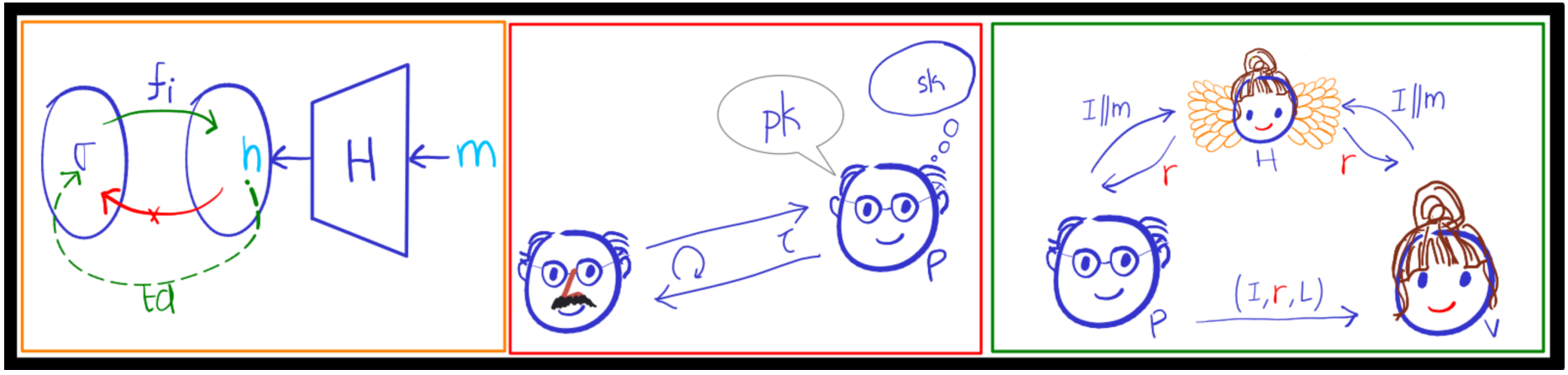



# CS409m: Introduction to Cryptography

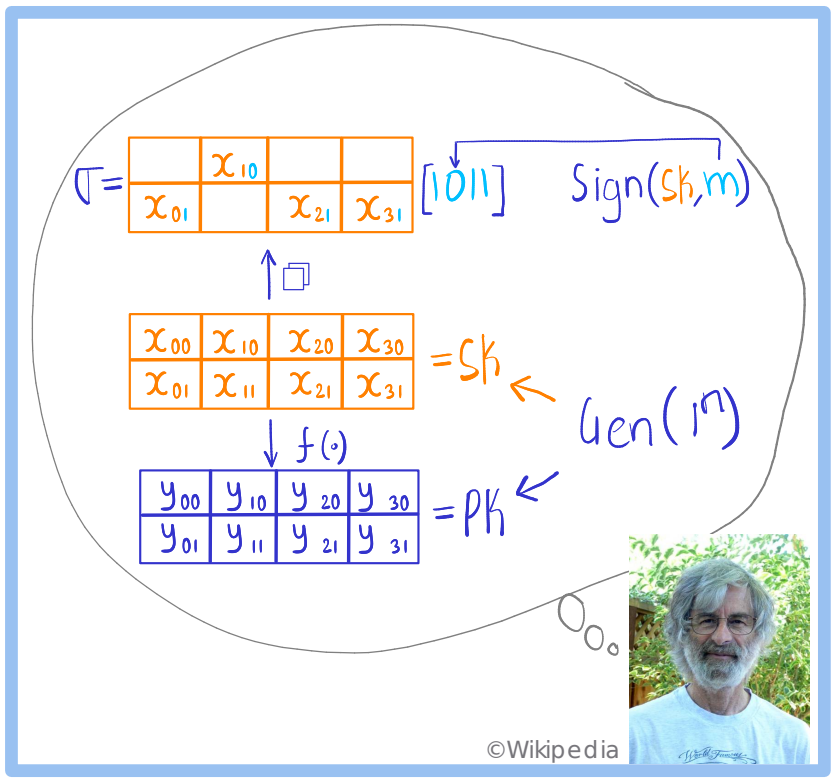
Lecture 17 (15/Oct/25)

Instructor: Chethan Kamath

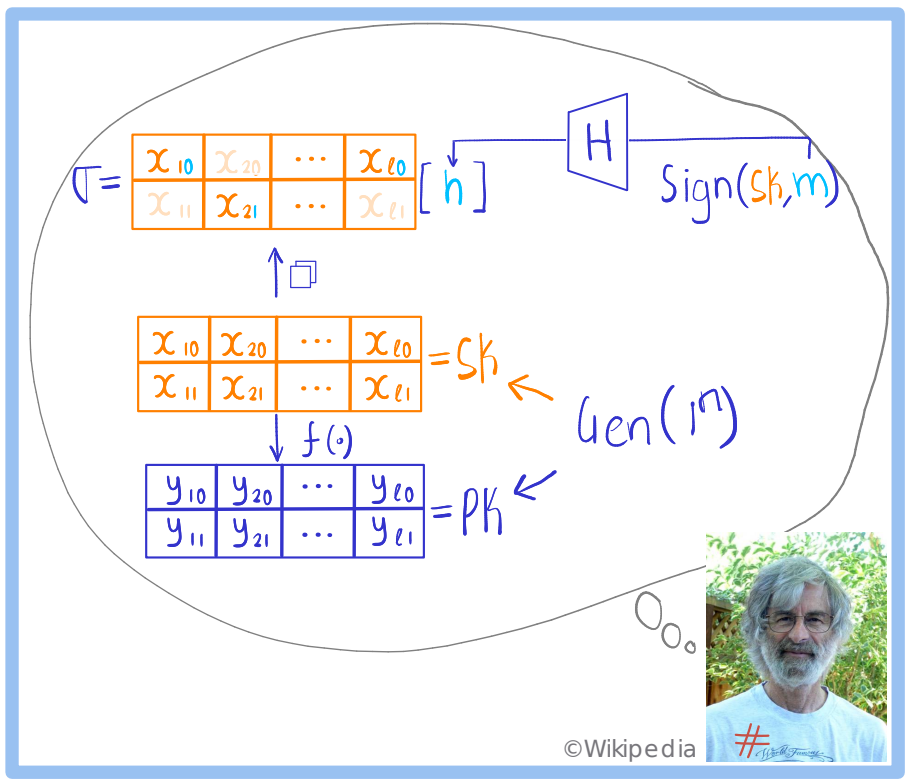
# Recall from Last Two Lectures...

- Primitive: digital signature (DS)
- Threat model: EU-CMA

## Lamport's One-Time DS




## Hash-then-Sign




Tools: OWF, CRHF and CRCF

# How to Construct CRCF in Theory?

- Based on DLP in $\mathbb{Z}_p^\times$: $\{H : (\mathbb{Z}_p^\times)^2 \times \mathbb{Z}_p^2 \to \mathbb{Z}_p^\times\}$, where

$$H_{g,h}(a, b) := g^a h^b \bmod p$$

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  - How to compute discrete-log of $h$ given a collision $((a, b), (a', b'))$?
- Based on subset-sum problem?

$$H_{a_1,\ldots,a_n}(x_1 \| \ldots \| x_n) := \sum_{i \in [1,n]} x_i a_i \bmod p$$

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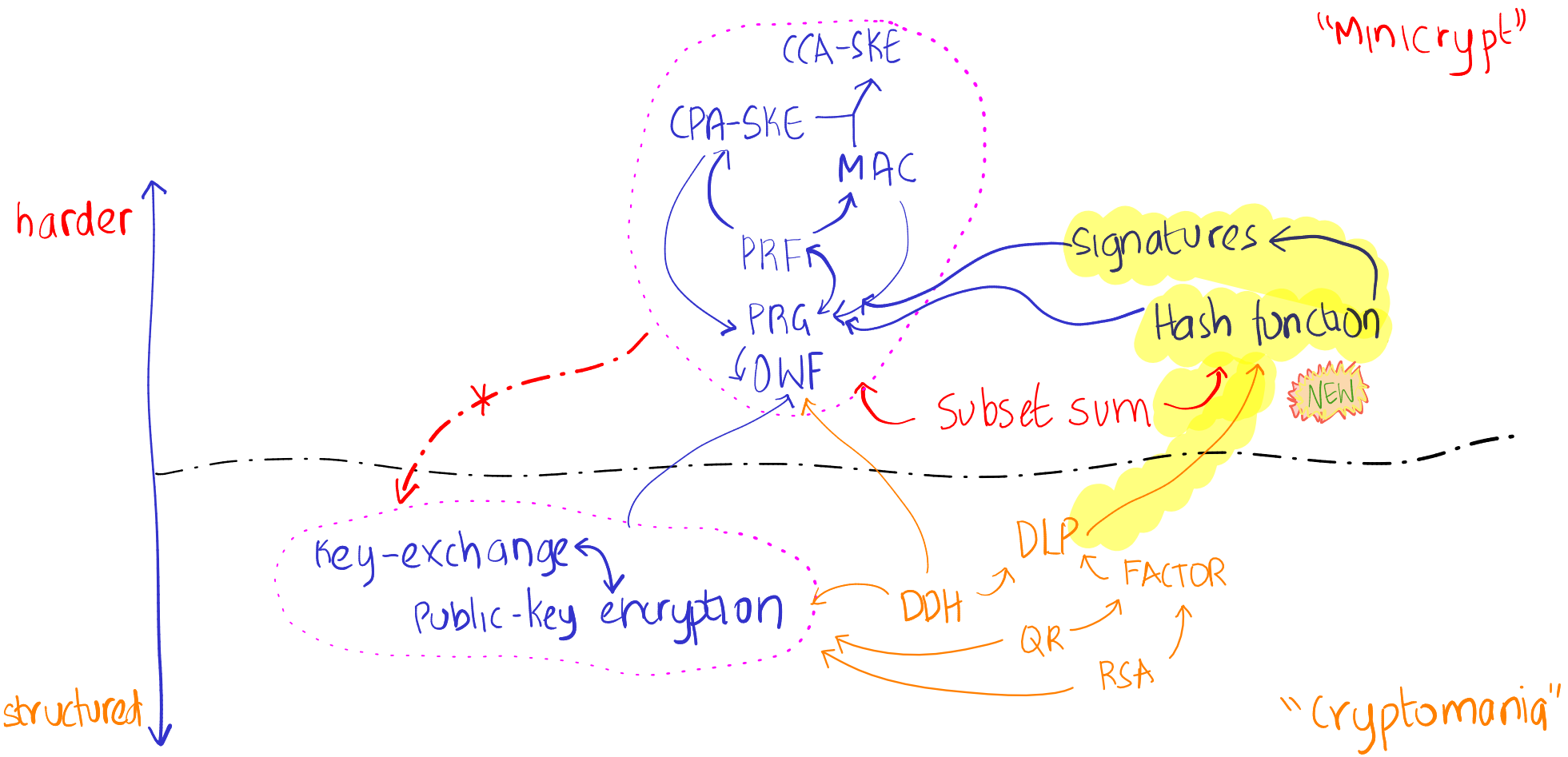
  - When is $H$ compressing? When we set $p < 2^n$
  - How to solve subset-sum given a collision?

