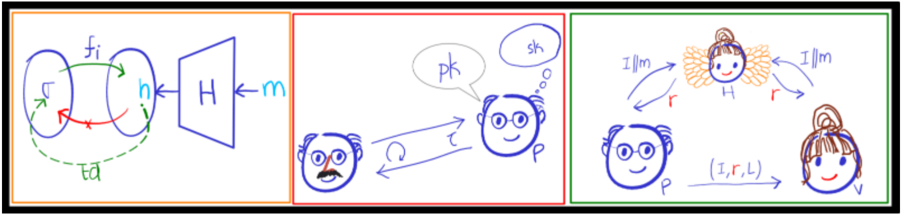


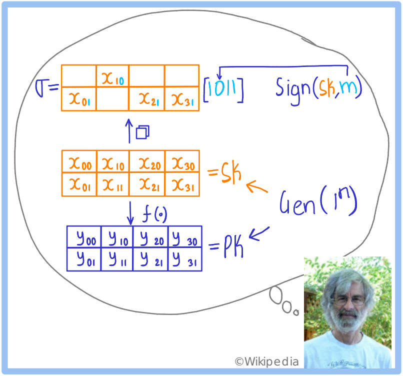
CS409m: Introduction to Cryptography

Lecture 17 (15/Oct/25)

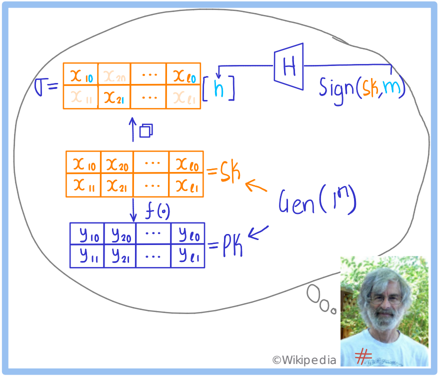
Instructor: Chethan Kamath

- Primitive: digital signature (DS)
- Threat model: EU-CMA

Lamport's One-Time DS



Hash-then-Sign



Tools: OWF, CRHF and CRCF

How to Construct CRCF in Theory?

- Based on DLP in $\mathbb{Z}_p^\times$: $\{H : (\mathbb{Z}_p^\times)^2 \times \mathbb{Z}_p^2 \to \mathbb{Z}_p^\times\}$, where

$$H_{g,h}(a, b) := g^a h^b \bmod p$$

? How to compute discrete-log of h given a collision $((a, b), (a', b'))$?

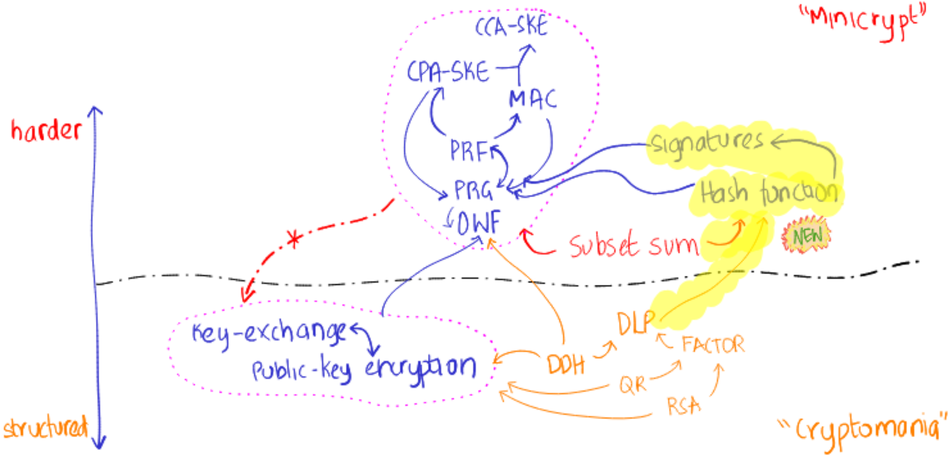
- Based on subset-sum problem?

$$H_{a_1,\ldots,a_n}(x_1 \| \ldots \| x_n) := \sum_{i \in [1,n]} x_i a_i \bmod p$$

? When is H compressing? When we set $p < 2^n$

? How to solve subset-sum given a collision?

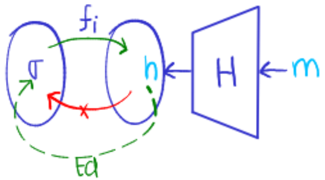
The Cryptographic Landscape

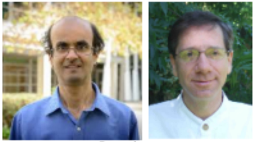


Plan for Today's Lecture...

