"Real world"

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"Real world"

"Random world"

Exercise 1

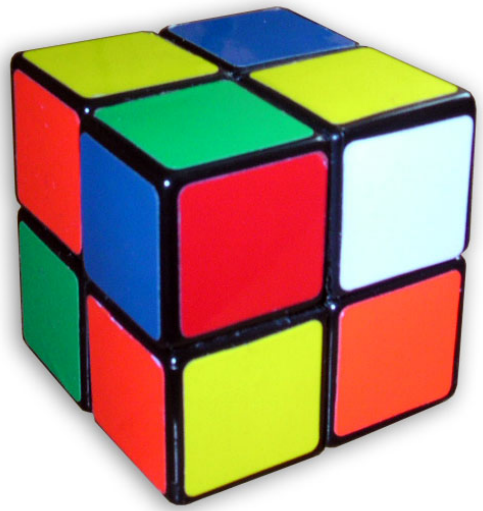
How can an unbounded eavesdropper Eve break secrecy?

Plan for Today's Lecture

- Task: key exchange
- Threat model: computational secrecy against eavesdroppers

Key Exchange

Basic Group Theory



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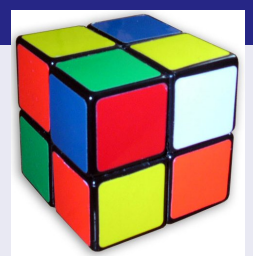
Abstracts properties of integers $\mathbb{Z} := (\{\cdots, -1, 0, 1, \cdots\}, +)$

Exercise 2

- Show that $\mathbb{Z} := (\{\cdots, -1, 0, 1, \cdots\}, +)$ is a group
- What is the group corresponding to $2 \times 2 \times 2$ Rubik's cube?
  - Describe the set $\mathcal{G}$ and the operation $\cdot$



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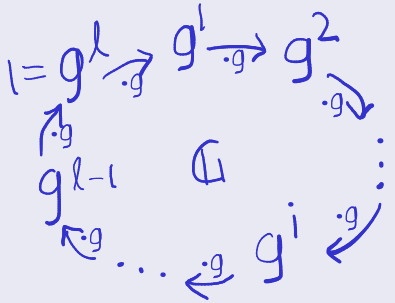

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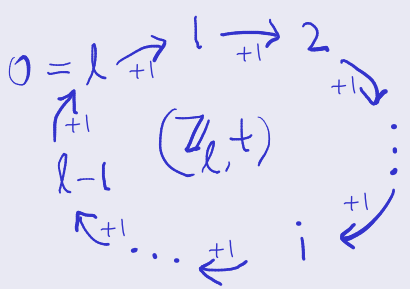

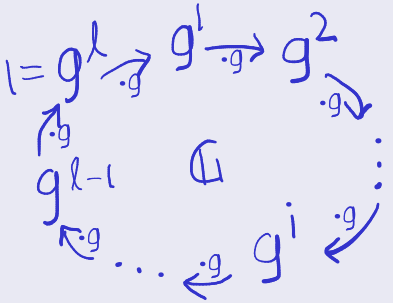

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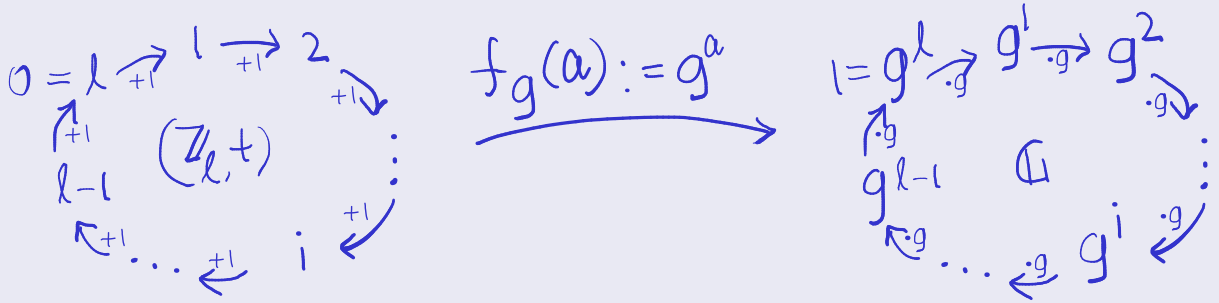

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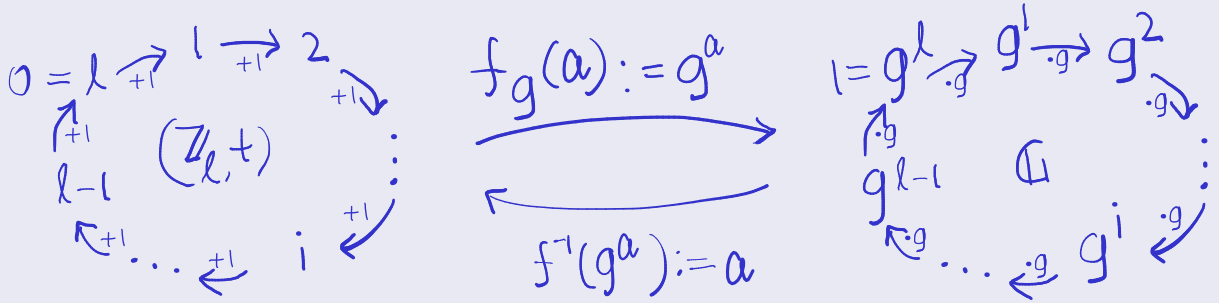

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Basic Group Theory...

Exercise 3 (Lagrange's theorem)

Prove that the order of an element divides order of the (finite) group.

Exercise 4

For a group $\mathbb{G}$ of order ℓ with generator g, show using group axioms that for all $a, b \in \mathbb{Z}_\ell$, $(g^a)^b = g^{ab} = (g^b)^a$

Exercise 5

Prove that a prime-order group is cyclic. Are all cyclic groups of prime order?

Some Examples of (Finite) Groups

Addition modulo prime p

$(\mathbb{Z}_p, +)$

$\{0, \cdots p-1\}$ $\quad g_1 + g_2 := g_1 + g_2 \pmod p$

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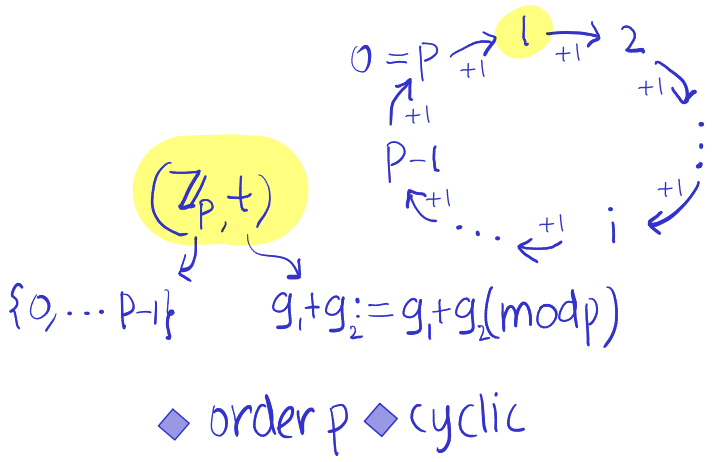

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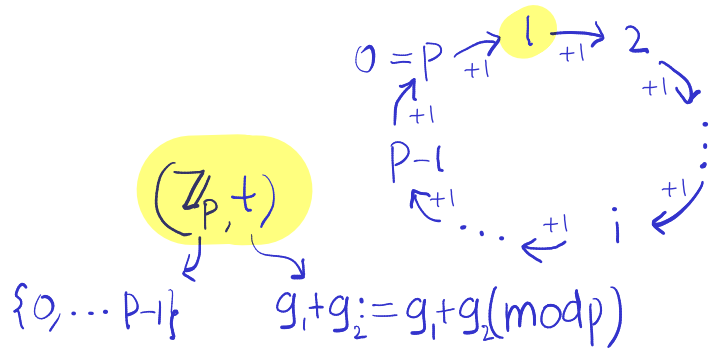

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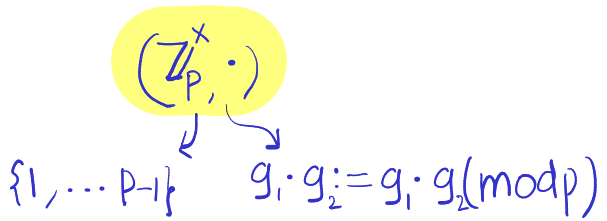

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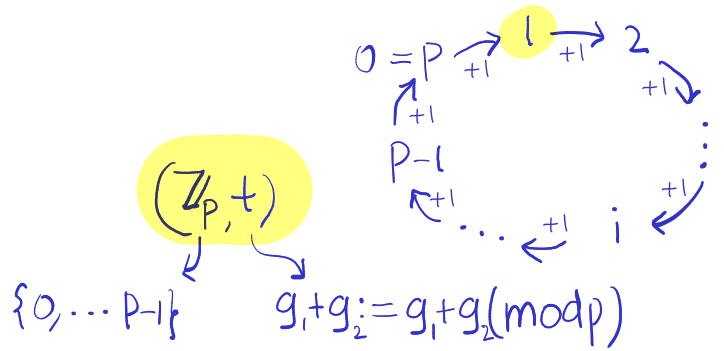

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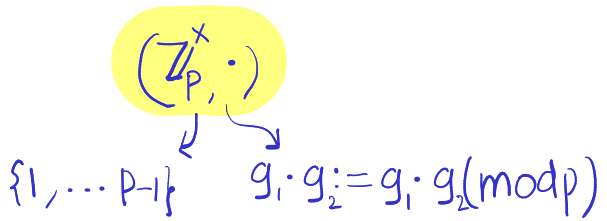

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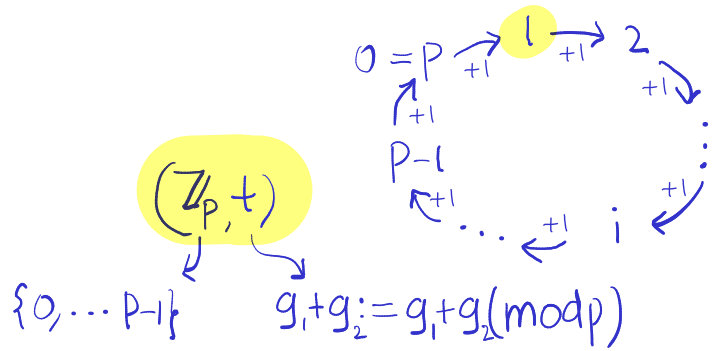

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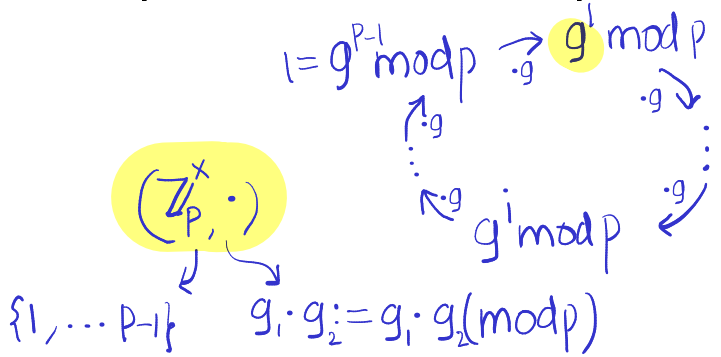

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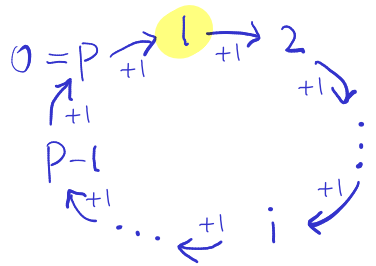

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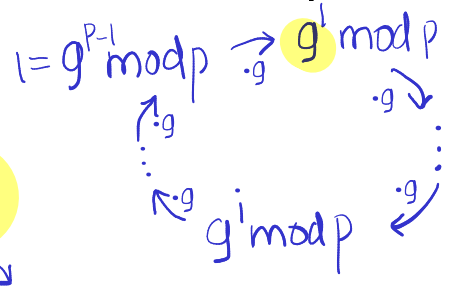

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Multiplication modulo $N = pq$

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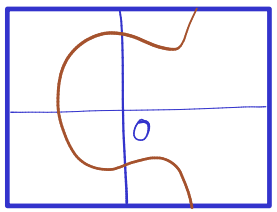

- $|p+1 - \text{order}| \leq 2\sqrt{p}$
- cyclic

What is Easy to Compute Over Cyclic Groups?...

Let's focus on $\mathbb{Z}_p^* = (\{0, \ldots, p-1\}, \cdot)$

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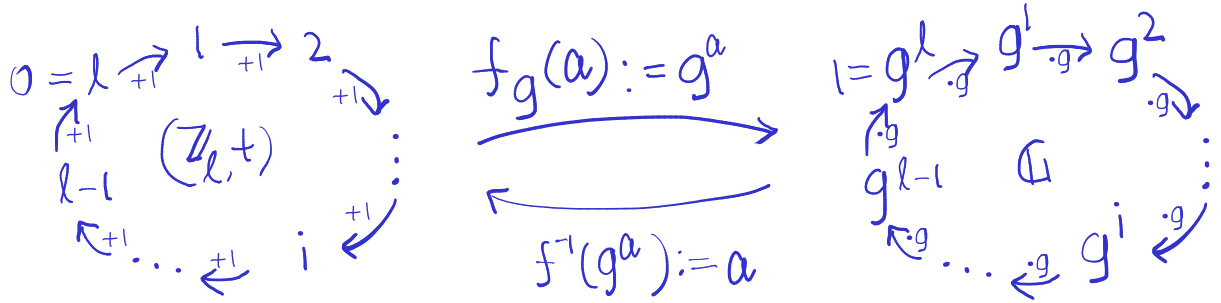
In general: group operation, exponentiation and inverse efficient

What is Hard to Compute Over Cyclic Groups?...