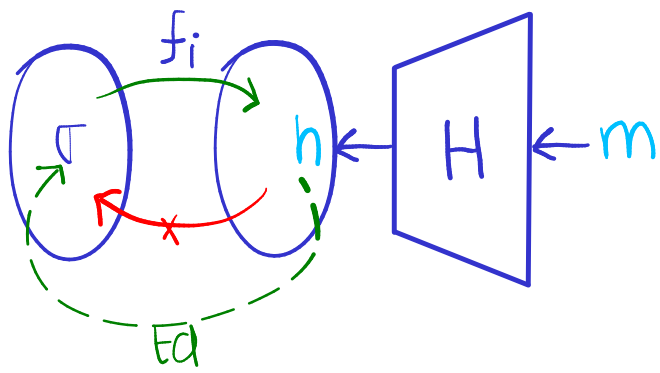
# The Cryptographic Landscape



# Plan for Today's Lecture...

- Task: *efficient* (many-time) digital signatures
- Threat model: EU-CMA

NEW Via Hash-then-Invert





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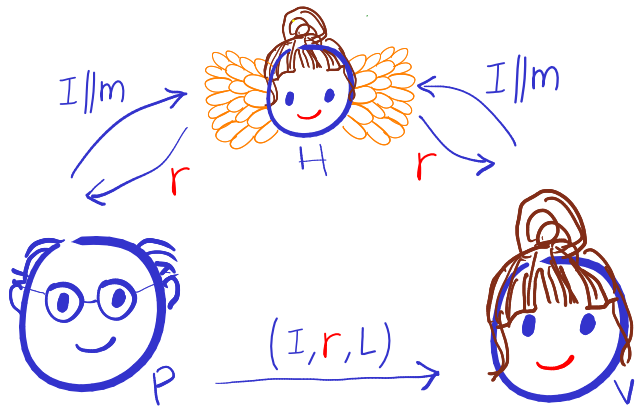
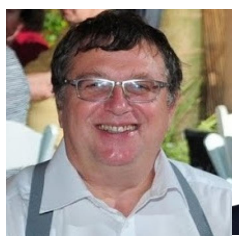
Via Identification Protocols NEW

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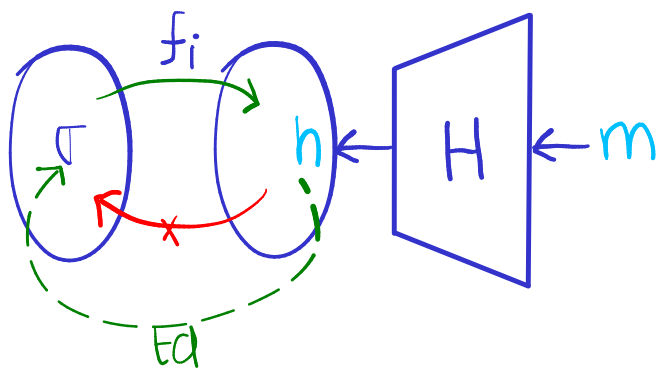


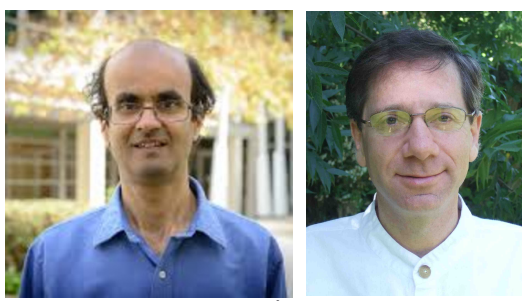

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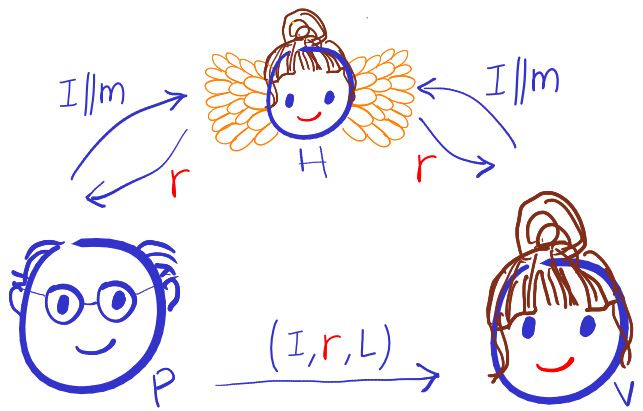


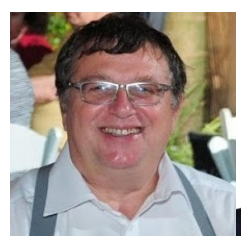

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Tools: Trapdoor perm., plug-and-pray, Fiat-Shamir Transform

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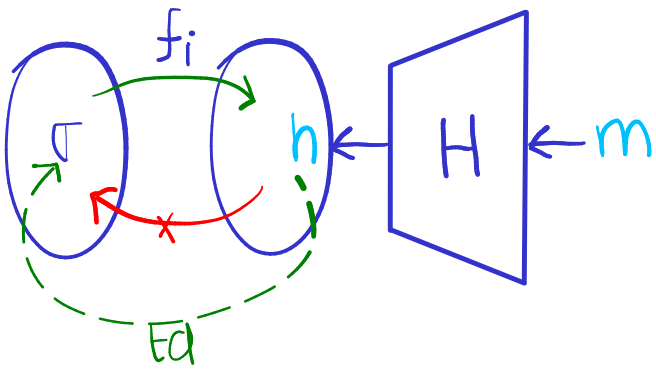


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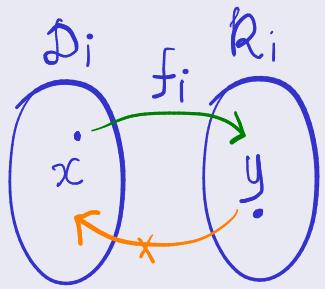
# *Collection* of OWFs

## Definition 1 (One-way function (OWF) collection)

A collection of functions $f := \{f_i : \mathcal{D}_i \to \mathcal{R}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ is one-way if

1. There is an efficient index-sampling algorithm Index
2. Each $f_i$ in collection is efficiently computable
3. For all PPT *inverters* Inv, the following is negligible:

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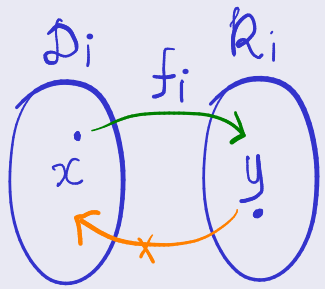


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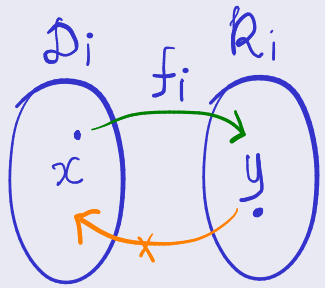


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Recall examples:

1. RSA function (power mod semiprime $N = pq$): $f_{N,e}(x) := x^e \bmod N$
2. Exponentiation function (modulo prime $p$): $f_{p,g}(x) := g^x \bmod p$

# Collection of *Trapdoor* OWP...

## Definition 2 (Trapdoor (one-way) permutation (TDP) collection)

A collection of permutations $f = \{f_i : \mathcal{D}_i \to \mathcal{D}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ is *trapdoor one-way* if

1. There is an efficient index+trapdoor sampling algorithm Index
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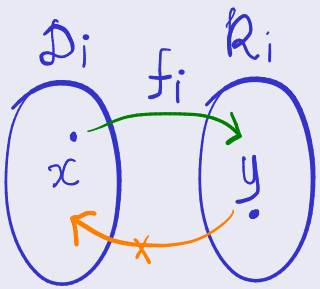


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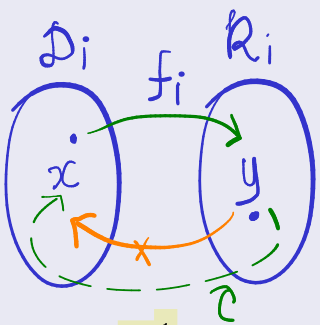


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4. $f_i^{-1}$ can be efficiently computed given trapdoor $\tau$ for $i$

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- RSA function $\left\{f_{N,e} : \mathbb{Z}_N^\times \to \mathbb{Z}_N^\times\right\}_{N,e}$, defined as

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  - $f_{N,e}$ is permutation when $GCD(e, (p-1)(q-1)) = 1$
  - One-way by RSA assumption

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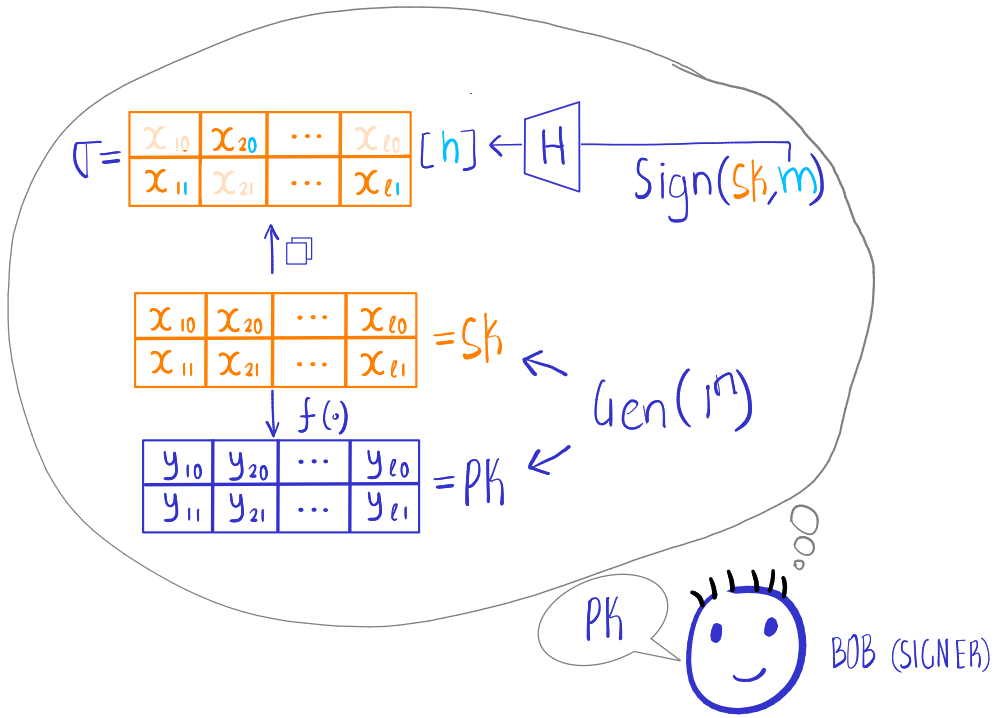
  - ⚠ We don't know a trapdoor!

# Recall: "Hash-Then-Sign" Paradigm

- 1) Compute "hash" $h = H(k, m)$ 2) sign $h$ using Lamport's OTDS



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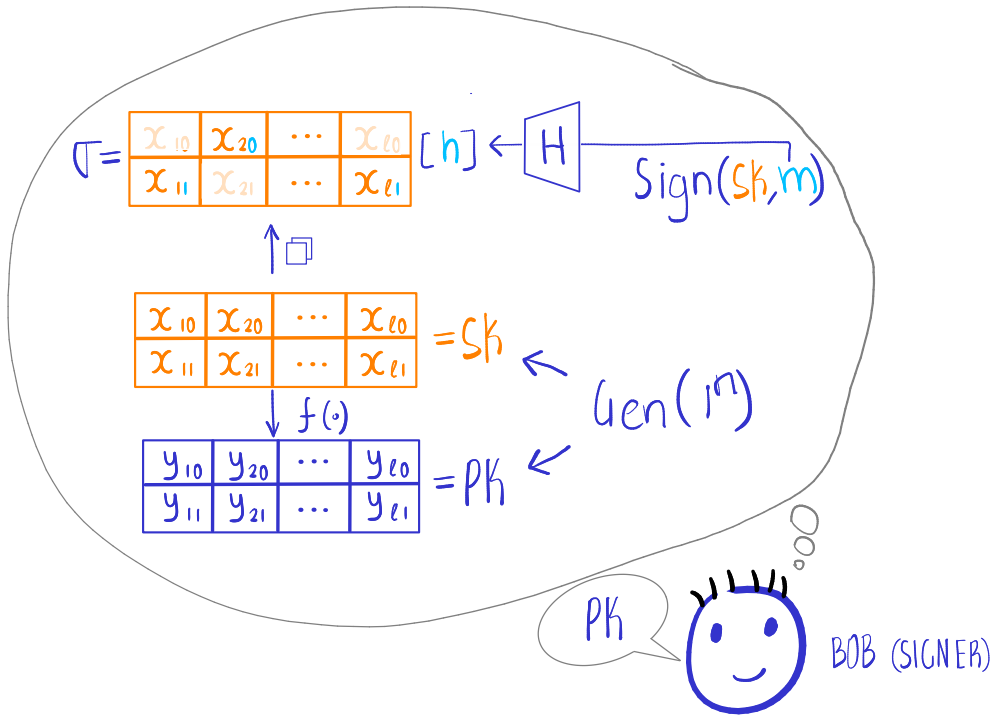


**Theorem 4 (from Lecture 16, rephrased)**

*If Lamport's scheme is OTDS and $H$ is CRHF then "hash-then-sign" scheme is a one-time EU-CMA for arbitrarily-long messages.*

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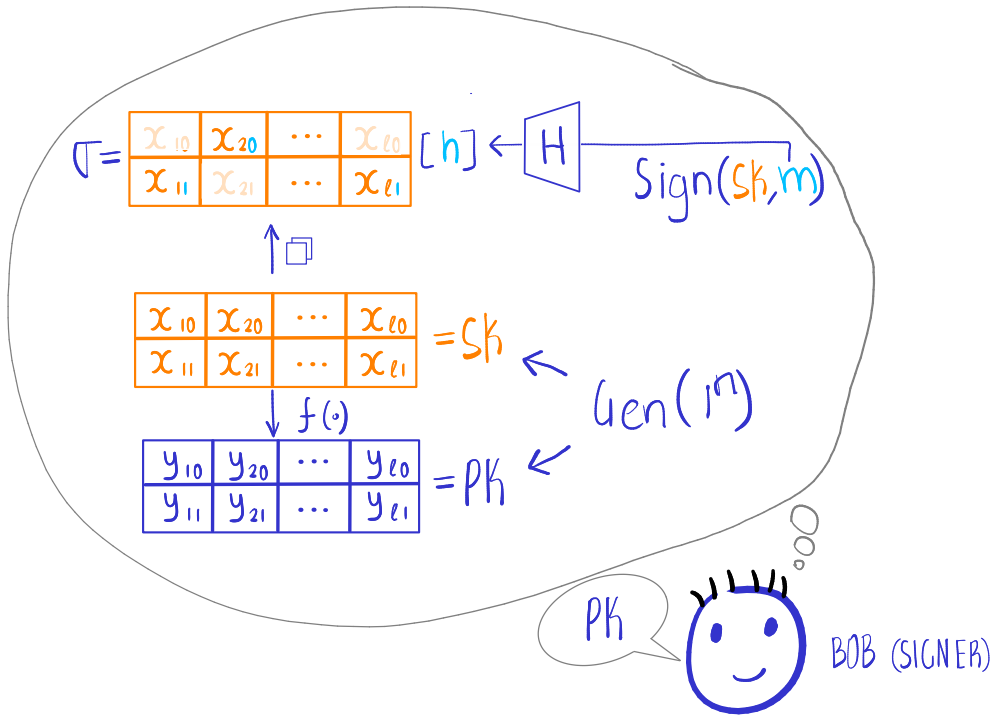