- Task: *efficient* (many-time) digital signatures
- Threat model: EU-CMA

Via Hash-then-Invert





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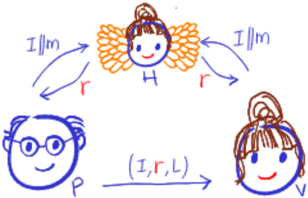
Via Identification Protocols



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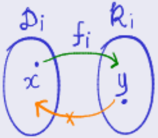
Tools: Trapdoor perm., plug-and-pray, Fiat-Shamir Transform

Collection of OWFs

Definition 1 (One-way function (OWF) collection)

A collection of functions $f := \{f_i : \mathcal{D}_i \to \mathcal{R}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ is one-way if

1. There is an efficient index-sampling algorithm Index
2. Each f_i in collection is efficiently computable
3. For all PPT *inverters* Inv, the following is negligible:



$$p(n) := \Pr_{\substack{i \leftarrow \mathsf{Index}(1^n) \\ x \leftarrow \mathcal{D}_i}} [\mathsf{Inv}(f_i(x)) \in f_i^{-1}(f_i(x))]$$

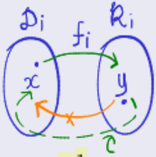
- One-way permutation (OWP): $\mathcal{D}_i = \mathcal{R}_i$ and f_i injective
- Recall examples:
  1. RSA function (power mod semiprime $N = pq$): $f_{N,e}(x) := x^e \bmod N$
  2. Exponentiation function (modulo prime p): $f_{p,g}(x) := g^x \bmod p$

Collection of *Trapdoor* OWP

Definition 2 (Trapdoor (one-way) permutation (TDP) collection)

A collection of permutations $f = \{f_i : \mathcal{D}_i \to \mathcal{D}_i\}_{i \in \mathcal{I} \subseteq \{0,1\}^*}$ is *trapdoor one-way* if

1. There is an efficient index+trapdoor sampling algorithm Index
2. Each f_i, $i \in \mathcal{I}$, is efficiently computable
3. For all PPT *inverters* Inv, the following is negligible:



$$p(n) := \Pr_{\substack{(i,\tau) \leftarrow \mathsf{Index}(1^n) \\ x \leftarrow \mathcal{D}_i}} [\mathsf{Inv}(f_i(x)) \in f_i^{-1}(f_i(x))]$$

4. f_i^{-1} can be efficiently computed given trapdoor τ for i

Collection of *Trapdoor* OWP...

- RSA function $\left\{f_{N,e} : \mathbb{Z}_N^\times \to \mathbb{Z}_N^\times\right\}_{N,e}$, defined as

$$f_{N,e}(x) := x^e \bmod N$$

  - $f_{N,e}$ is permutation when $GCD(e, (p-1)(q-1)) = 1$
  - One-way by RSA assumption
  - ★ The trapdoor is $d := e^{-1} \bmod (p-1)(q-1)$

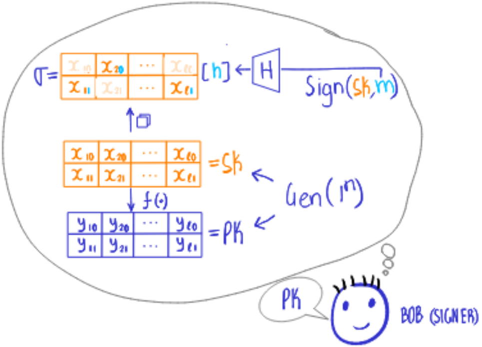
- Exponentiation function $\{f_{p,g}(x) : \mathbb{Z}_p^\times \to \mathbb{Z}_p^\times\}_{p,g}$, defined as

$$f_{p,g}(x) := g^x \bmod p$$

  - $f_{p,g}$ is permutation
  - One-way by discrete-log assumption
  - ⚠ We don't know a trapdoor!

Recall: "Hash-Then-Sign" Paradigm

- 1) Compute "hash" $h = H(k, m)$ 2) sign h using Lamport's OTDS



Theorem 4 (from Lecture 16, rephrased)

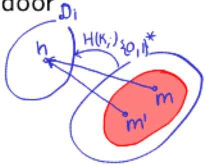
If Lamport's scheme is OTDS and H is CRHF then "hash-then-sign" scheme is a one-time EU-CMA for arbitrarily-long messages.

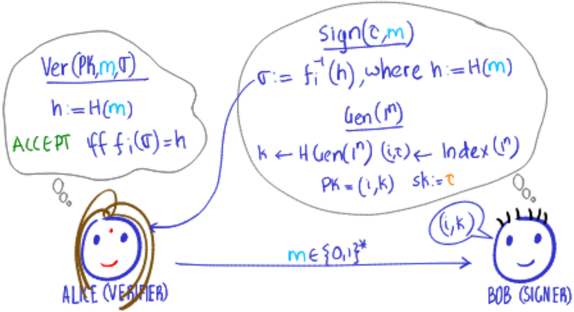
? How can a TDP be useful here? To *replace* Lamport's OTS!