- Recall the exponentiation map for cyclic group



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... holds if solving the DLog problem in $\mathbb{G}$ w.r.to S is hard for all PPT inverters Inv. That is, for all Inv, the following is negligible:

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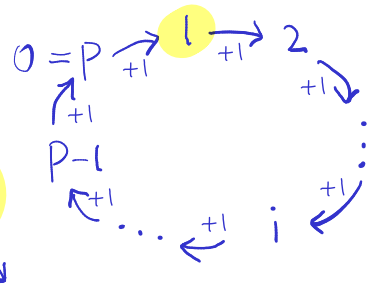

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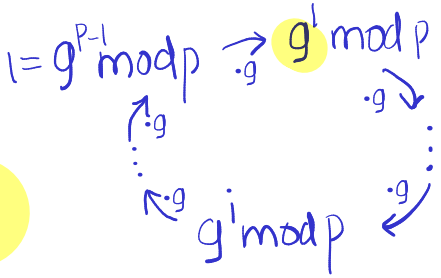

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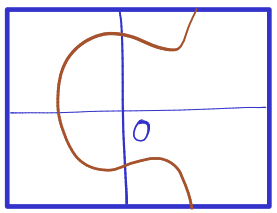
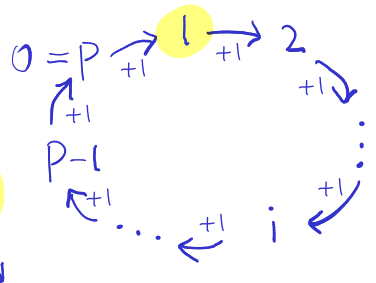


◆ $|p+1 - \text{order}| \leq 2\sqrt{p}$ ◆ cyclic

What is Hard to Compute Over Cyclic Groups?...

Addition modulo prime p



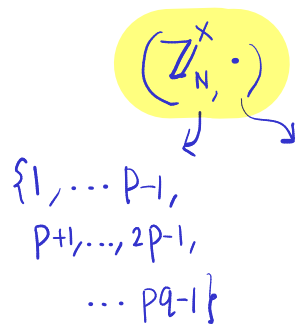
- order p
- cyclic

Multiplication modulo prime p

- order $p-1$
- cyclic

Multiplication modulo $N = pq$

primes



- order $(p-1)(q-1)$
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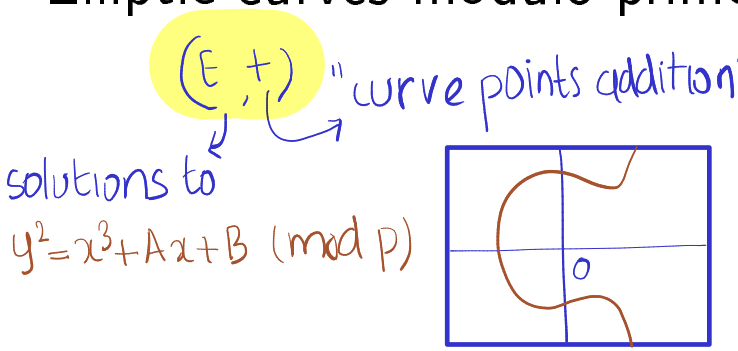
Elliptic curves modulo prime p



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What is Hard to Compute Over Cyclic Groups?...

? Easy! Division modulo p !

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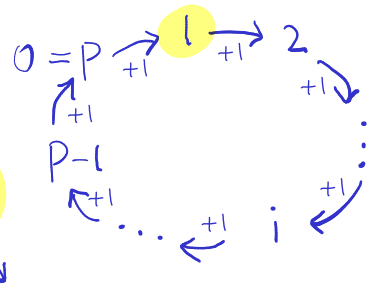
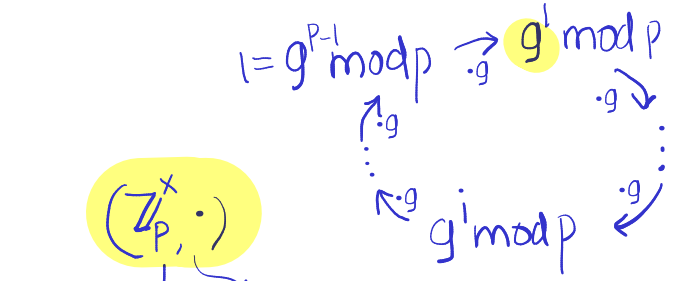


◆ order p ◆ cyclic

Multiplication modulo prime p



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Multiplication modulo $N = pq$

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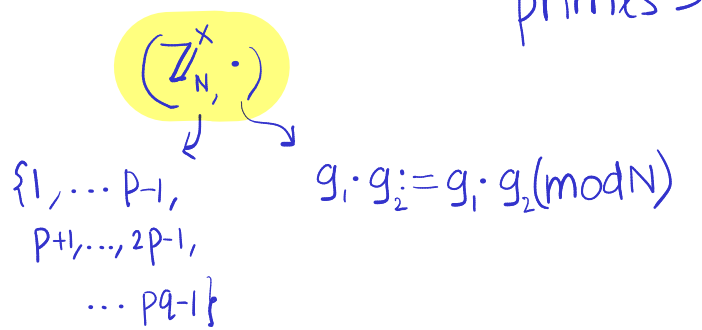

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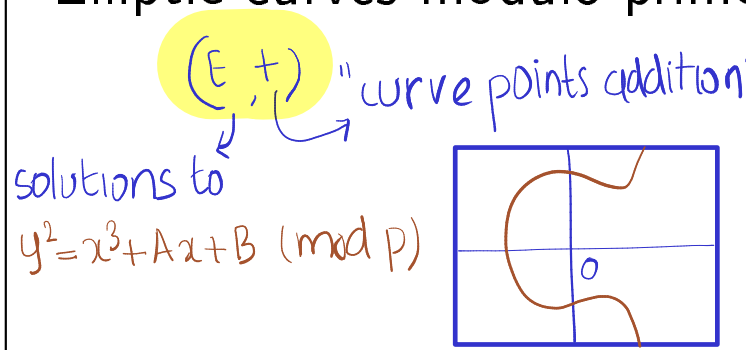

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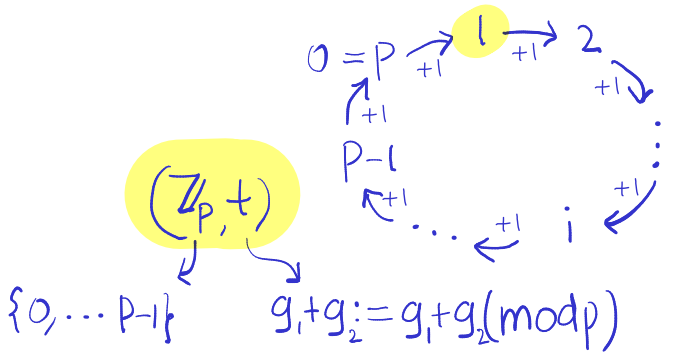

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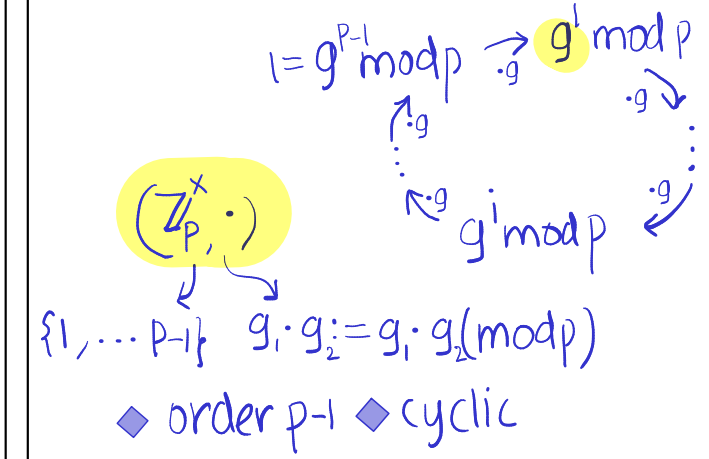

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Multiplication modulo $N = pq$

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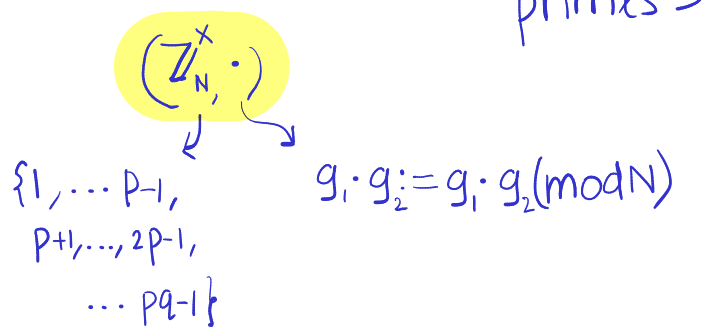

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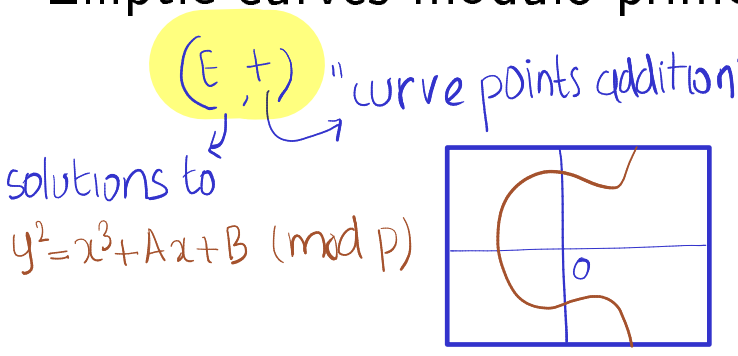

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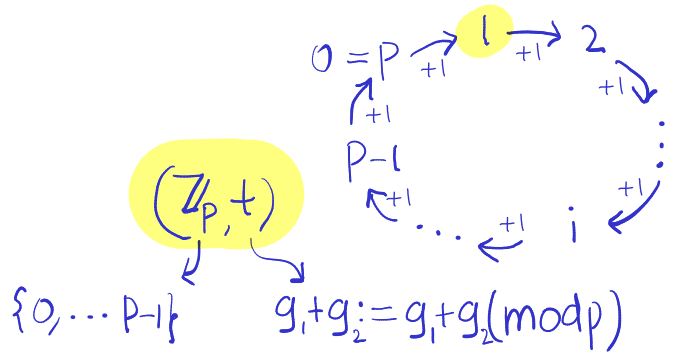

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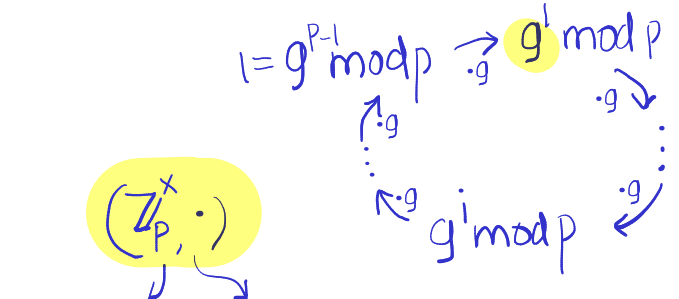

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Multiplication modulo $N = pq$

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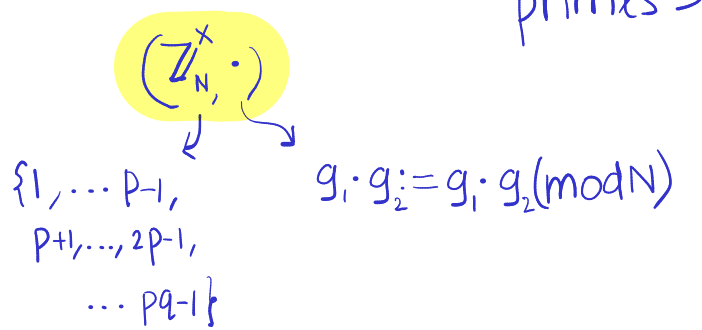

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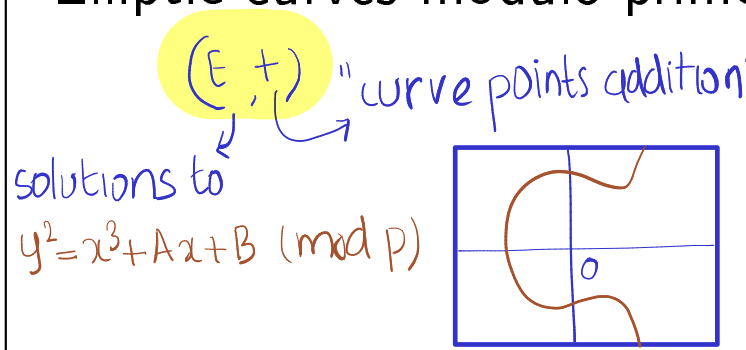

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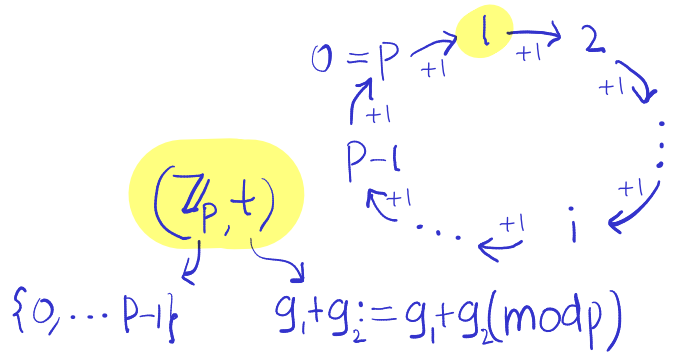

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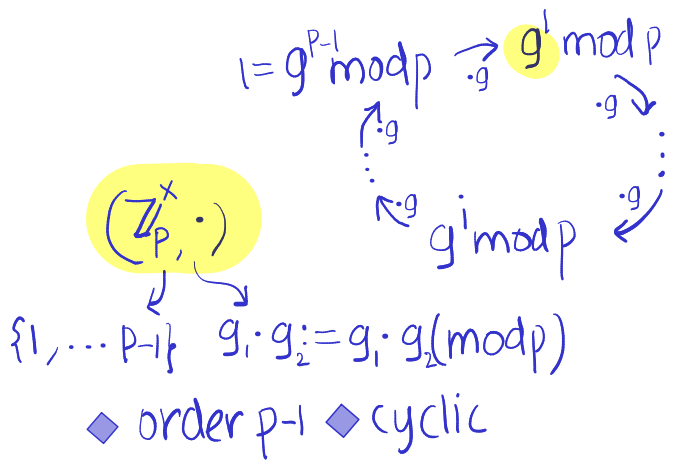