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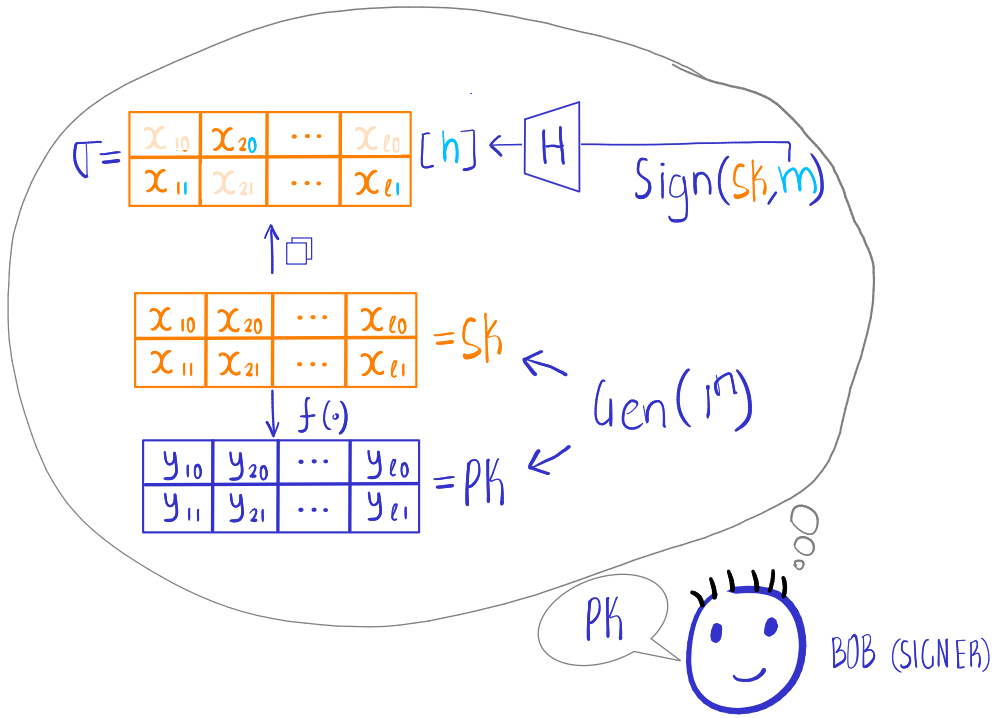


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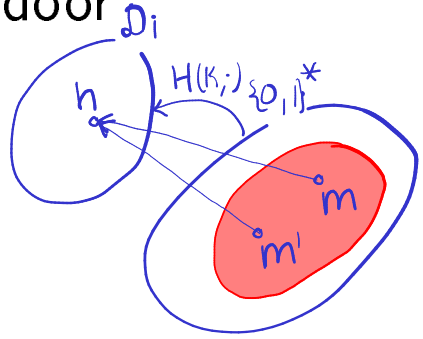
How can a TDP be useful here? To *replace* Lamport's OTS!

# TDP-based Signature: "Hash-then-Invert"

1) Compute "hash" $h = H(k, m)$ 2) *invert* $h$ using trapdoor

- "Full domain" hash function $H : \mathcal{K} \times \{0,1\}^* \to \mathcal{D}_i$



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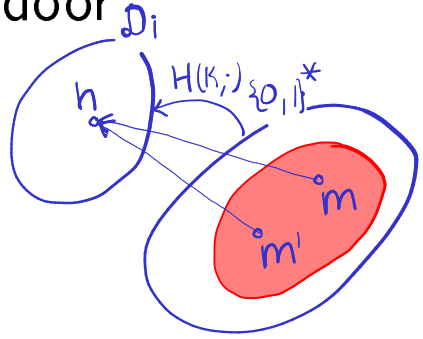



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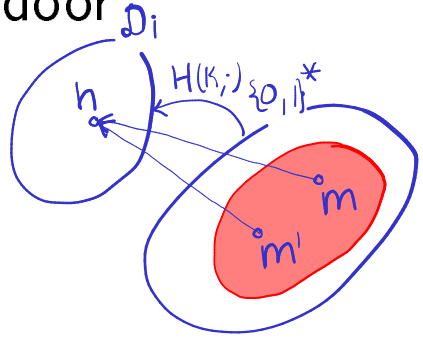


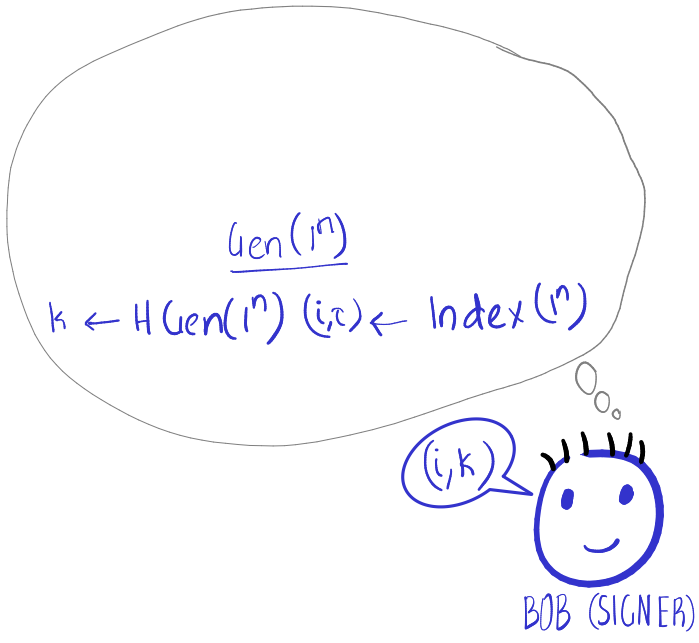

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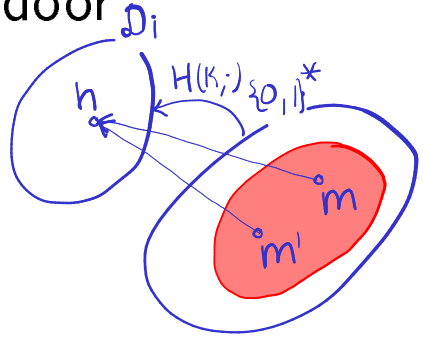


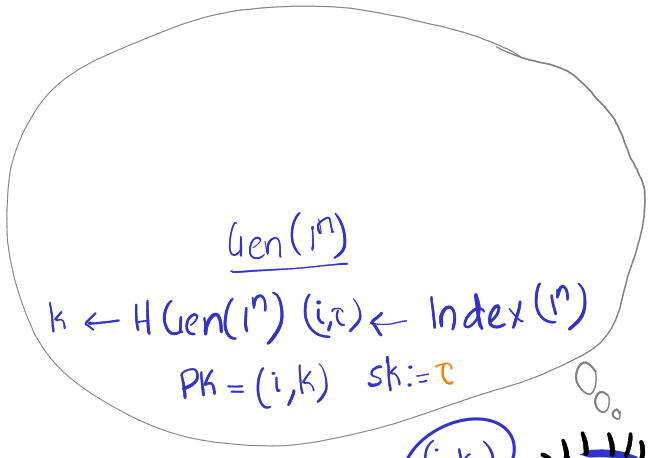

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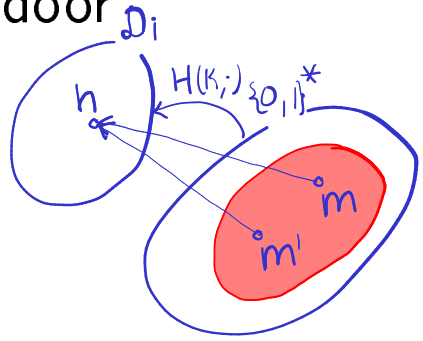


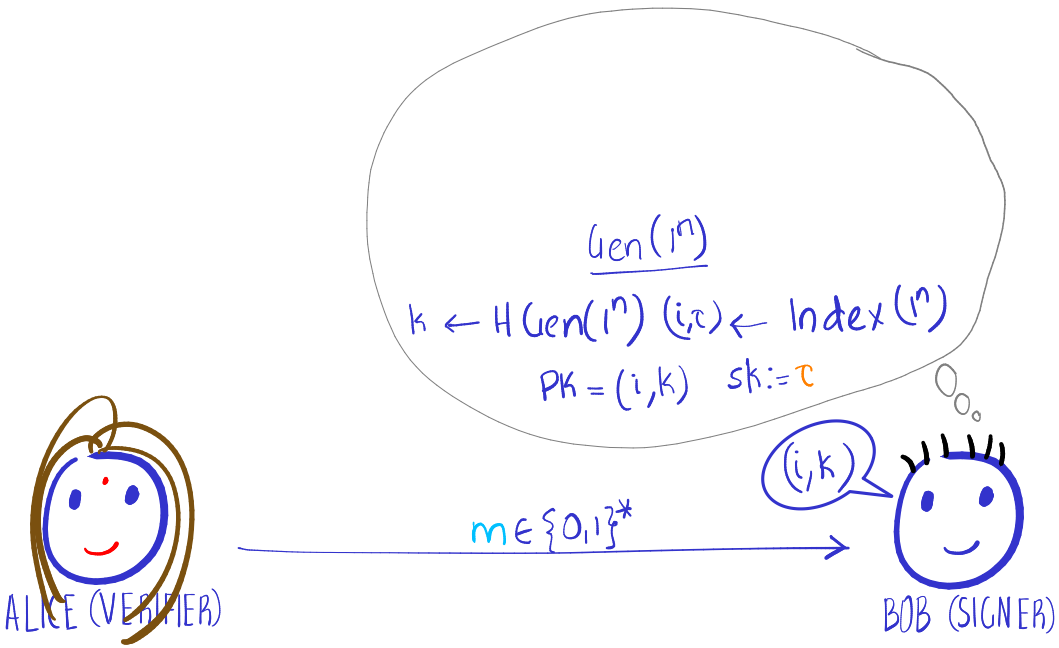

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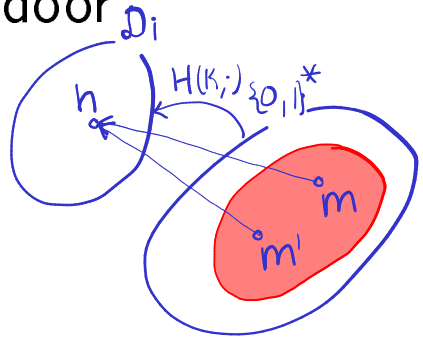


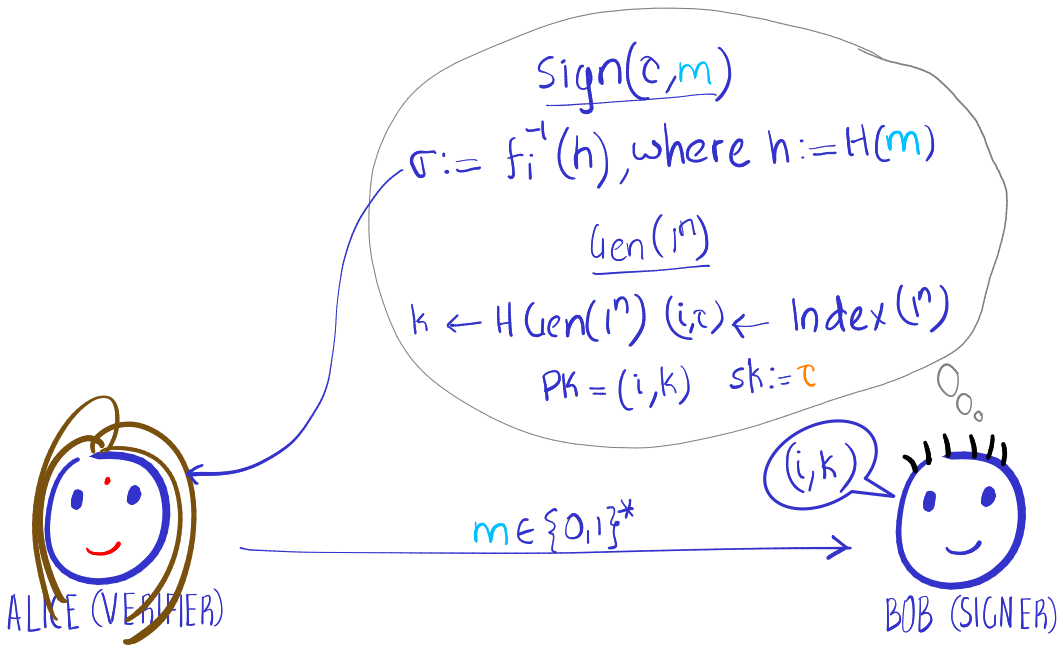

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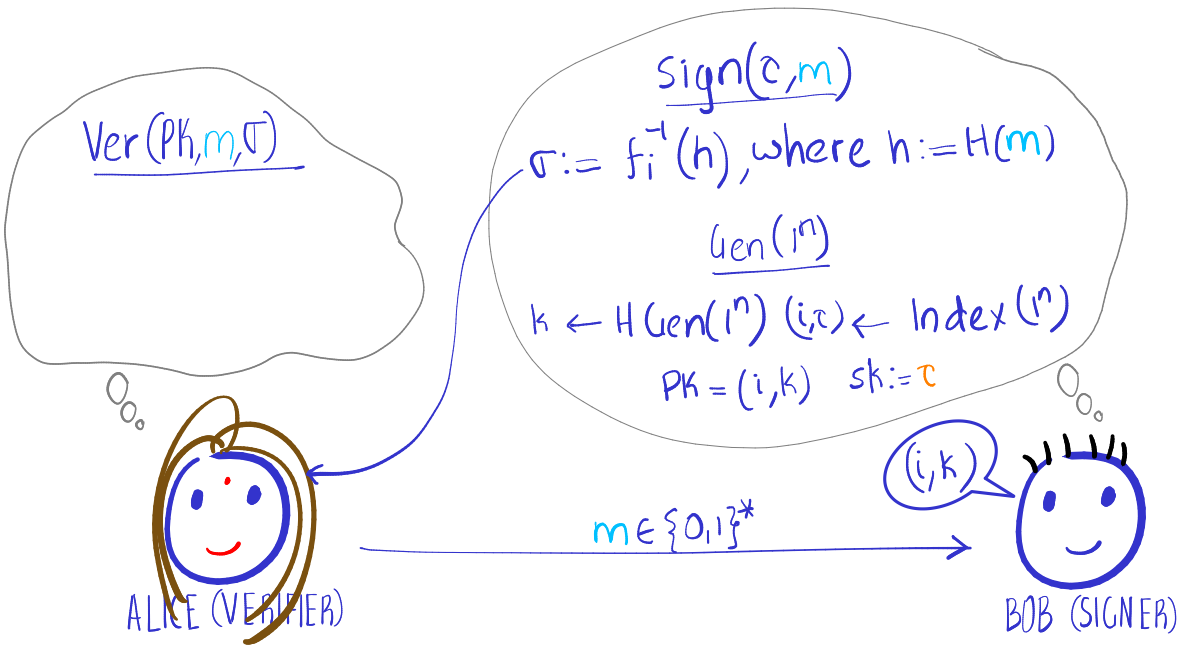


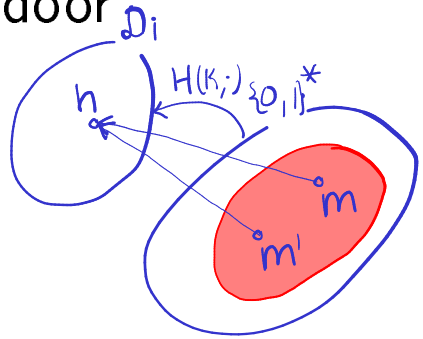

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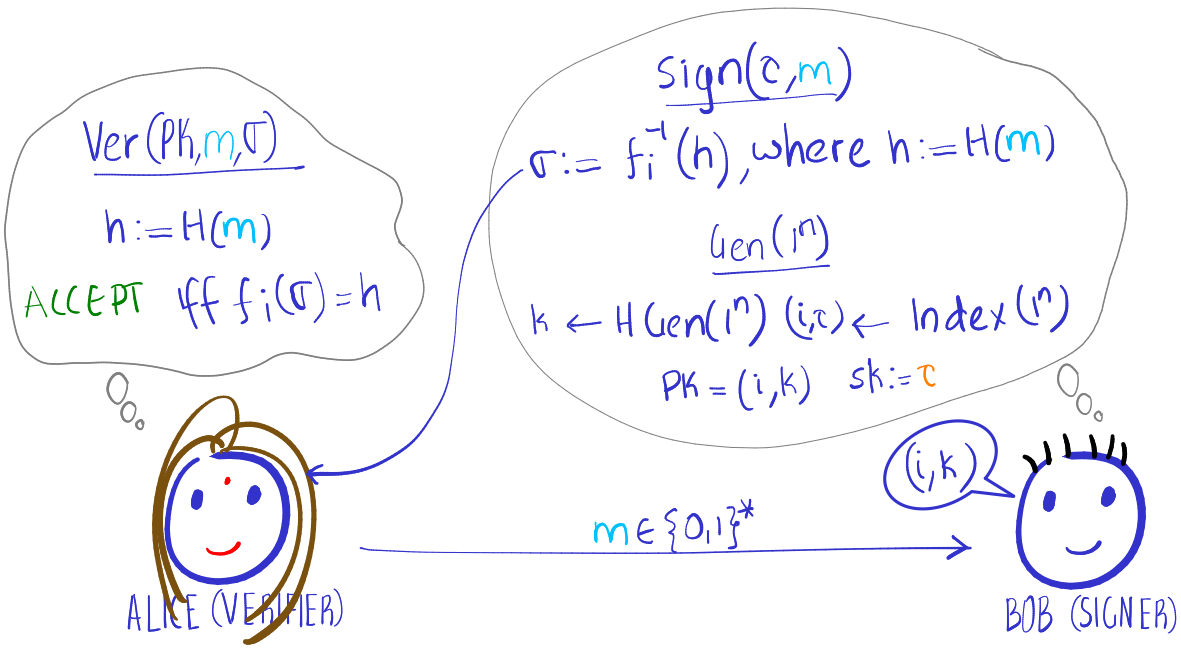


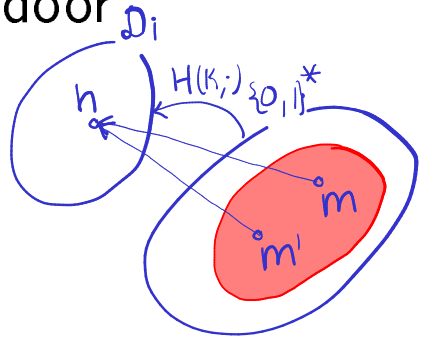

# TDP-based Signature: "Hash-then-Invert"

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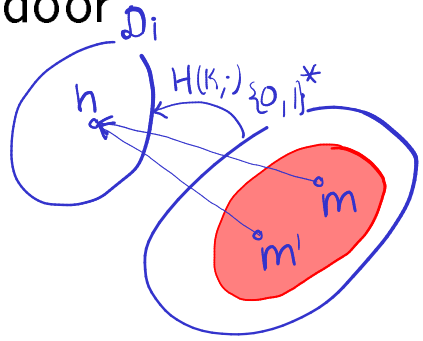


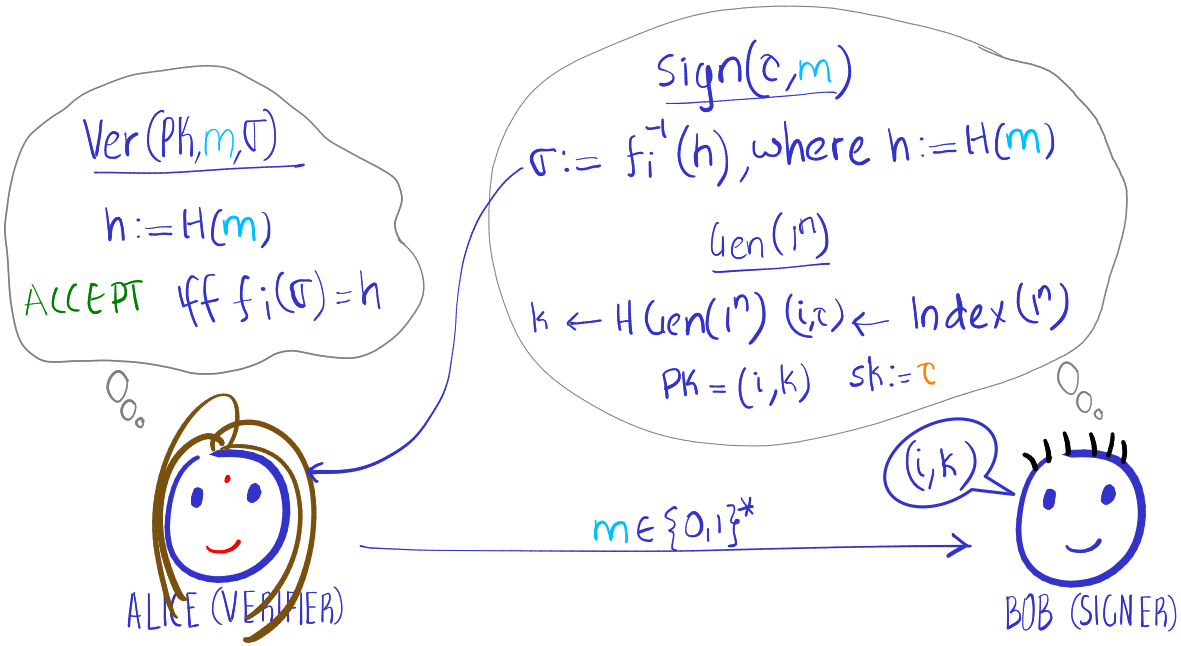


Efficiency, when using RSA function (i.e., RSA-FDH)

$$f_{N,e}(x) := x^e \bmod N$$

- Public key: $(N, e)$ and description of $H$
- Signature: one element of $\mathbb{Z}_N^\times$
- Signing/verification: one exponentiation + hash evaluation

# Let's Prove Security of "Hash-then-Invert" ...

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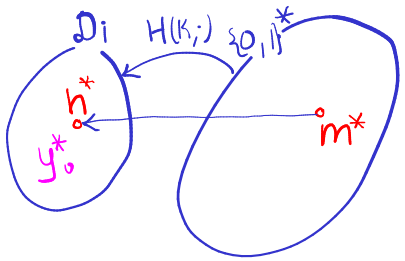

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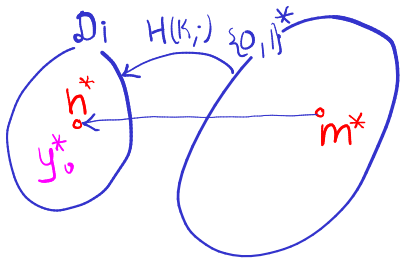

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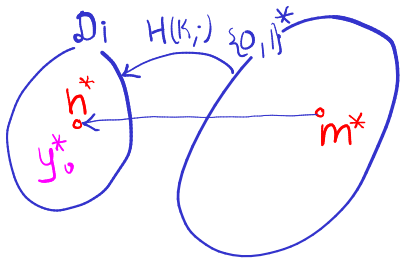


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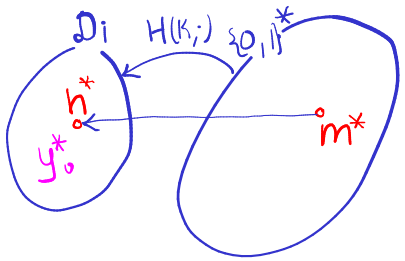


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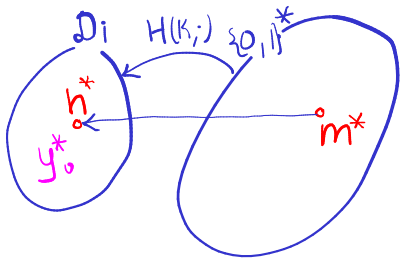


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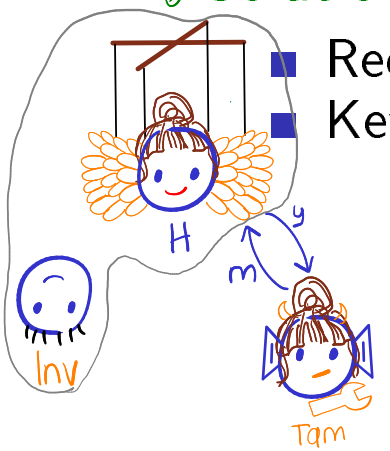


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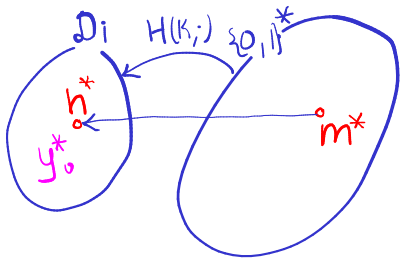


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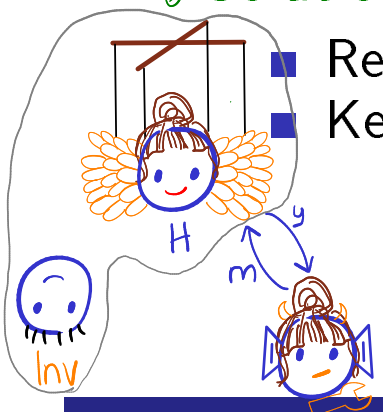