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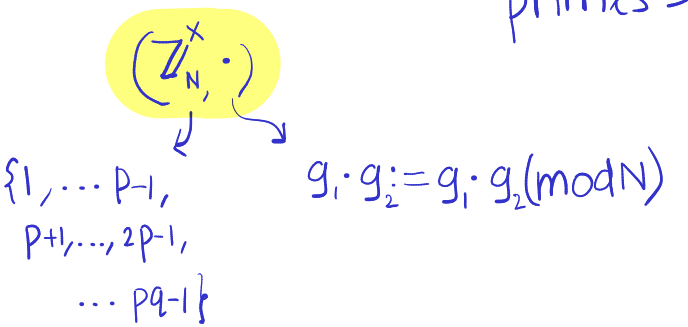

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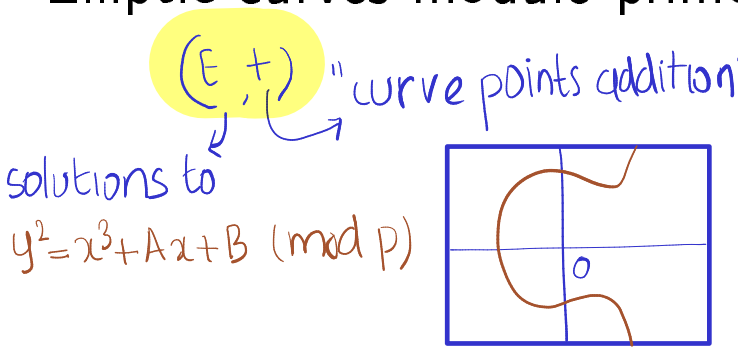
Hard in its cyclic subgroup

Elliptic curves modulo prime p



◆ $|p+1 - \text{order}| \leq 2\sqrt{p}$ ◆ cyclic

Believed hard; no known sub-exp. algo

Efficient Group Samplers Exist

E.g.: $(\mathbb{Z}_p^*, p, g) \leftarrow \mathsf{S}(1^n)$

1. Sample random prime p such that $\|p\| \approx n$
   1. Sample random *integer* p such that $\|p\| \approx n$
   2. Test whether p prime using (say) Miller-Rabin text
      - ⏱ Randomised test, runs in roughly $\tilde{O}(n^3)$ time for negligible error

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   1. Sample a random $g \in \{0, \ldots, p-1\}$
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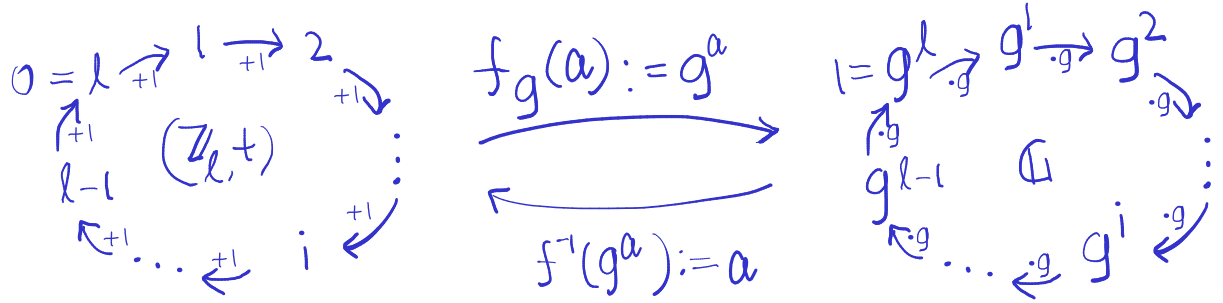
- Note: sample random generator $\implies$ sample random group element via "isomorphism"



Plan for Today's Lecture

- Task: key exchange
- Threat model: computational secrecy against eavesdroppers

Key Exchange

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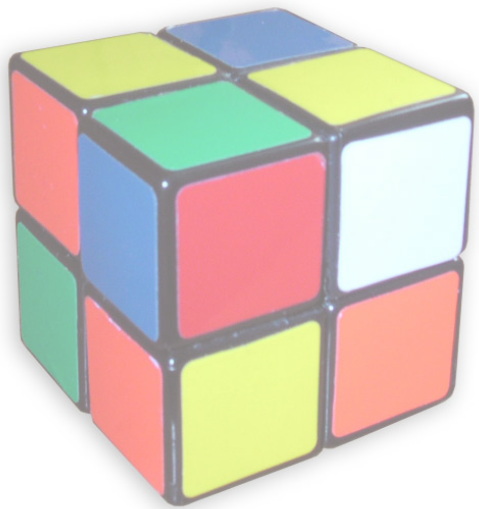
Basic Group Theory



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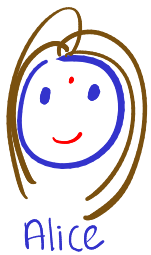
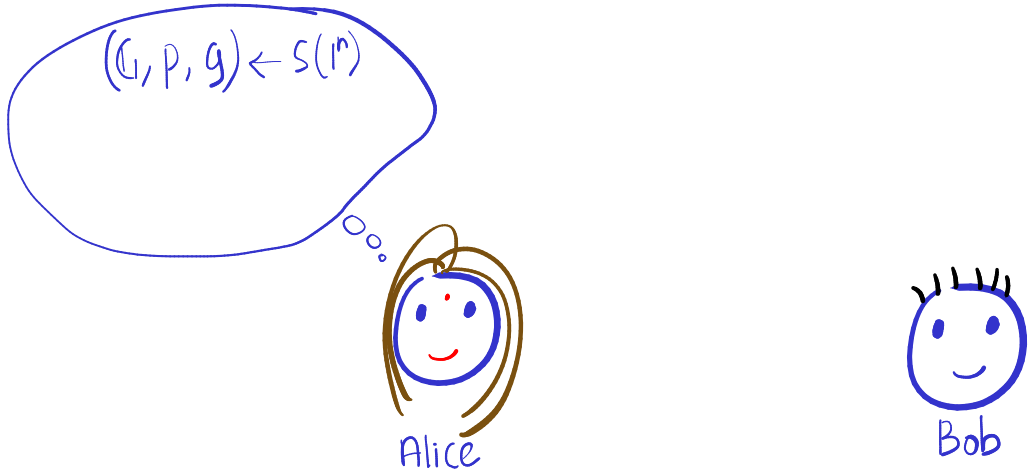
Diffie-Hellman Key-Exchange Protocol



Protocol 1

Diffie-Hellman Key-Exchange Protocol



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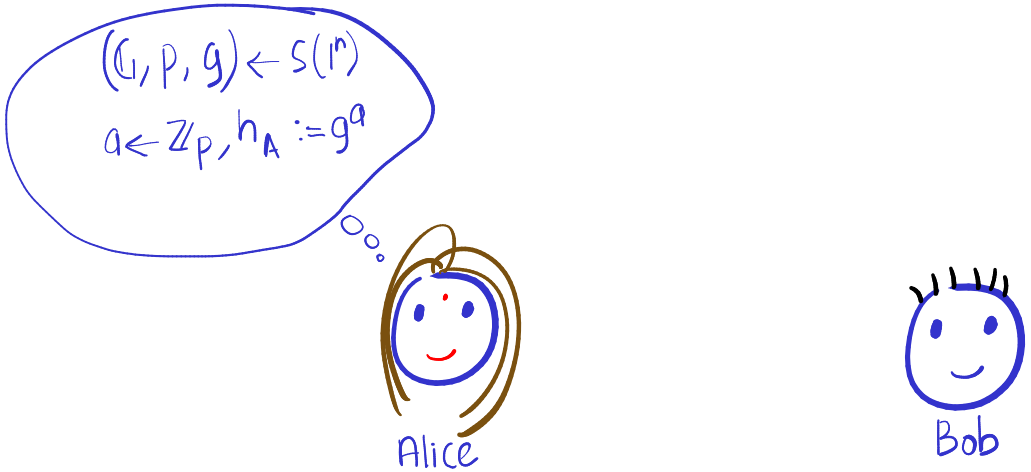

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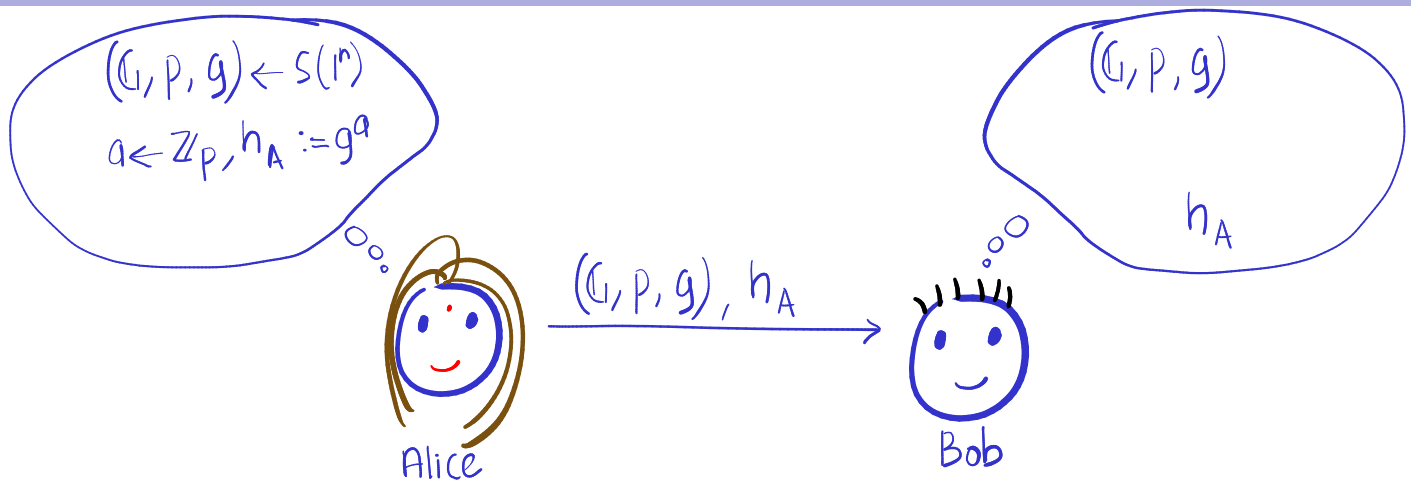

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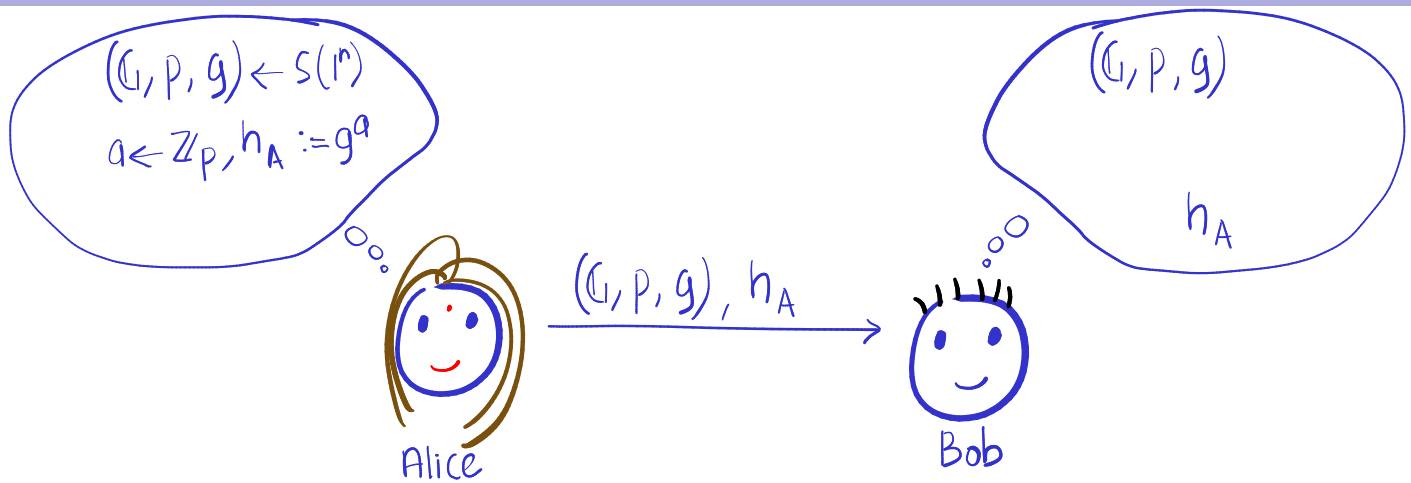

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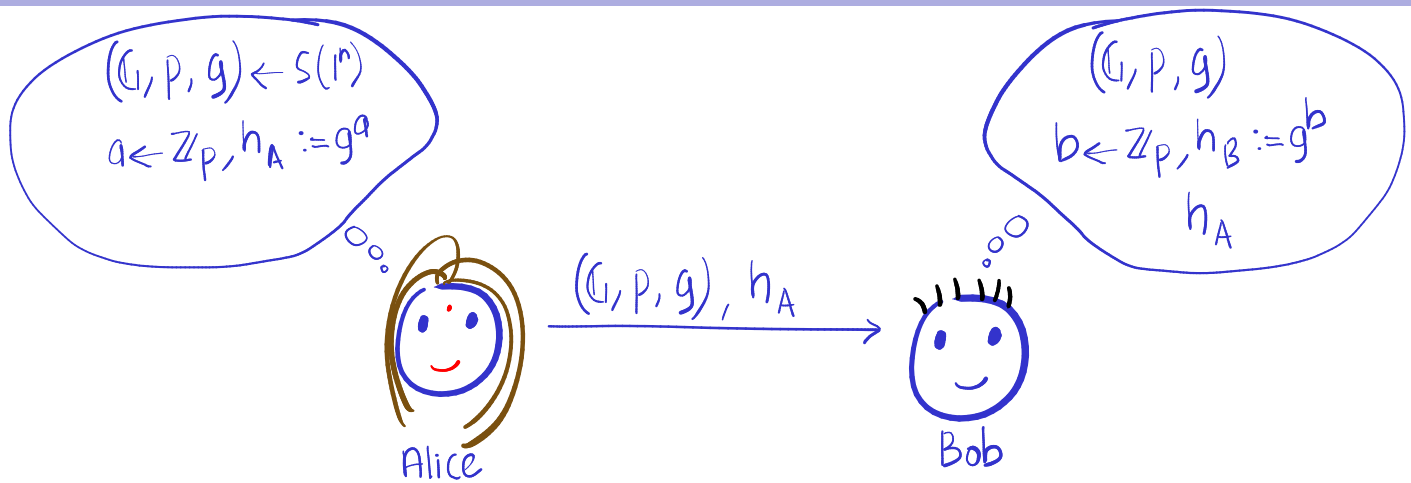
Diffie-Hellman Key-Exchange Protocol



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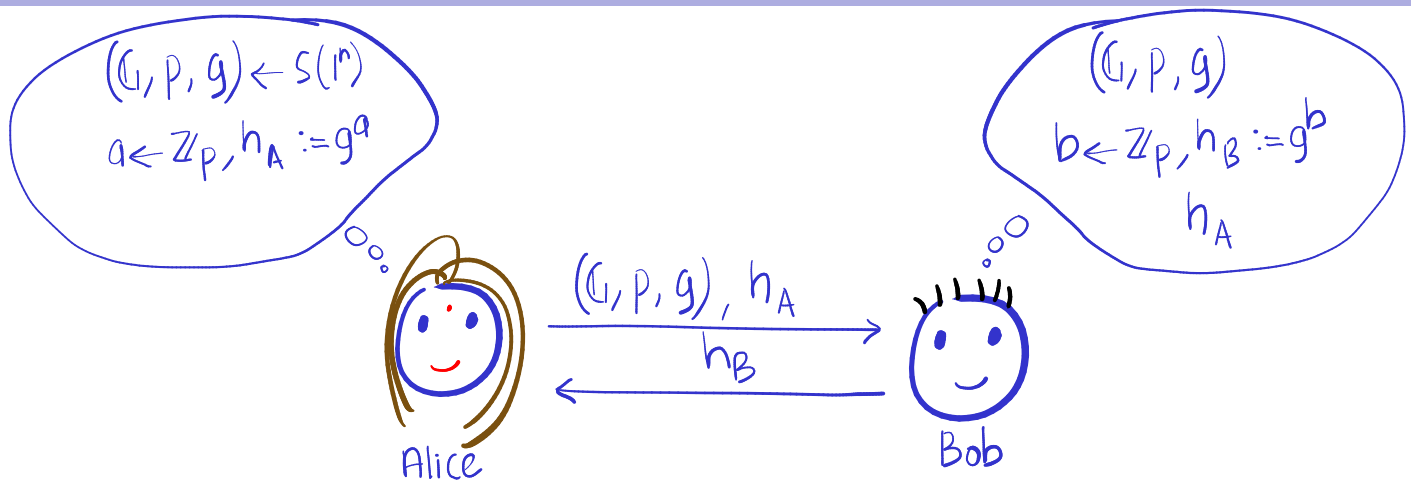

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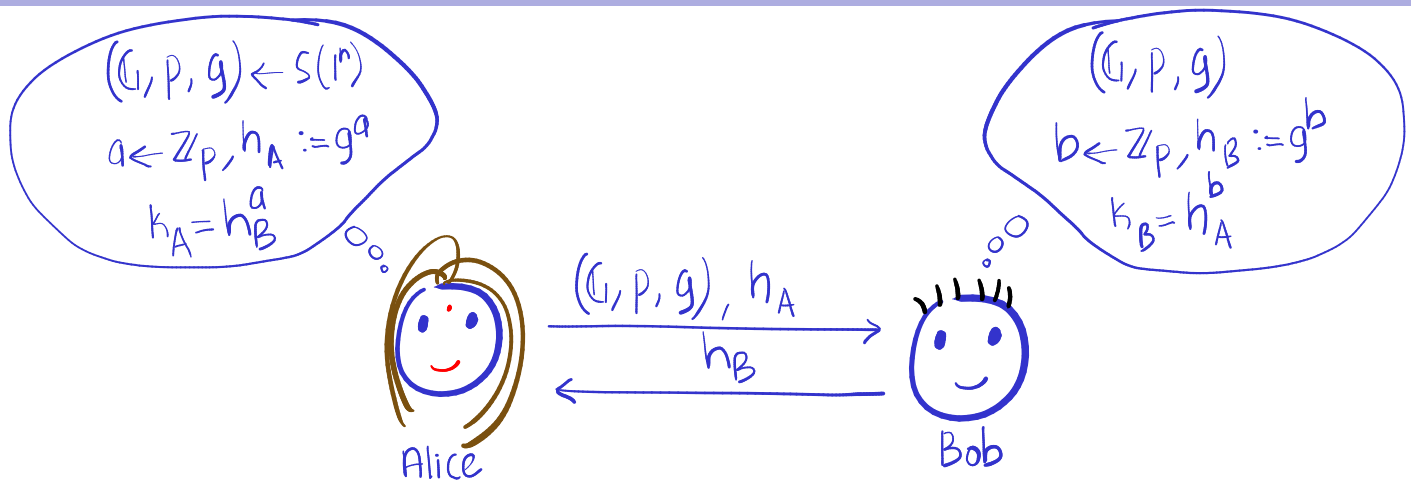

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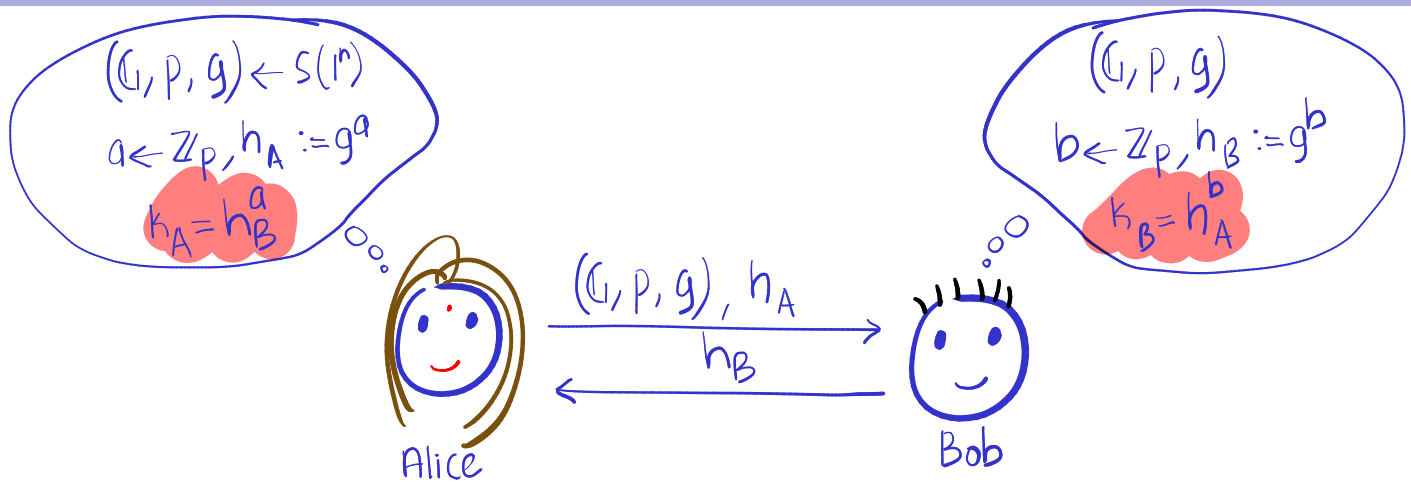

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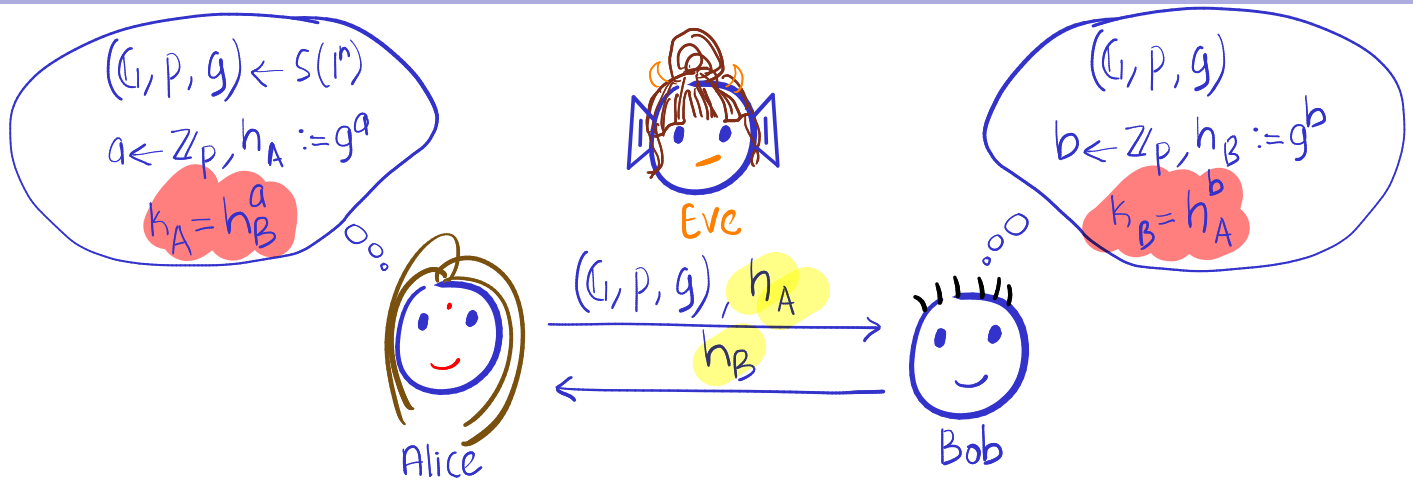
2. Alice←Bob: Send $h_B := g^b$ for $b \leftarrow \mathbb{Z}_\ell$
3. Alice outputs $k_A := (h_B)^a$; Bob outputs $k_B := (h_A)^b$

- Correctness of key generation:

$$k_A = h_B^a = (g^b)^a = g^{ab} = (g^a)^b = h_A^b = k_B$$

When is it Secret Against Eavesdroppers?...



- What does Eve see? The transcript is $(h_A := g^a, h_B := g^b)$

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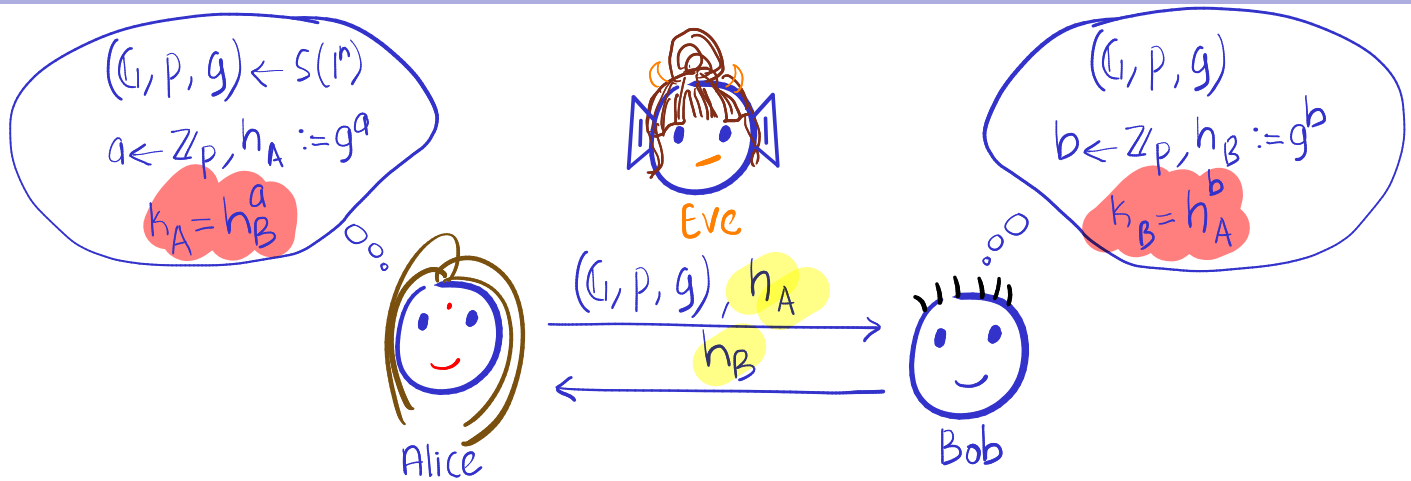

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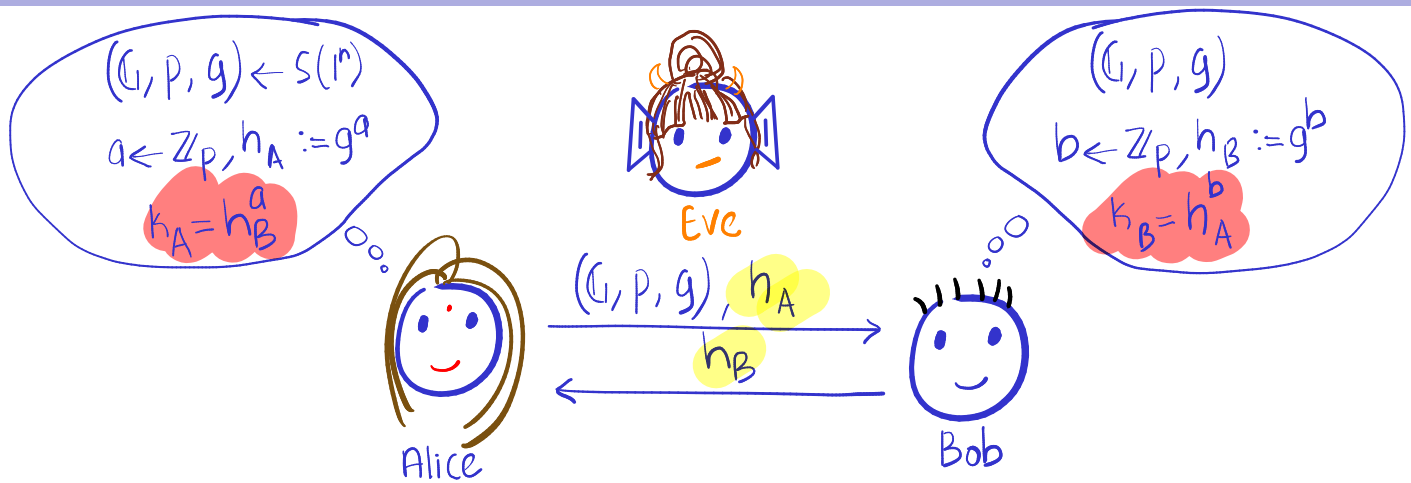