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**Theorem 5**

*If $f$ is a TDP and $H$ is a random oracle then "hash-then-invert" is EU-CMA for arbitrarily-long messages.*

- Proof uses "plug and pray" (on the whiteboard)

# Plan for Today's Lecture...

- Task: *efficient* (many-time) digital signatures
- Threat model: EU-CMA

NEW Via Hash-then-Invert




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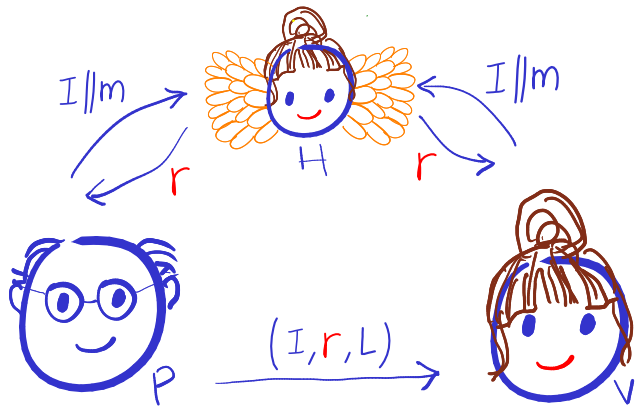
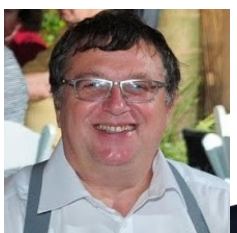
Via Identification Protocols NEW



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Tools: Trapdoor perm., plug-and-pray, Fiat-Shamir Transform

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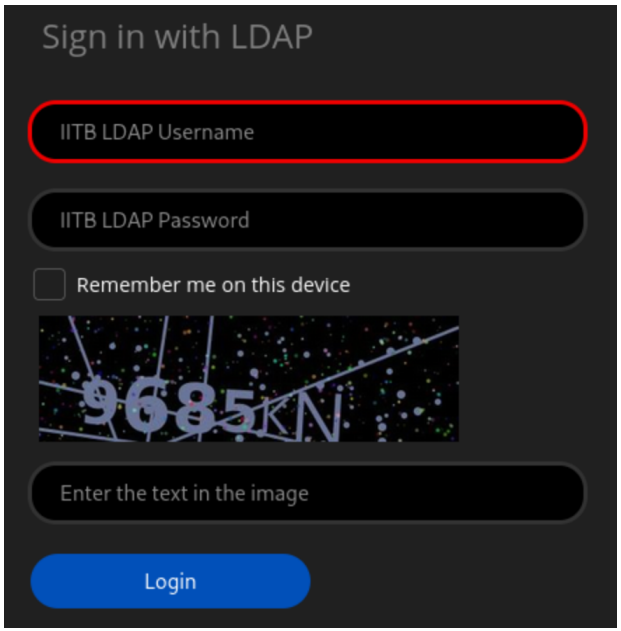

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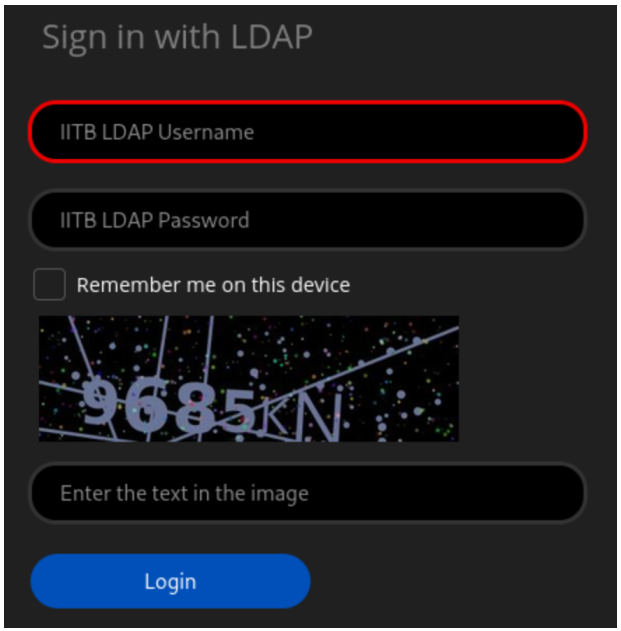


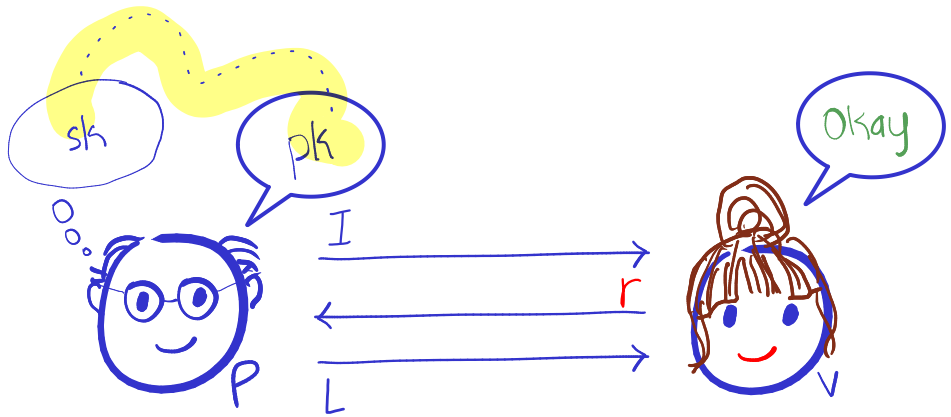


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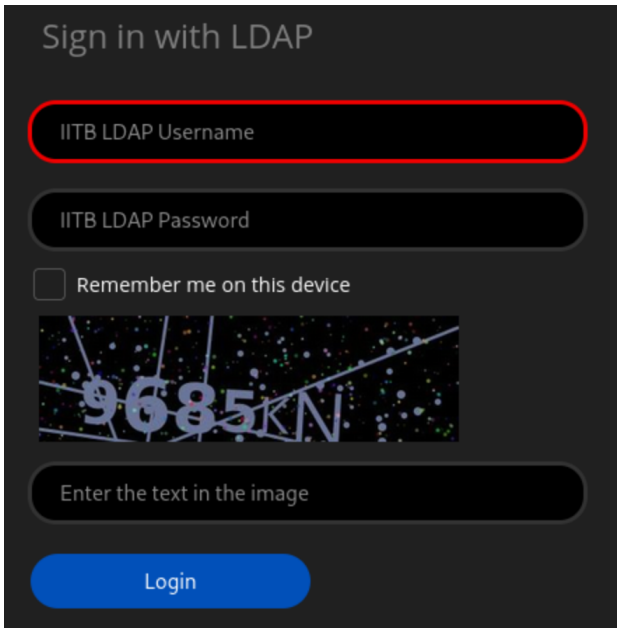


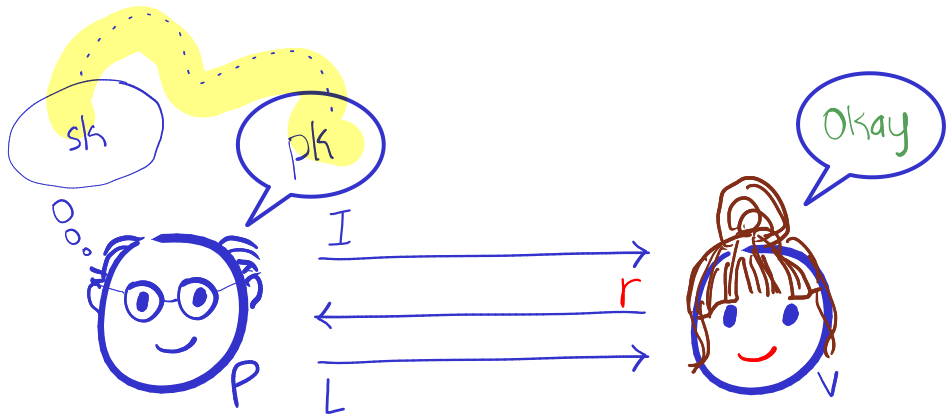


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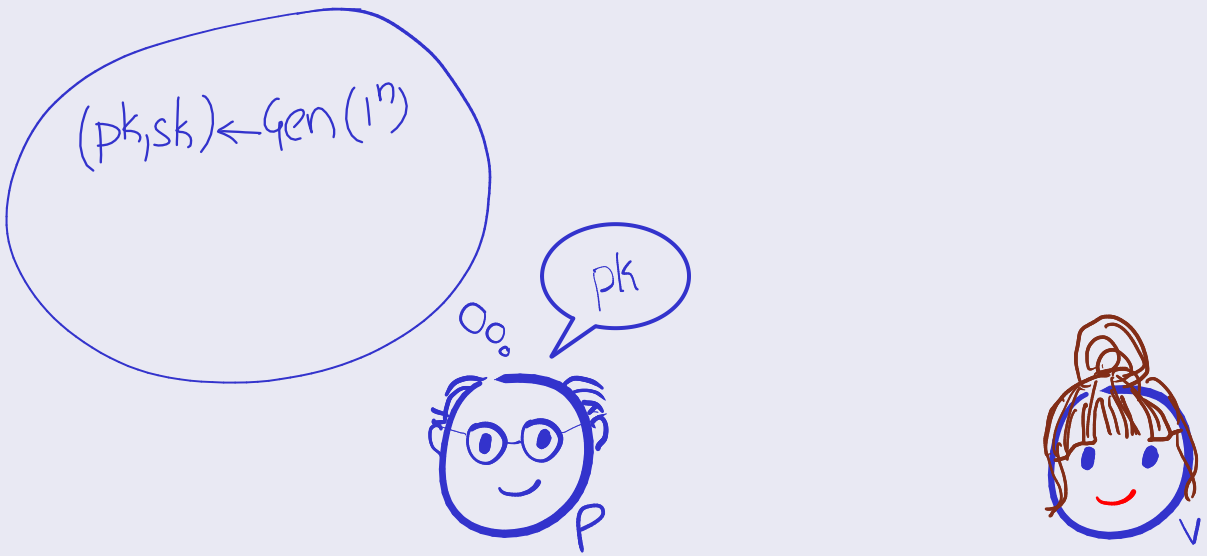
- E.g.: SSHing into a remote server

# Let's Look at the Syntax

## Definition 3 ((Three-Round) Identification (ID) Protocol...)

... $\Pi$ is a triple of PPT algorithms $(\mathsf{Gen}, \mathsf{P} = (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V})$ with the following syntax:



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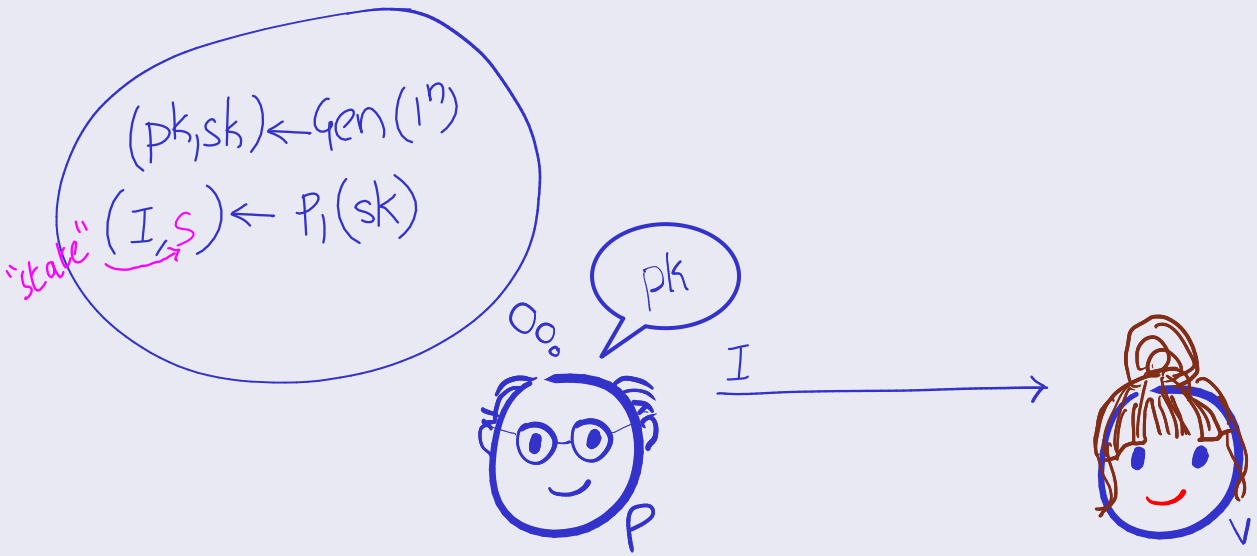