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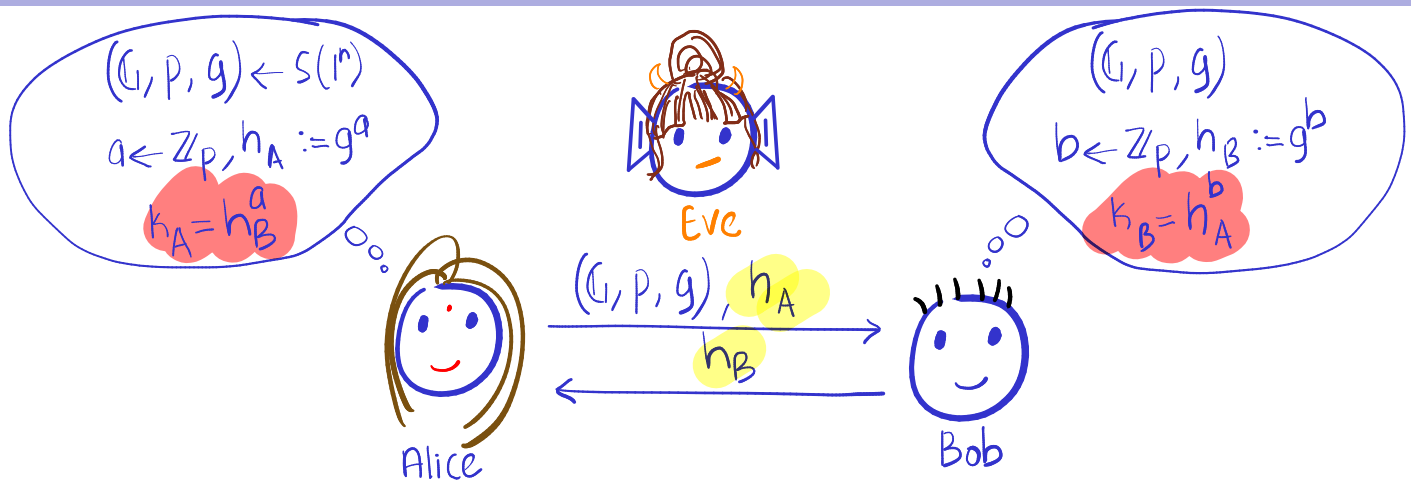

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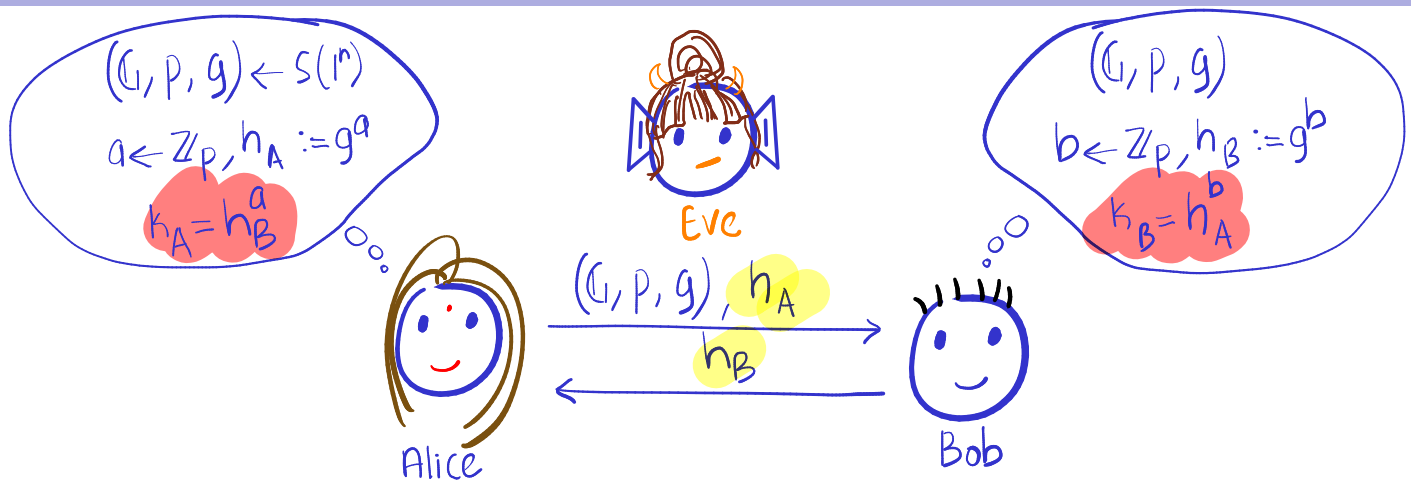

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⚠ No, what if Eve can compute g^{ab} given g^a and g^b?

- This is the "computational Diffie-Hellman" (CDH) problem

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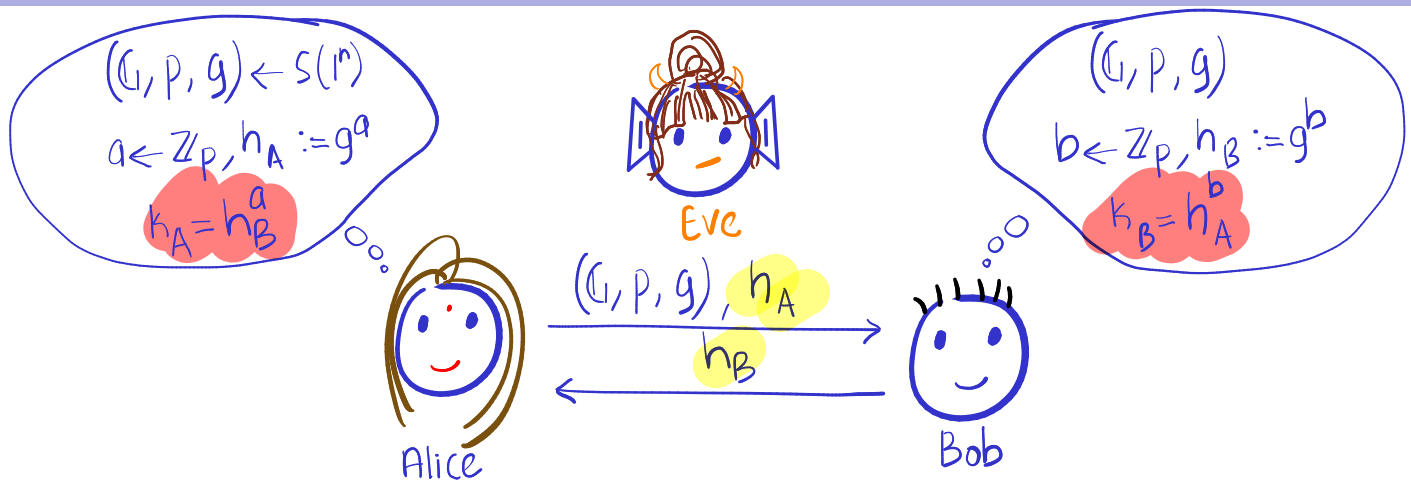

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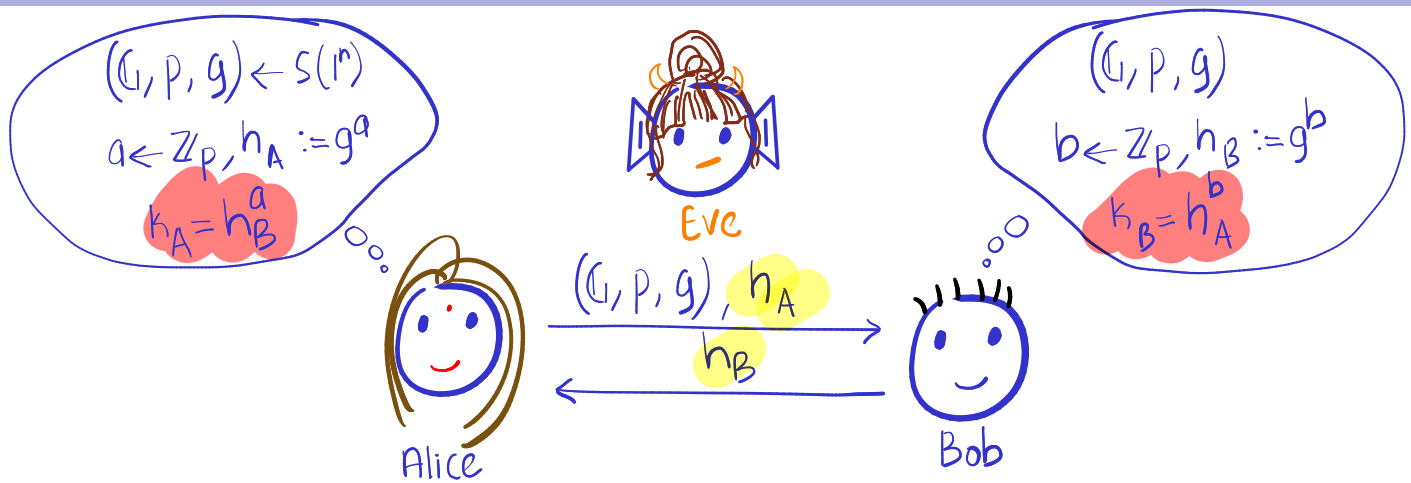

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❓ Is CDH problem being hard sufficient?

⚠ What if Eve can distinguish g^{ab} from random group elements?

- There exist such groups!

When is it Secret Against Eavesdroppers?...

Assumption 2 (Decisional DH (DDH) assumption in $\mathbb{G}$ w.r.to S $\cdots$)

$\cdots$ holds if for all PPT distinguishers D, the following is negligible:

$$\Pr_{\substack{(\mathbb{G},\ell,g)\leftarrow \mathsf{S}(1^n)\\ a,b\leftarrow\mathbb{Z}_\ell}}[\mathsf{D}(g^a, g^b, g^{ab}) = 0] - \Pr_{\substack{(\mathbb{G},\ell,g)\leftarrow \mathsf{S}(1^n)\\ a,b,r\leftarrow\mathbb{Z}_\ell}}[\mathsf{D}(g^a, g^b, g^r) = 0]$$

"Real world" "Random world"

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Theorem 1

Diffie-Hellman key-exchange is computationally secret against eavesdroppers under the DDH assumption in $\mathbb{G}$ w.r.to S.

Proof.

Secrecy requirement is same as the assumption! □

When is it Secret Against Eavesdroppers?...

Assumption 2 (Decisional DH (DDH) assumption in $\mathbb{G}$ w.r.to S $\cdots$)

$\cdots$ holds if for all PPT distinguishers D, the following is negligible:

$$\Pr_{\substack{(\mathbb{G},\ell,g)\leftarrow \mathsf{S}(1^n)\\ a,b\leftarrow\mathbb{Z}_\ell}}[\mathsf{D}(g^a, g^b, g^{ab}) = 0] - \Pr_{\substack{(\mathbb{G},\ell,g)\leftarrow \mathsf{S}(1^n)\\ a,b,r\leftarrow\mathbb{Z}_\ell}}[\mathsf{D}(g^a, g^b, g^r) = 0]$$

"Real world" "Random world"

Theorem 1

Diffie-Hellman key-exchange is computationally secret against eavesdroppers under the DDH assumption in $\mathbb{G}$ w.r.to S.

Proof.

Secrecy requirement is same as the assumption! □

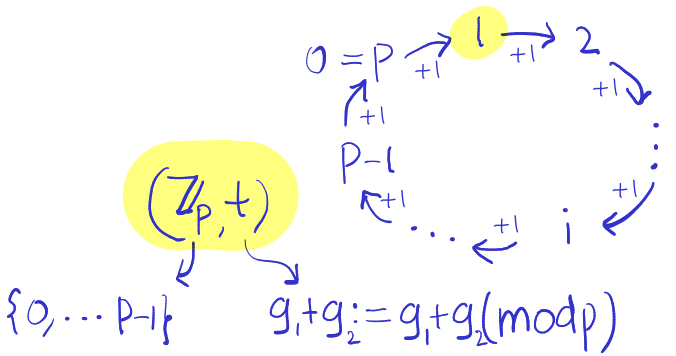
Exercise 6

But I did slightly cheat! Figure out where.

Where is DDH Assumption Known to Hold?

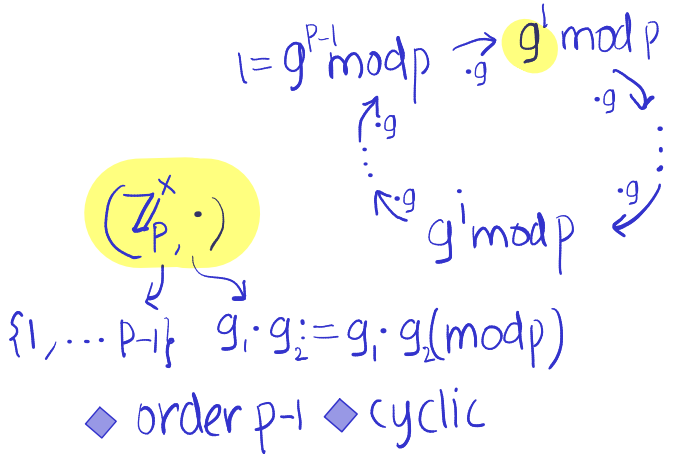
?

Addition modulo prime p



◆ order p ◆ cyclic

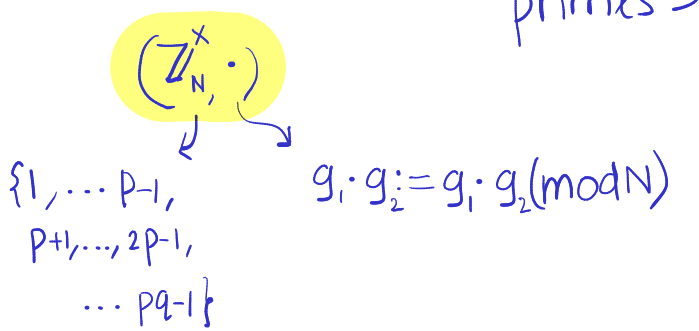
Multiplication modulo prime p



◆ order $p-1$ ◆ cyclic

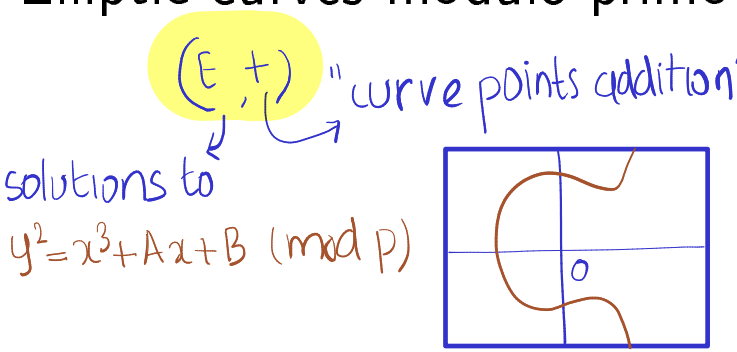
Multiplication modulo $N = pq$

primes



◆ order $(p-1)(q-1)$ ◆ not cyclic

Elliptic curves modulo prime p

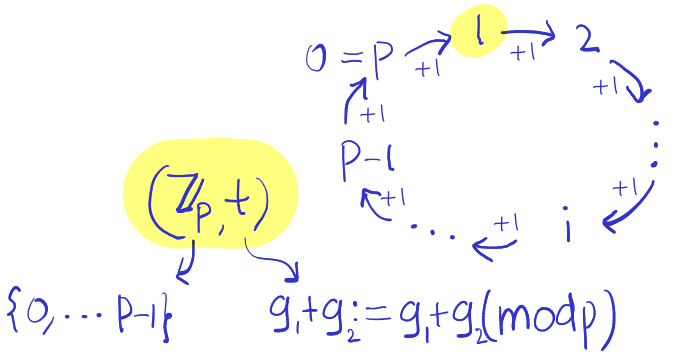


◆ $|p+1 - \text{order}| \leq 2\sqrt{p}$ ◆ cyclic

Where is DDH Assumption Known to Hold?

? Easy! Since DLog is easy ⚠

Addition modulo prime p



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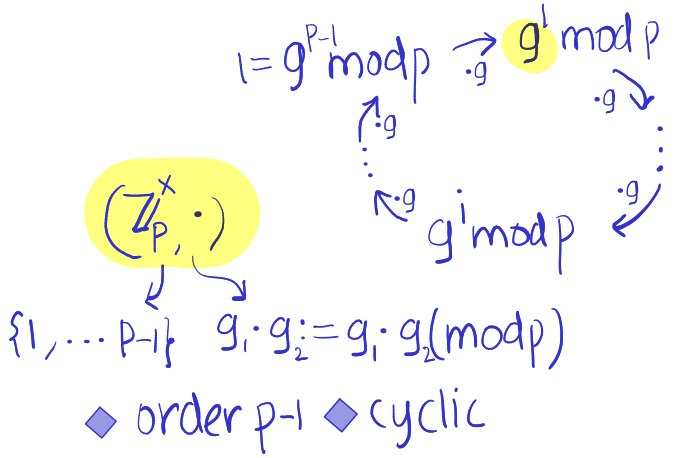

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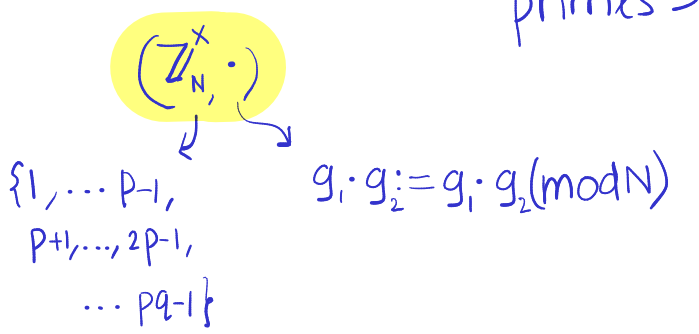

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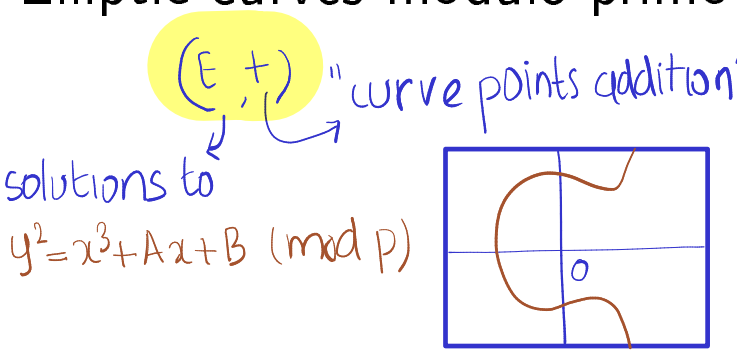

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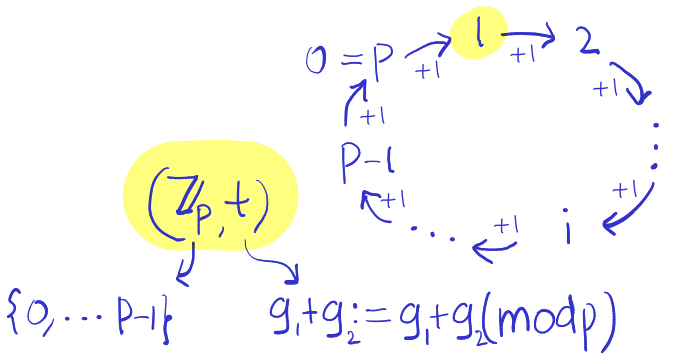

- order p
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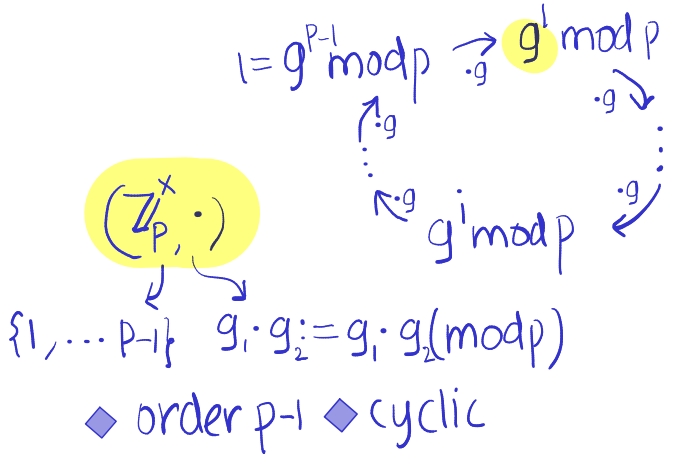
Multiplication modulo prime p



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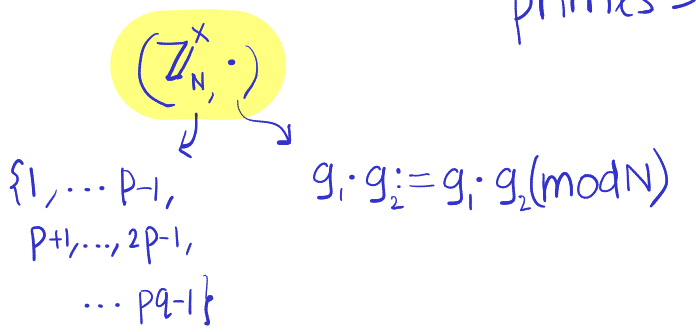

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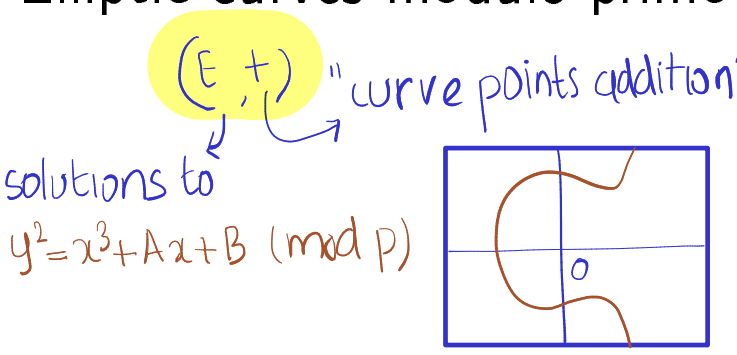

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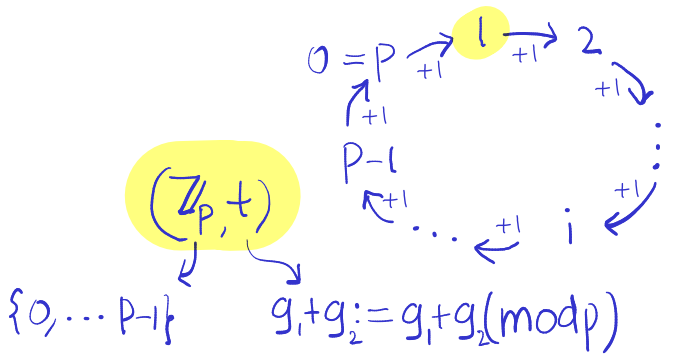

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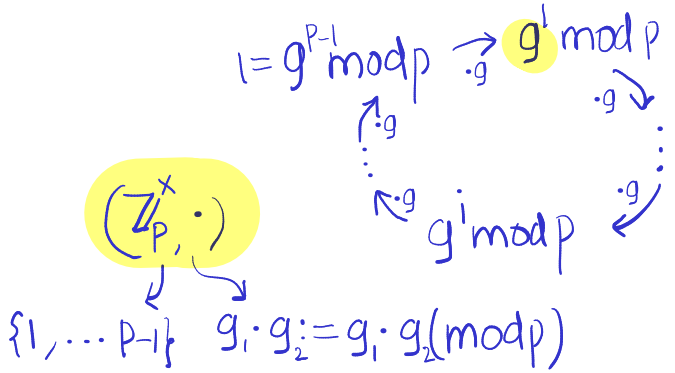
Easy! See Assignment 4 ⚠

Multiplication modulo prime p



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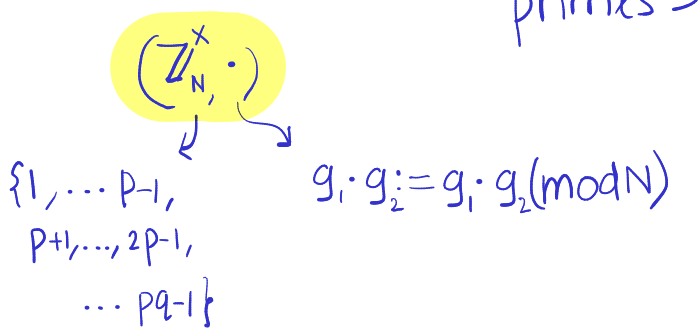

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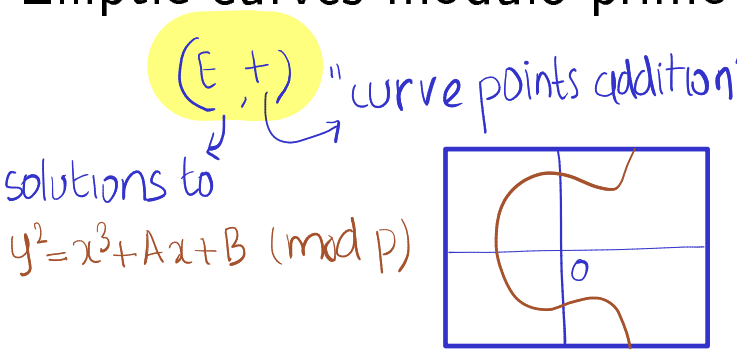

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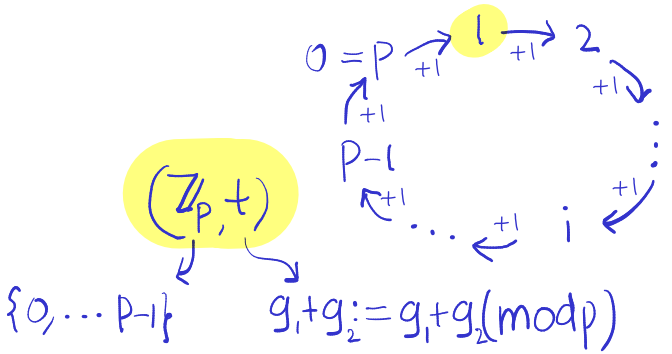

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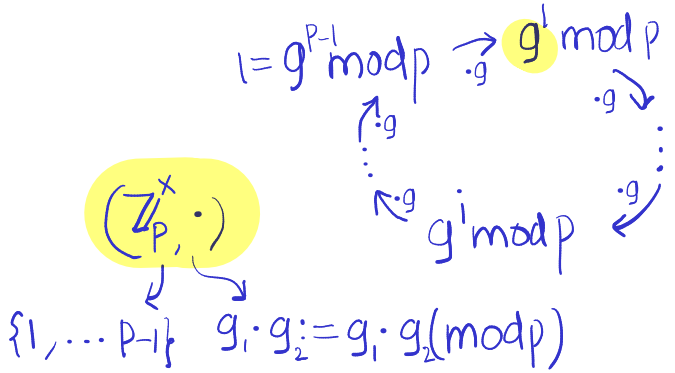

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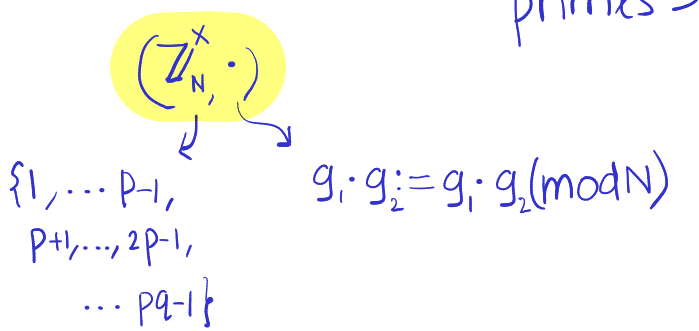

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Hard in its cyclic subgroup

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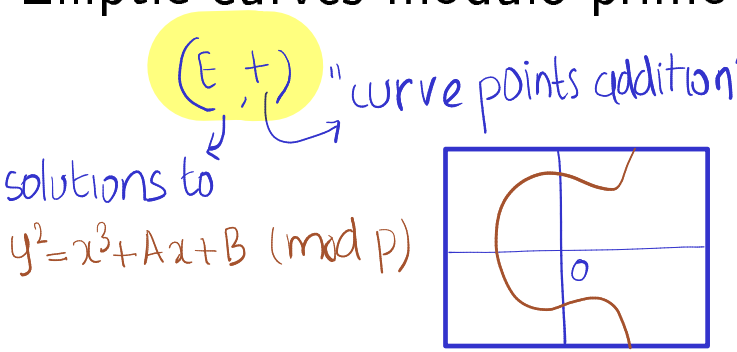

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Believed hard

What About Secrecy Against Active Eve?