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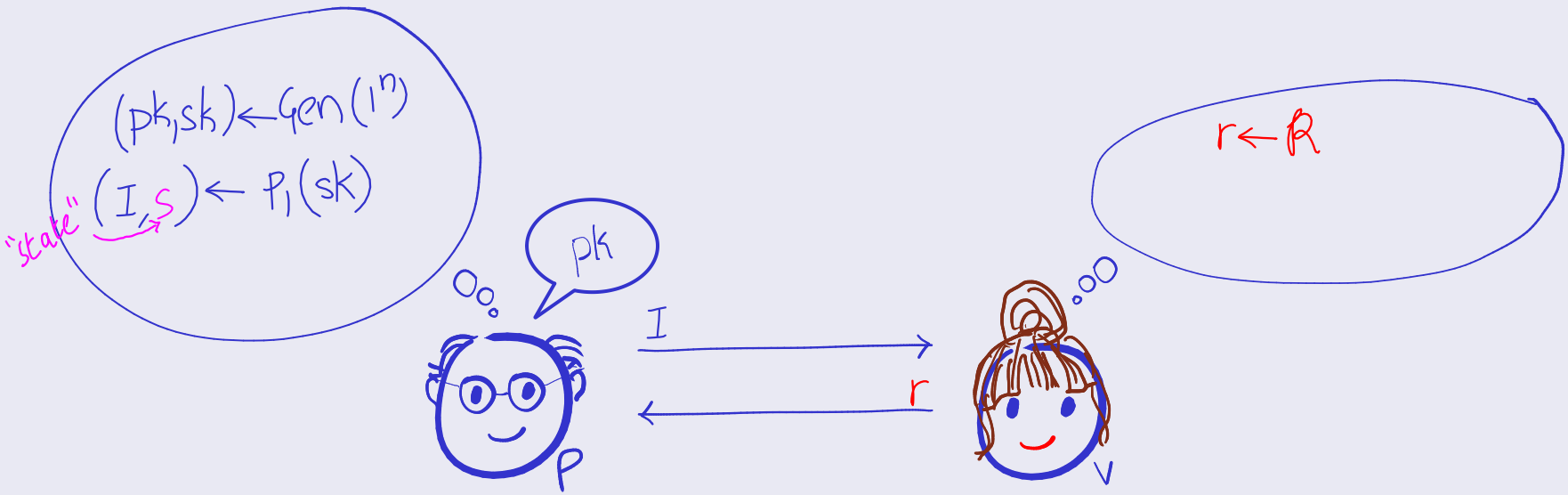

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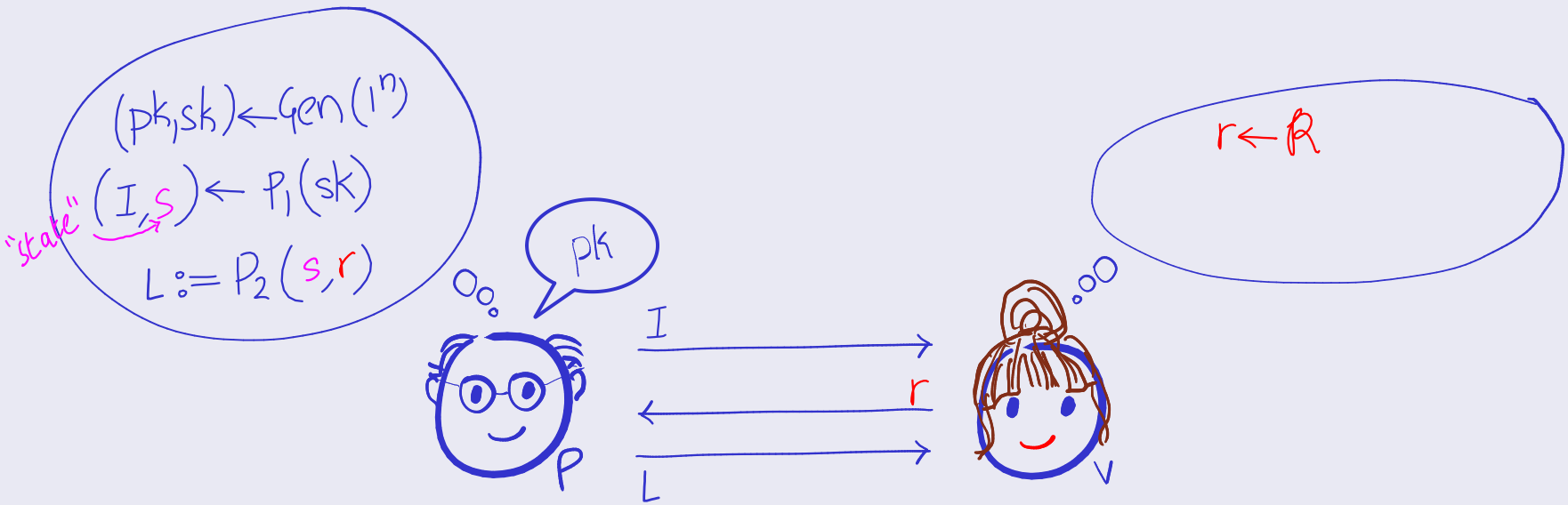

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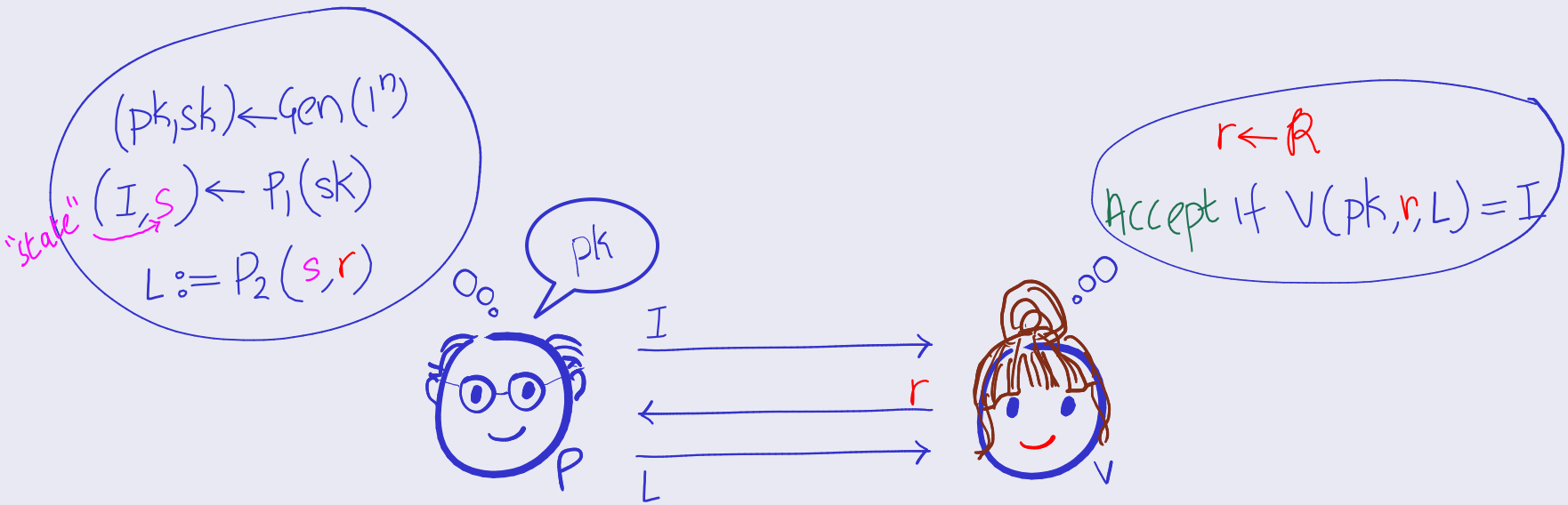

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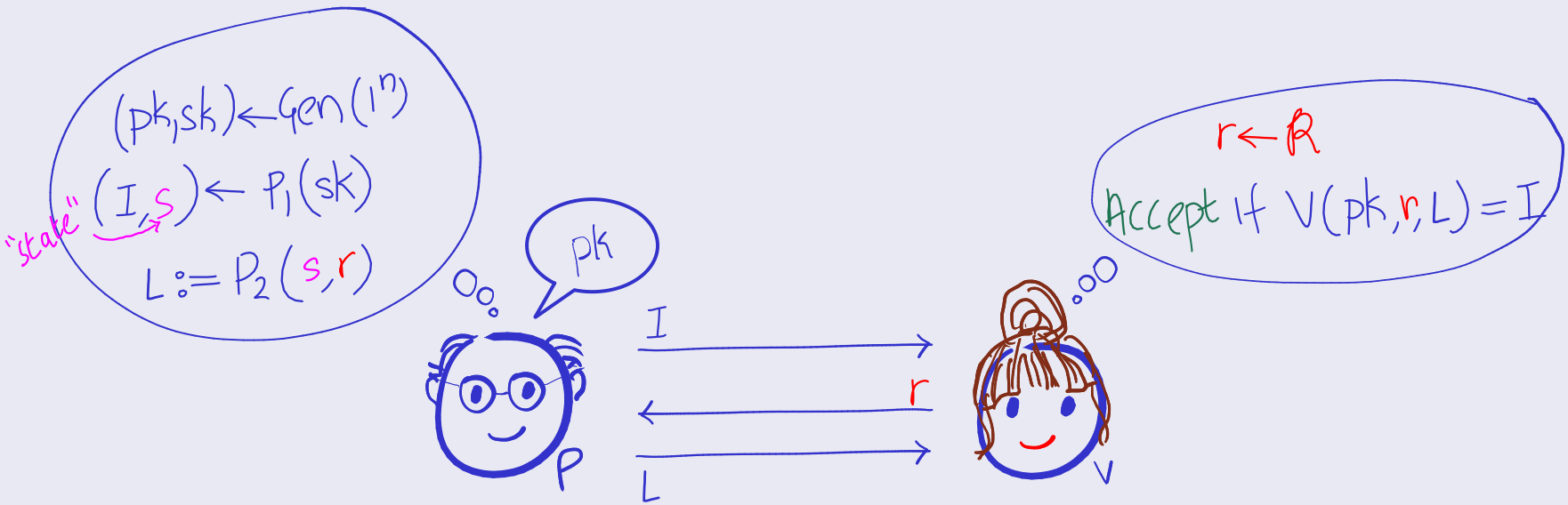


## Exercise 1

Define the correctness requirement

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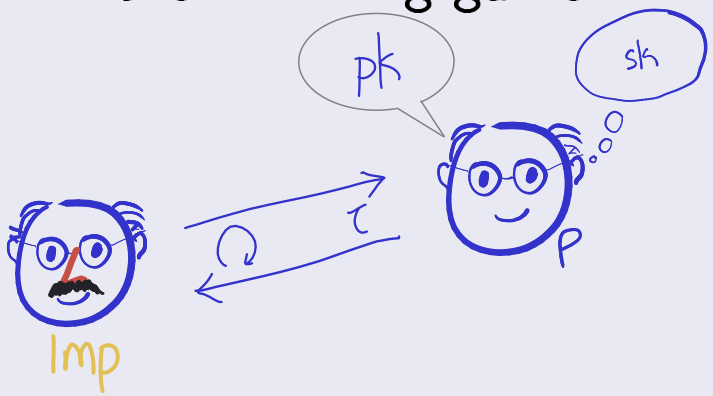

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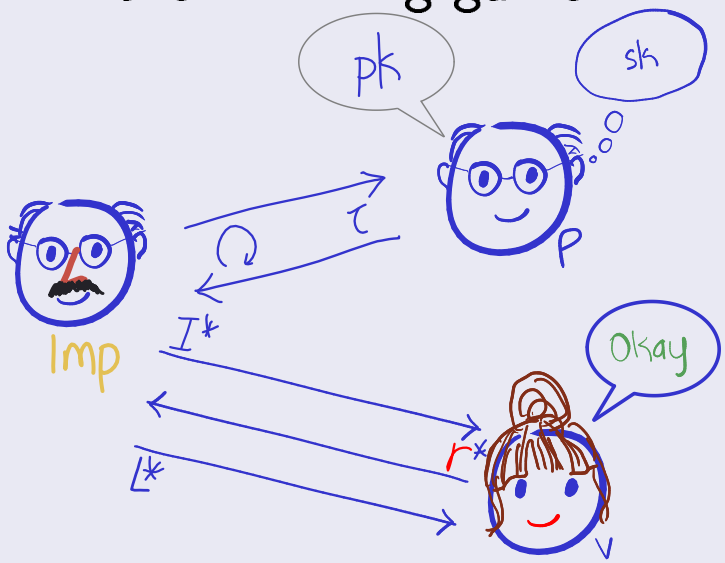
3. When Imp sends $I^*$, respond with $r^* \leftarrow \mathcal{R}$
4. Imp responds with $L^*$ and breaks $\Pi$ if $\mathsf{V}(pk, r^*, L^*) = I^*$



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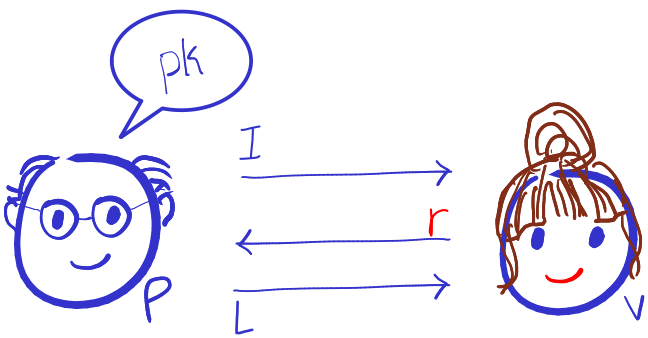
Intuition: ID protocol "proves" P's knowledge of $sk$



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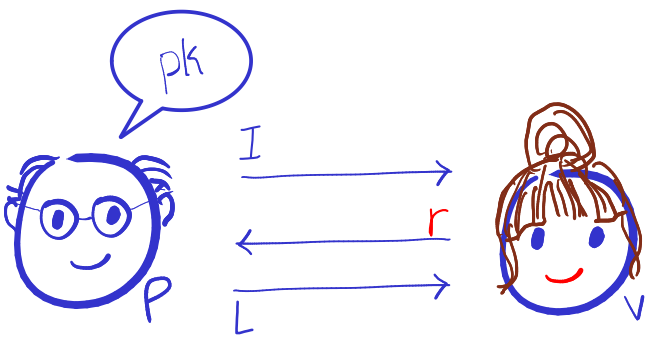

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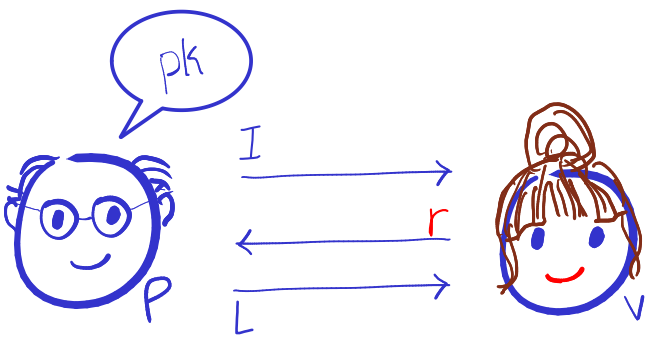

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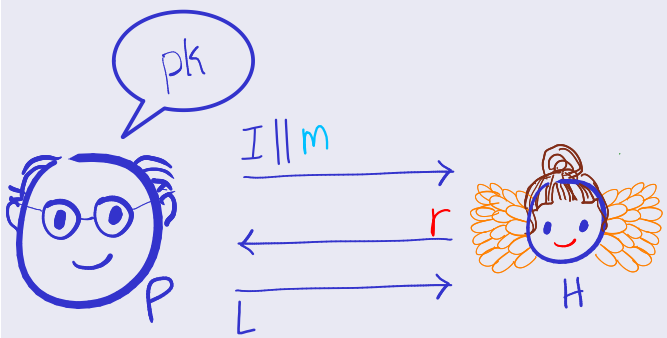


- $\mathsf{Sign}^H(sk, m)$: output $\sigma := (I, r, L)$, where $(I, s) \leftarrow \mathsf{P}_1(sk)$, $r := H(I \| m)$ and $L := \mathsf{P}_2(s, r)$

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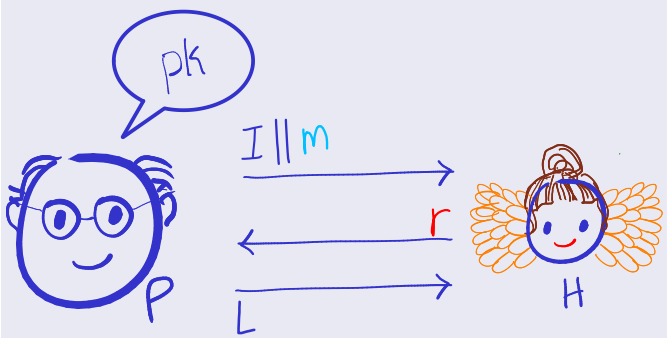


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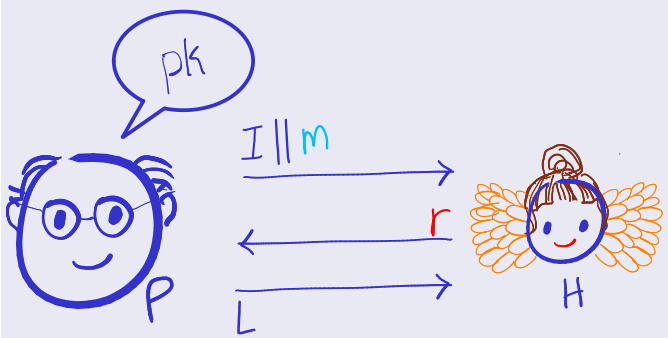


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## Theorem 6

*If $\Pi$ is secure against passive attacks, then $\Sigma$ is EU-CMA secure if $H$ is modelled as random oracle*

# Schnorr's ID Protocol...

Recall Elgamal PKE over group $\mathbb{G}$ of prime order $\ell$

- Gen: public key is $h := g^x$ and secret key is $x \in \mathbb{Z}_\ell$

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  - Gen: public key is $h := g^x$ and secret key is $x \in \mathbb{Z}_\ell$
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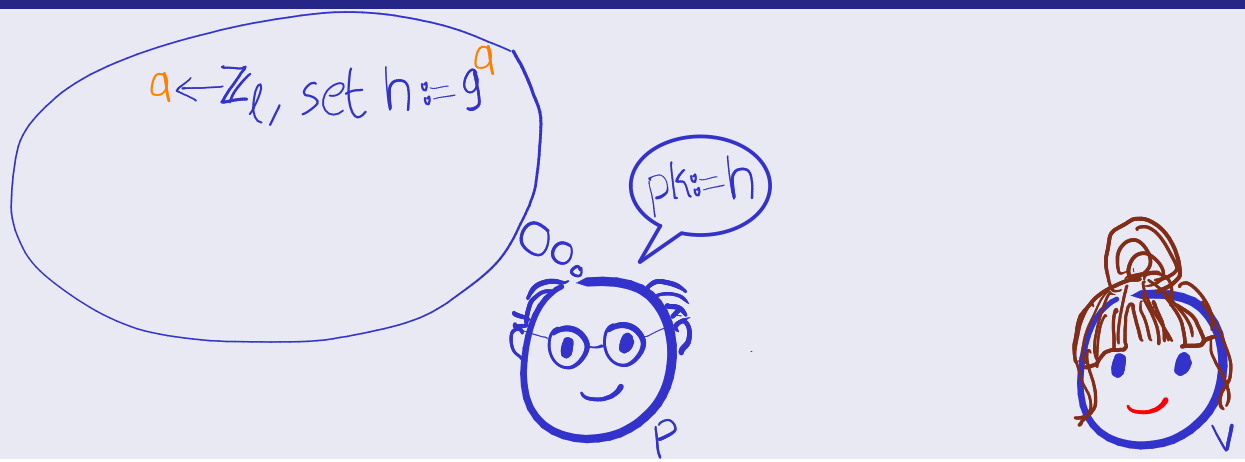

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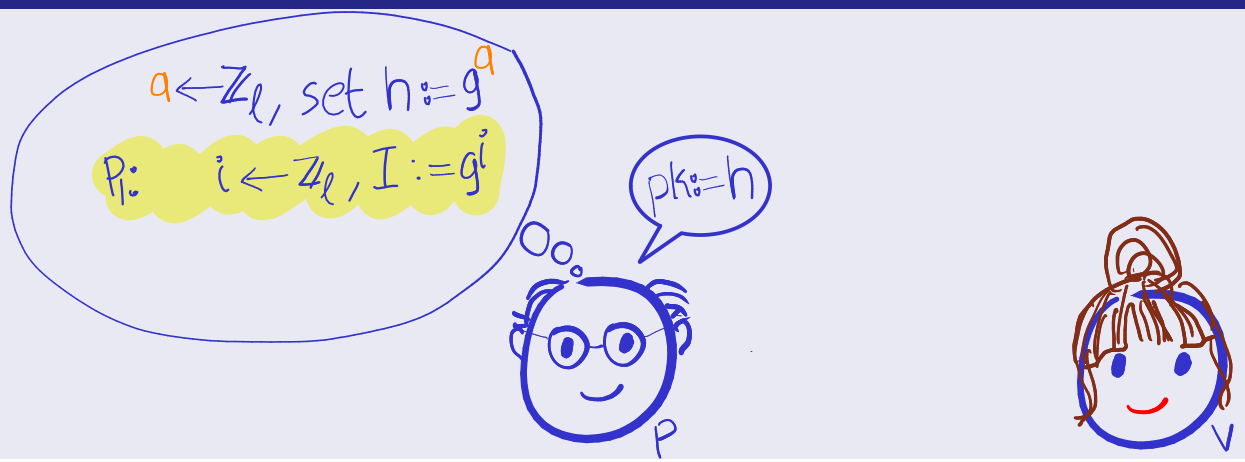

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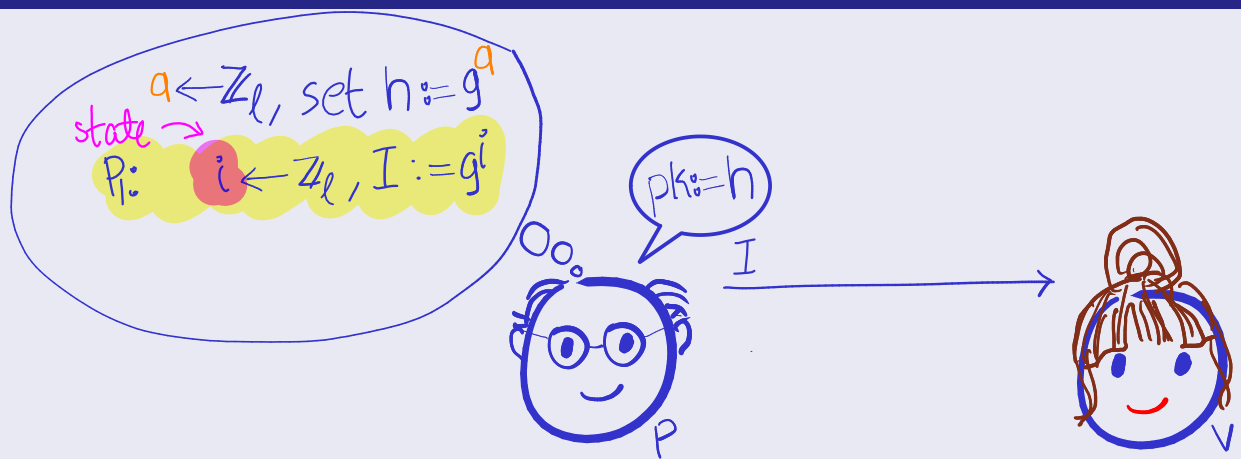

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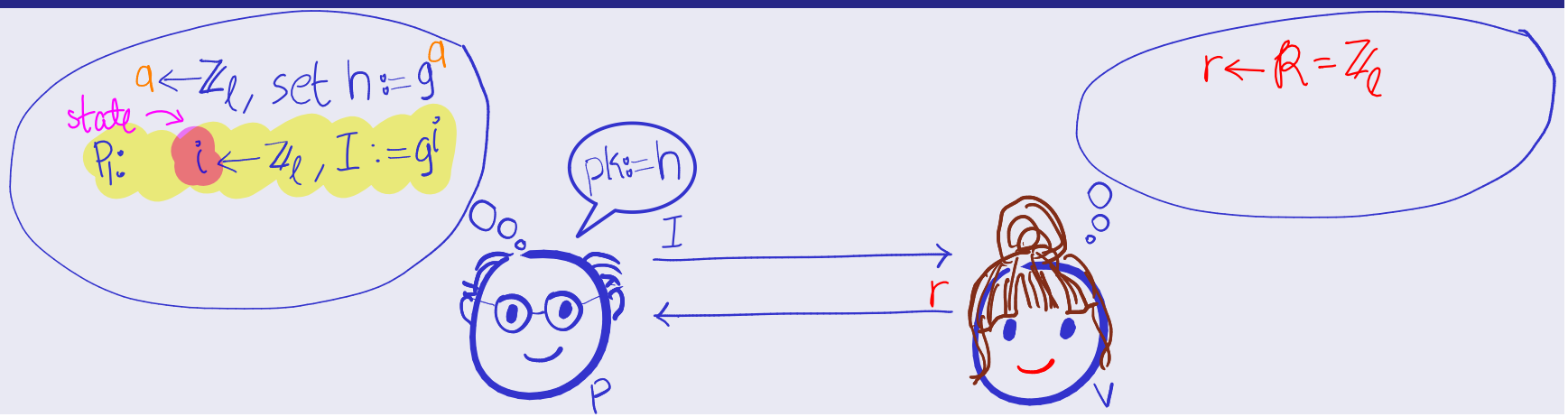