- What if Eve is an *active* adversary?
  - Recall that active Eve can intercept/tamper messages

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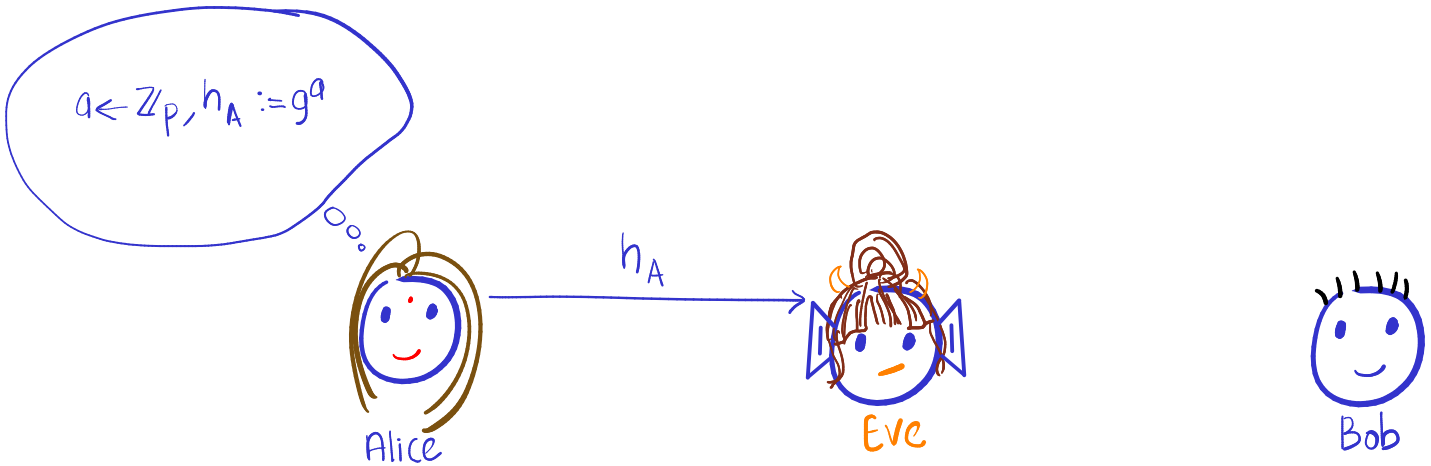

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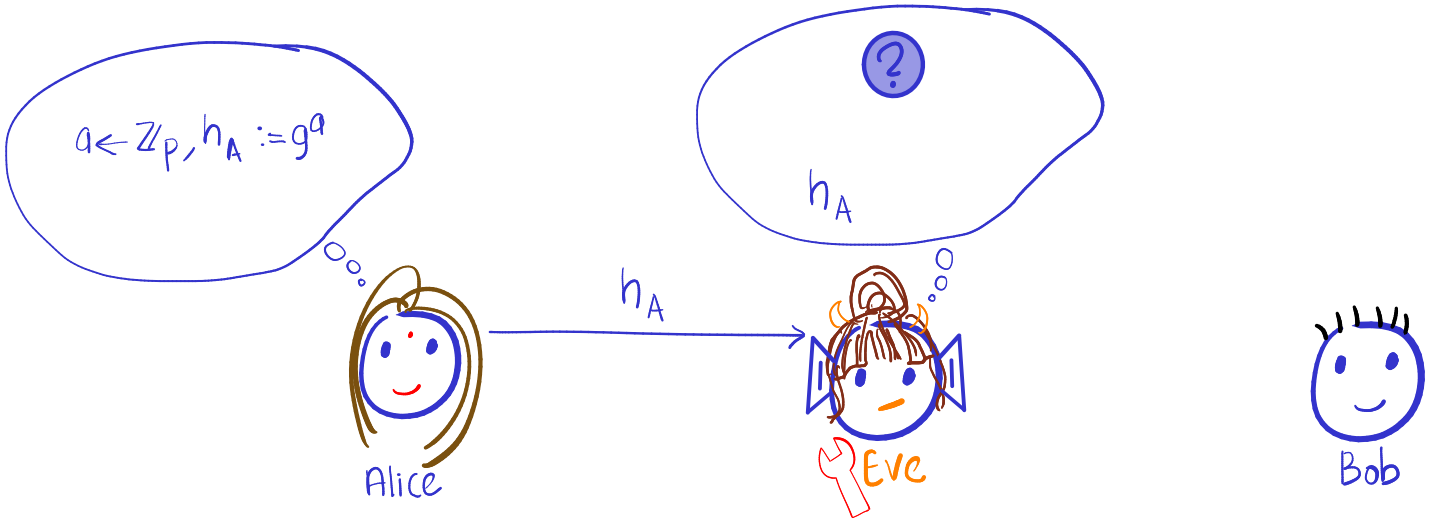

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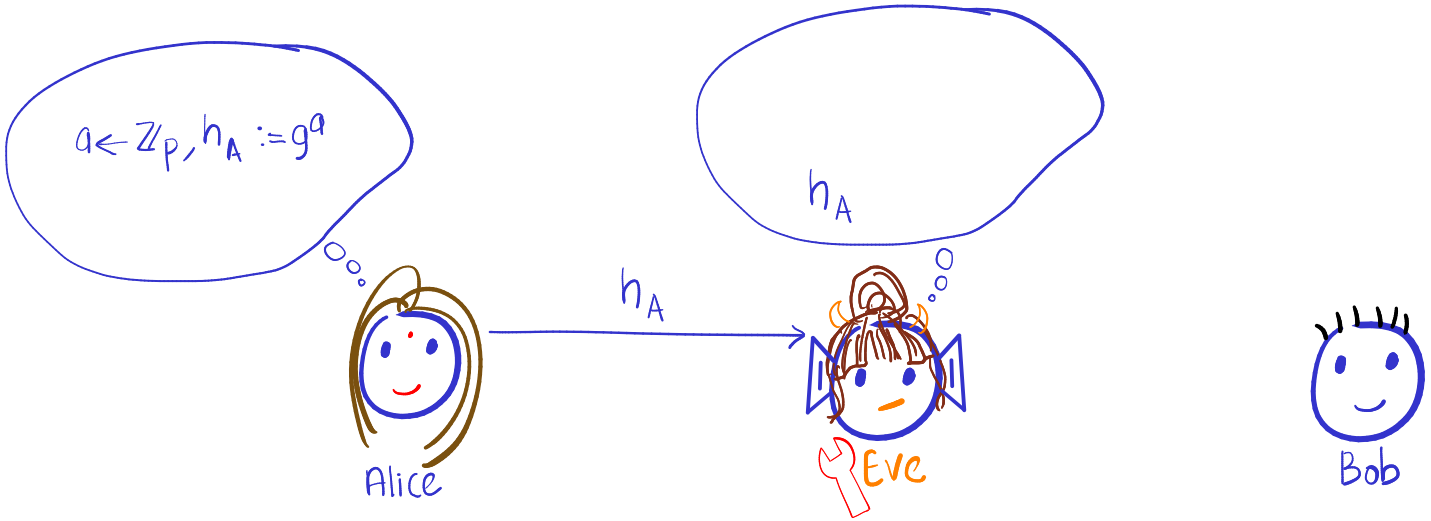

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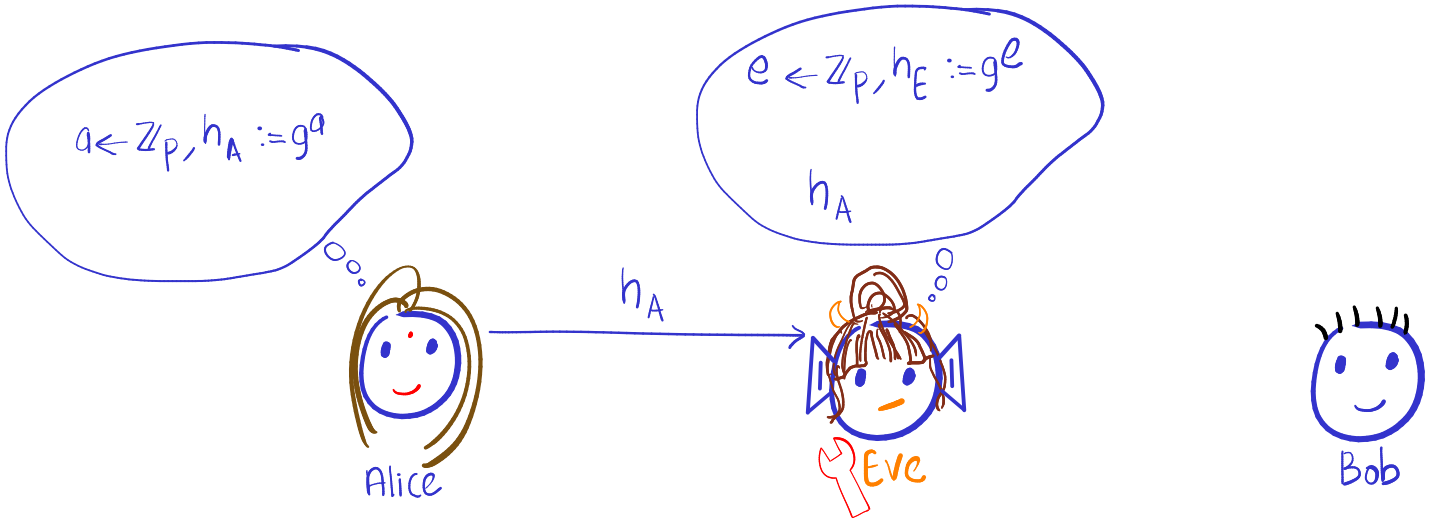

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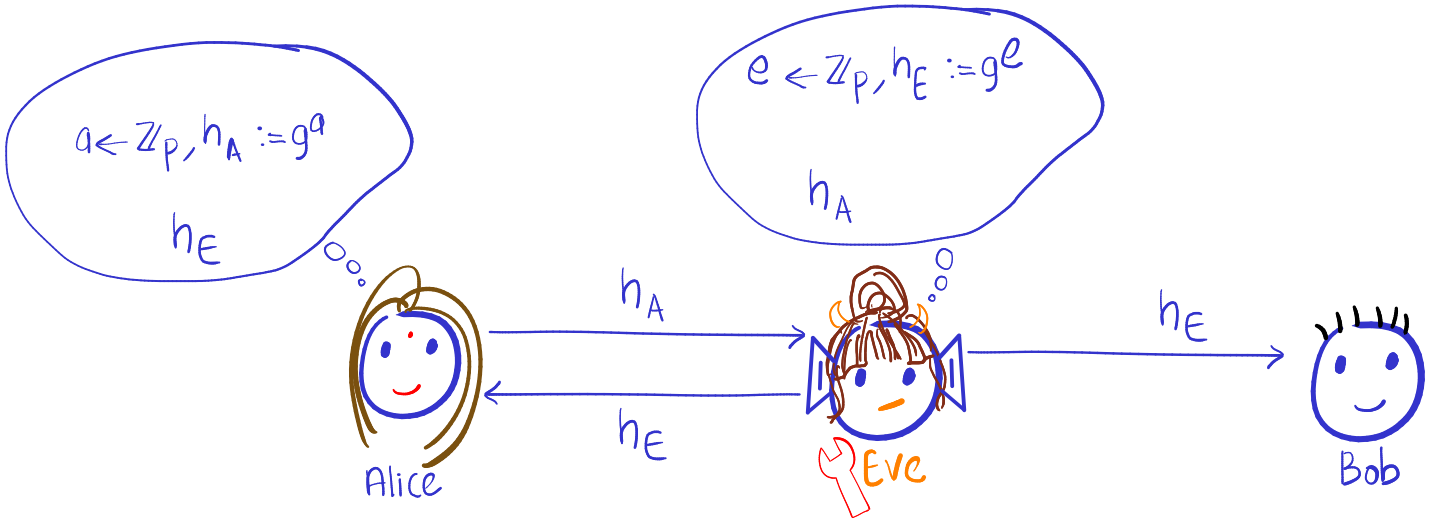

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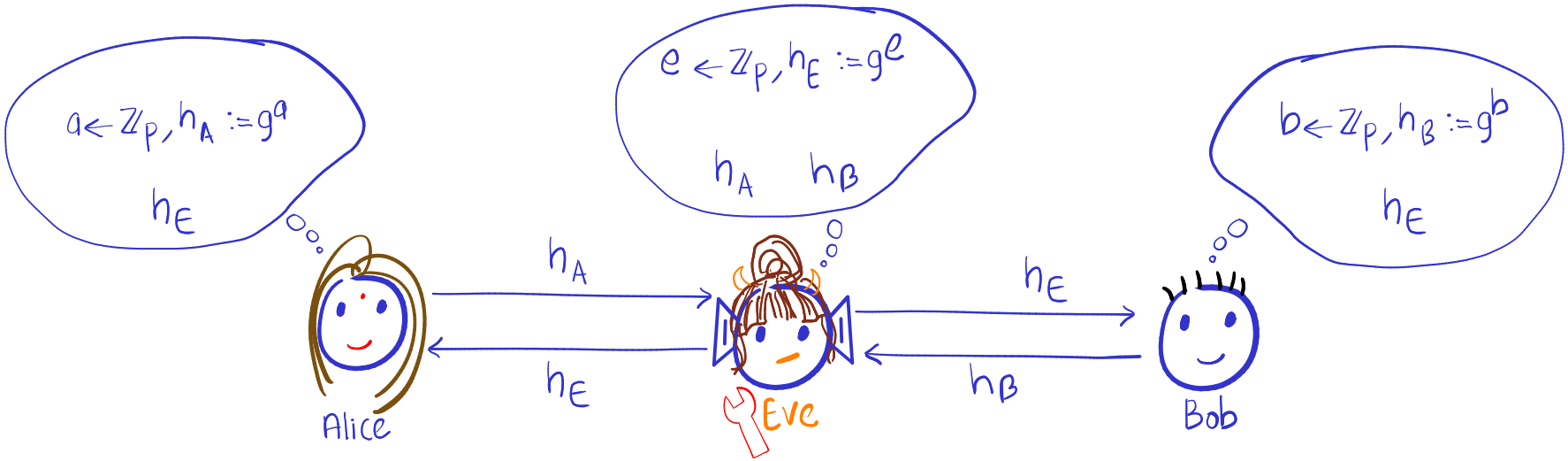

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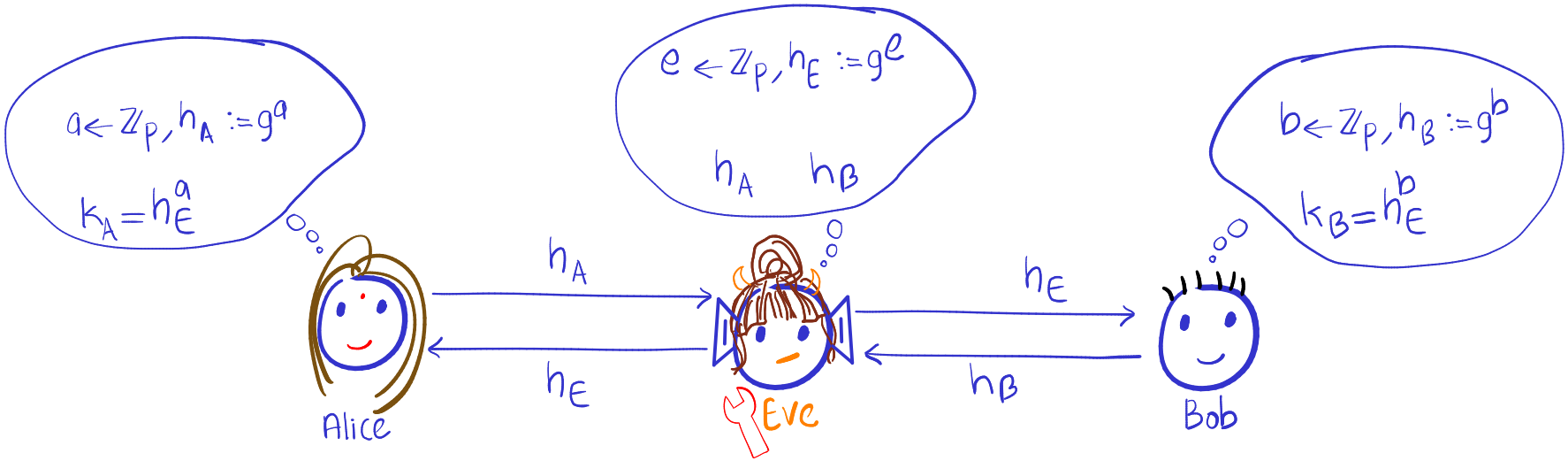

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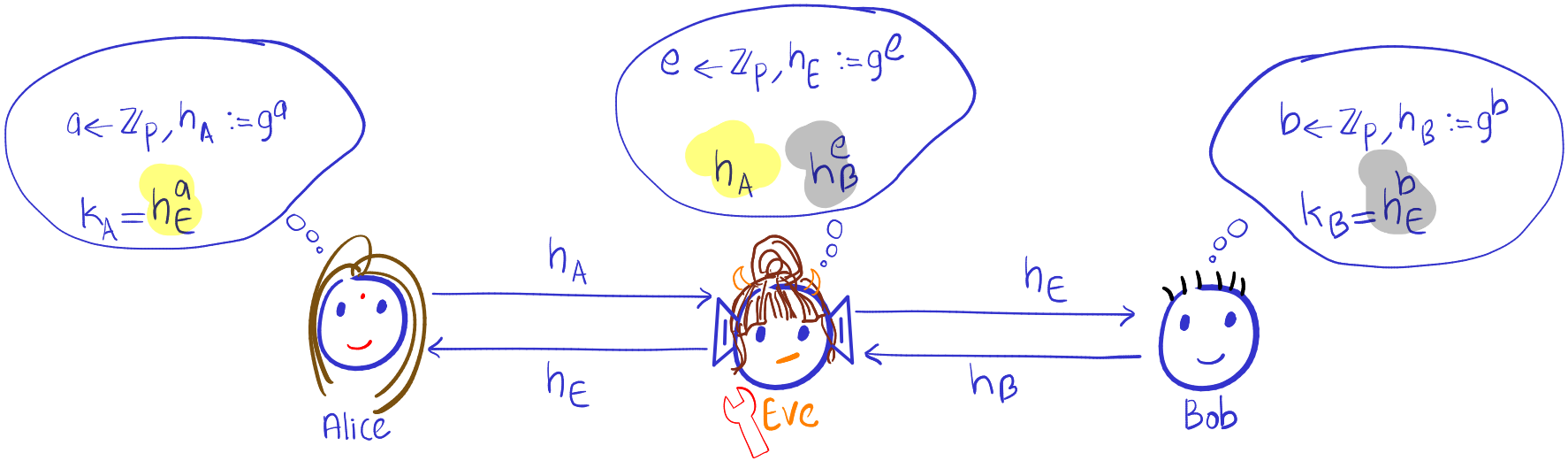

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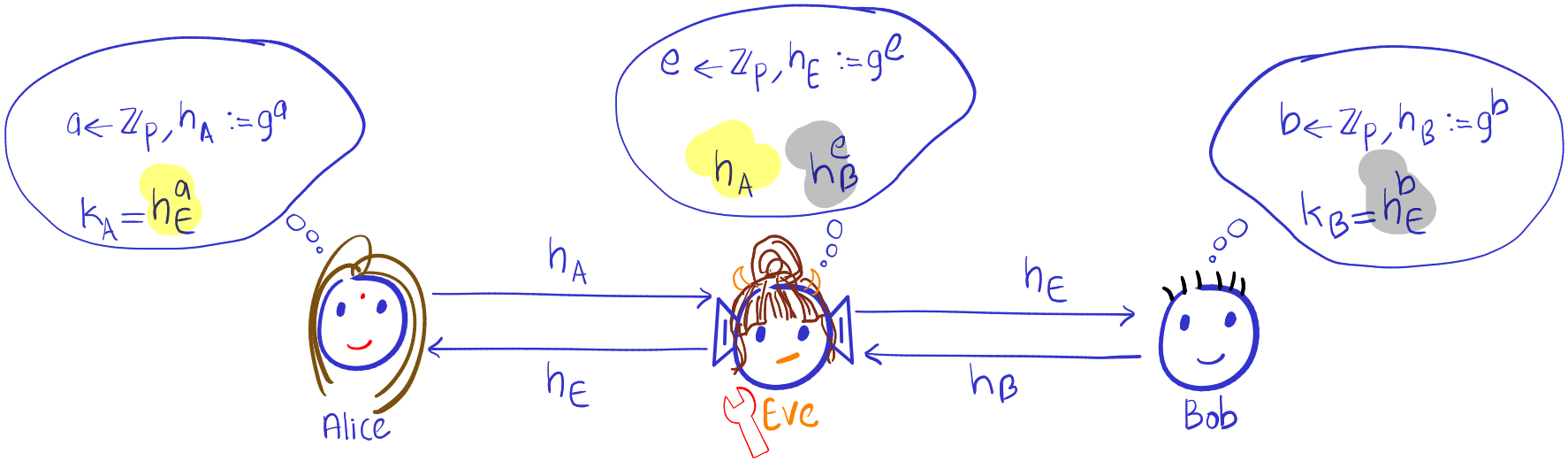

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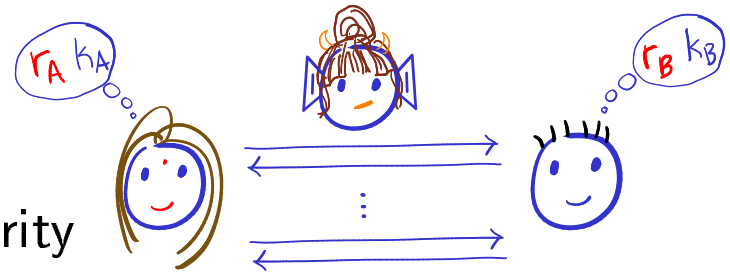
⚠ Insecure against active adversary

Recap/Next Lecture

- Key exchange against eavesdroppers
  - Modelled key exchange setting and security



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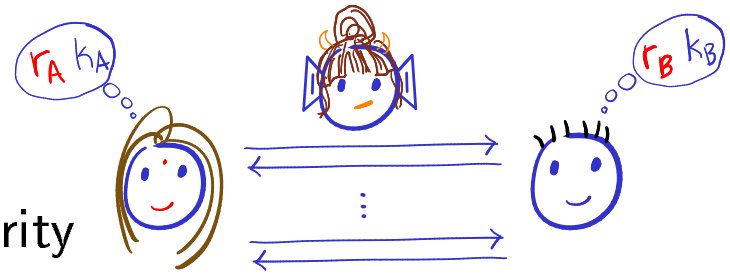

  - Diffie-Hellman key exchange protocol
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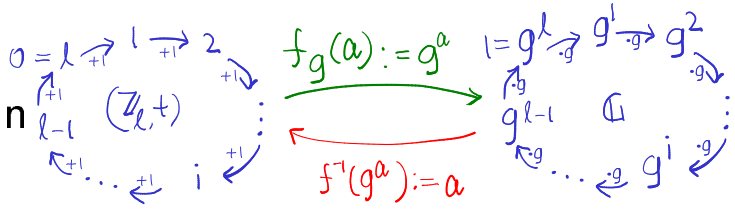

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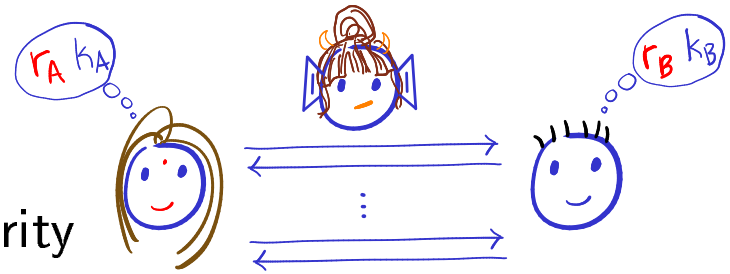

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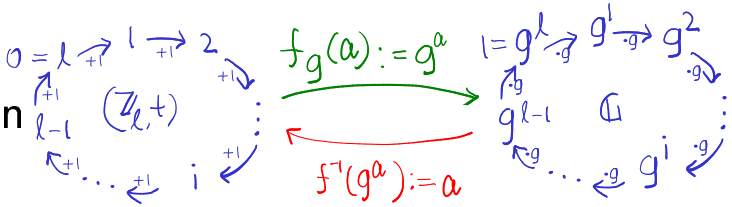
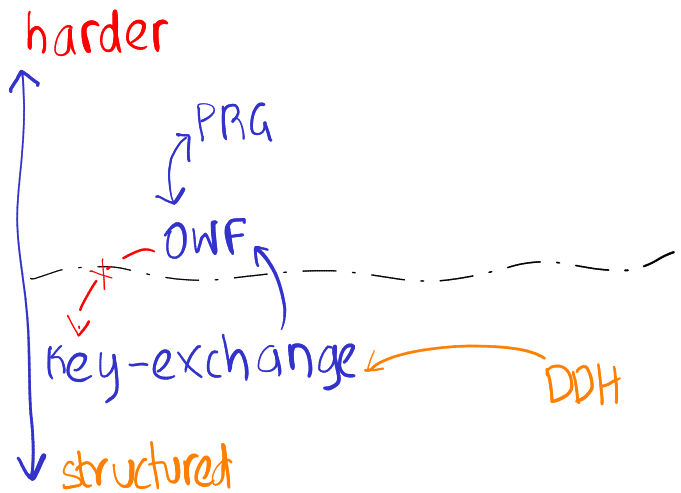


- Takeaway: structure vs hardness
  1. Structure is useful for protocol design and proofs
  2. Also makes it susceptible to algorithms



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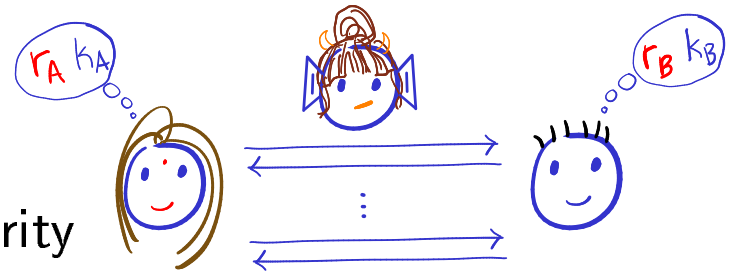

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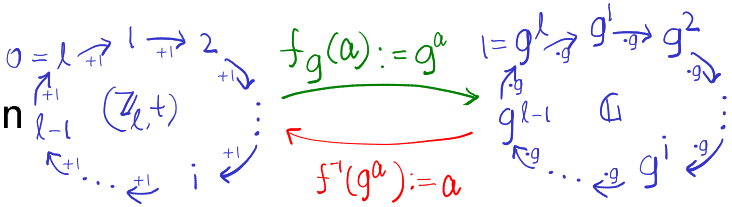

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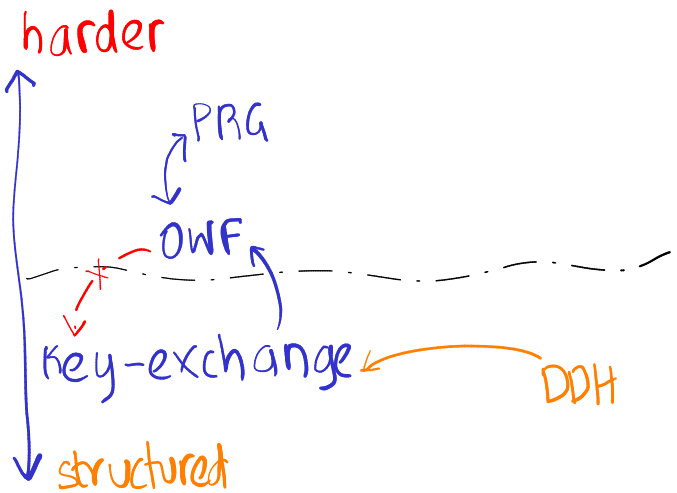

- Next lecture: public-key encryption (PKE)
  - Syntax and security
  - Relationship with key-exchange
  - Elgamal PKE

References

1. Basic group theory and algorithmic number theory can be found in [KL14, Appendix B]. MIT 6875 handout is also an excellent resource.
2. More motivation about groups can be found in Keith Conrad's expository paper on the topic
3. [KL14, Chapter 11] for more details on key exchange
4. Read the seminal paper by Diffie and Hellman [DH76] for a description of the namesake key-exchange. In general this paper is a very insightful read.
5. Boneh's survey [Bon98] is an excellent source on the DDH problem.


Dan Boneh.

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