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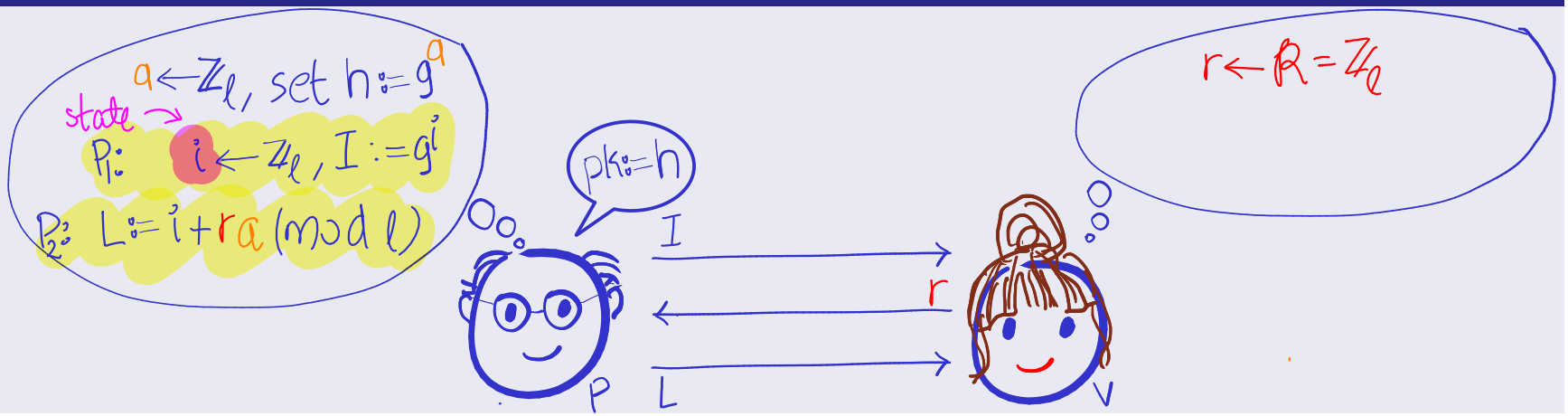

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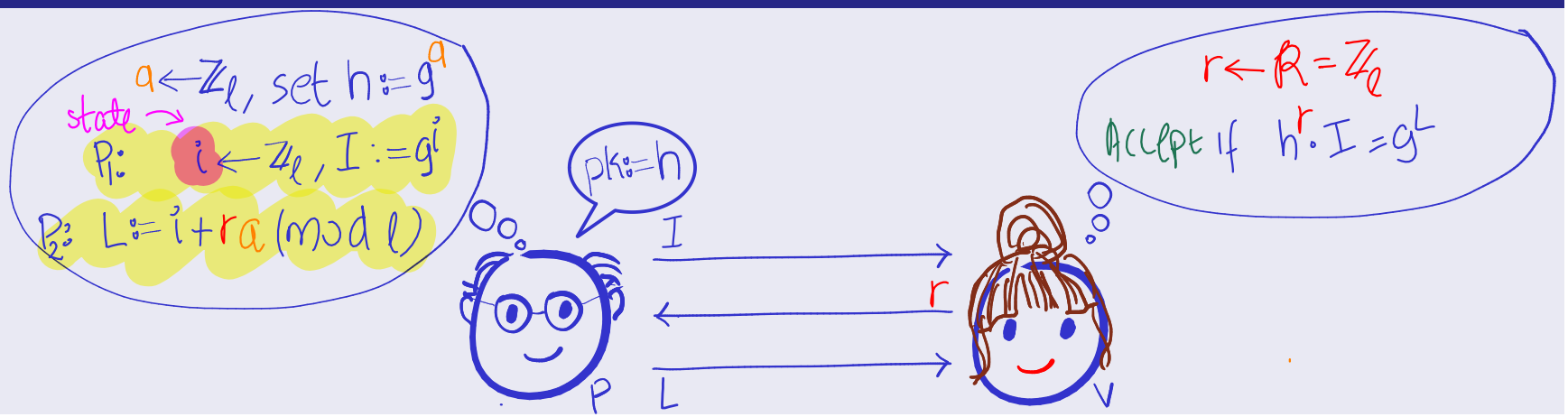

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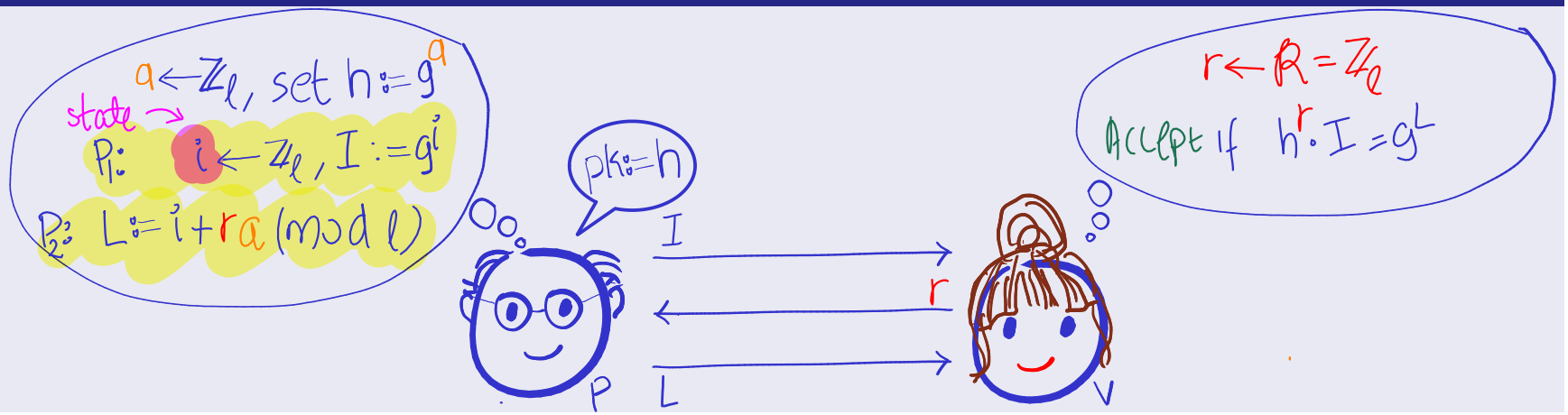


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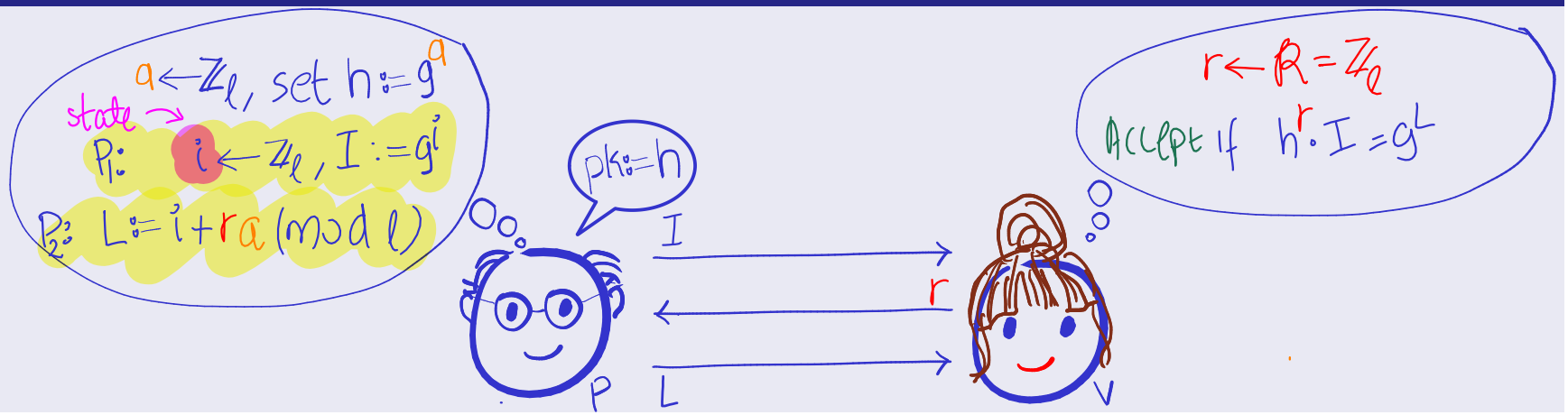


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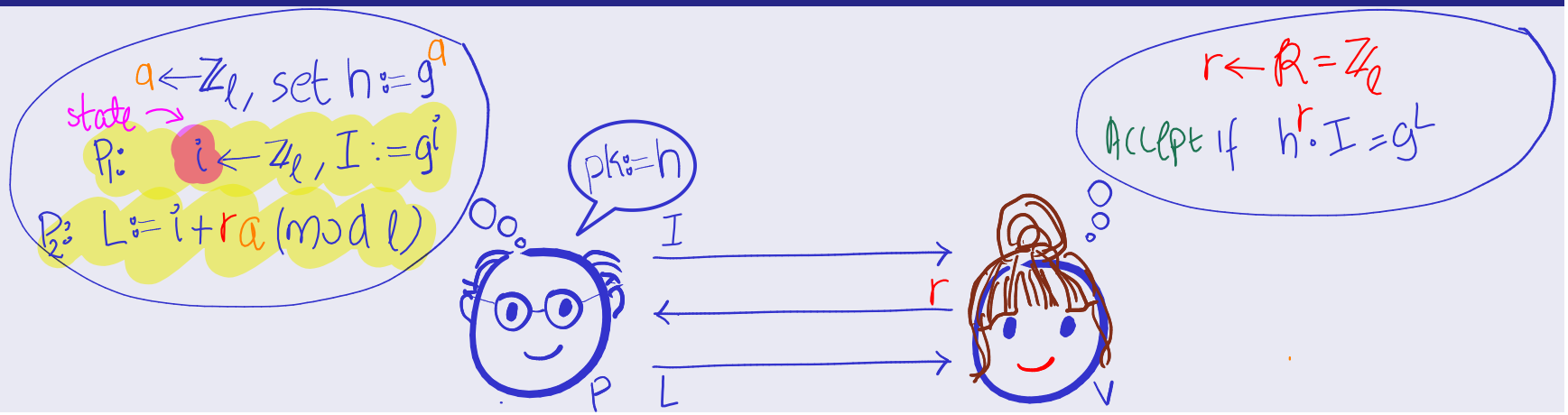


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  - Can Imp compute correct $L$ without knowing $x$? Seems not
    - It is possible to "extract" $x$ from Imp if fools V a lot!
  - Does transcript reveal $x$? No, it can be "simulated"!

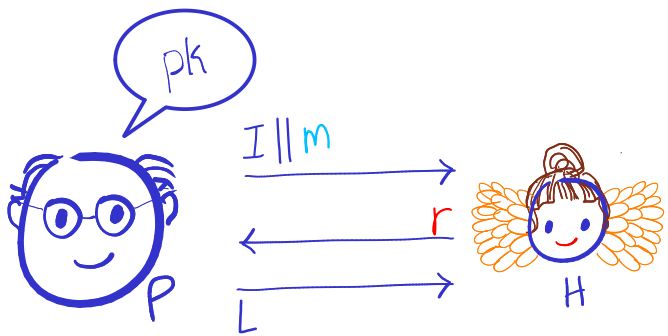
1. Sample $r, s \leftarrow \mathbb{Z}_\ell$
2. Output $(g^s \cdot h^{-r}, r, s)$

# Schnorr's ID Protocol...

## Theorem 7

*If discrete-log assumption holds in $\mathbb{G}$, then $\Pi$ is secure against passive attacks*

- Schnorr signature: apply Fiat-Shamir transform to $\Pi$




## Exercise 2

Formally describe Schnorr signature

# Recap/Next Module

- Efficient digital signatures
  - RSA-FDH: via hash-then-invert
    - Tools: trapdoor permutation, random oracle programming
  - Schnorr signature: via identification protocol
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  - Other efficient signatures: ECDSA and EdDSA
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- Next Module: Applications!
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  - eVoting
  - TLS/SSL
  - Secure messaging
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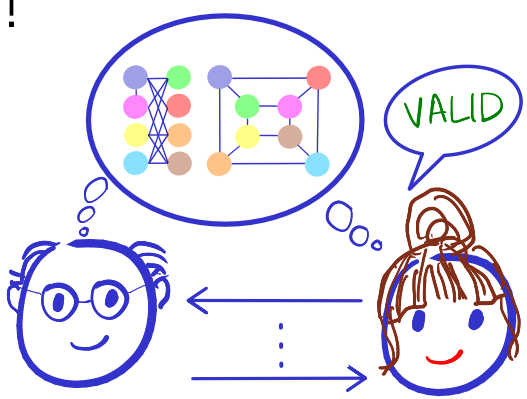


helios
Trust the vote.

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1. Details about trapdoor permutation (TDP) can be found in [KL14, Chapter 15.1.1]. As a primitive, they were introduced by Yao [Yao82].
2. You can read about RSA-FDH in [KL14, Chapter 13.4.2]. A tighter analysis was done later on by Coron [Cor00].
3. Identification protocols and its connections to signatures are discussed in [KL14, Chapter 13.5.1]. In particular, proof of Theorem 6 can be found in [KL14, Theorem 13.10]. Schnorr's identification protocol for DLP is then covered in [KL14, Chapter 13.5.2]. The protocol is originally from [Sch90].


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