TDP-based Signature: "Hash-then-Invert"

1) Compute "hash" $h = H(k, m)$ 2) invert h using trapdoor

- "Full domain" hash function $H : \mathcal{K} \times \{0,1\}^* \to \mathcal{D}_i$





Efficiency, when using RSA function (i.e., RSA-FDH)

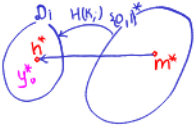
$$f_{N,e}(x) := x^e \bmod N$$

- Public key: (N, e) and description of H
- Signature: one element of $\mathbb{Z}_N^\times$
- Signing/verification: one exponentiation + hash evaluation

Let's Prove Security of "Hash-then-Invert"

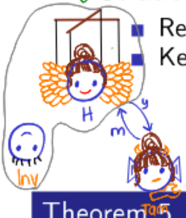
Is H being CRHF sufficient to prove security? Seems not, problem:

1. Need to invert a *particular challenge* y^*
2. Forger could forger *any* message m^*



Exploit H to "link" forgery (σ^*, m^*) and challenge y^*

Solution: model H as a *random-oracle*



- Recall: H is a *random function* that all parties have *oracle* access to
- Key idea: reduction "controls" H (programming):
  - Constructs H by on-the-fly/lazy sampling
  - A "fresh" query $m \in \{0,1\}^*$ replied with $y \leftarrow \mathcal{D}_i$
  - A "repeat" query m responded *consistently* with y (in table)

Theorem 5

If f is a TDP and H is a random oracle then "hash-then-invert" is EU-CMA for arbitrarily-long messages.

- Proof uses "plug and pray" (on the whiteboard)

Plan for Today's Lecture...

- Task: *efficient* (many-time) digital signatures
- Threat model: EU-CMA

NEW Via Hash-then-Invert

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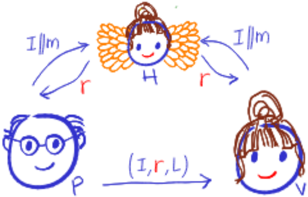
Via Identification Protocols NEW



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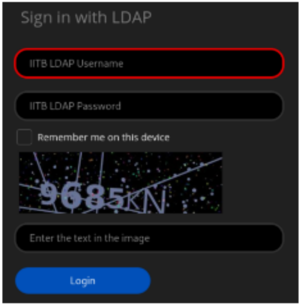
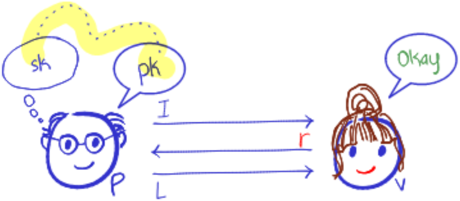
Tools: Trapdoor perm., plug-and-pray, Fiat-Shamir Transform

Identification in Public-Key Setting

How do you (the prover) *identify* yourself to IITB webmail server (the verifier)? LDAP identity+password (via SSO)



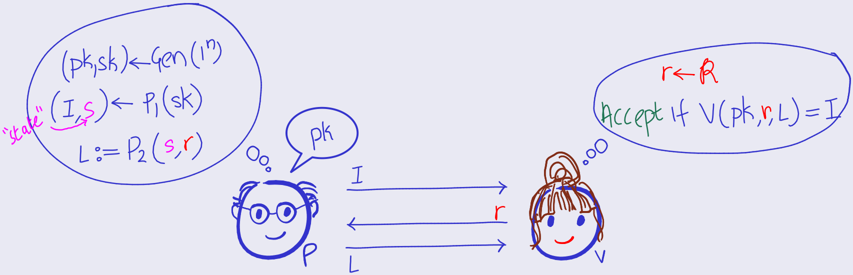


- Identification *in the public-key setting*:
  - Verifier (V) knows only the public key of the prover (P)
  - Verifier must be convinced it is communicating with P *rather than an imposter*
- E.g.: SSHing into a remote server

Let's Look at the Syntax

Definition 3 ((Three-Round) Identification (ID) Protocol...)

... Π is a triple of PPT algorithms $(\mathsf{Gen}, \mathsf{P} = (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V})$ with the following syntax:



Exercise 1

Define the correctness requirement

How to Model Security?

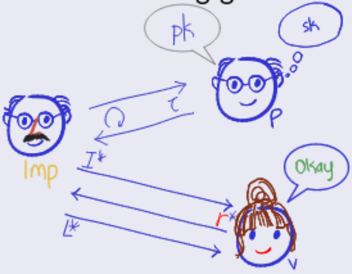
- The impostor (who doesn't know the secret key):

  - May see several transcripts: via Auth_{sk} oracle
  - Should not be able to fool the verifier into accepting *in the protocol*

Definition 4 (Security against passive attack)

An ID protocol $\Pi := (\mathsf{Gen}, \mathsf{P} = (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V})$ is secure against passive attacks if no PPT adversary Imp can break Π in the following game with a non-negligible probability.

1. Give pk to Imp, for $(pk, sk) \leftarrow \mathsf{Gen}(1^n)$
2. Imp queries to $\mathsf{Auth}_{sk}()$ to get τs
3. When Imp sends I^*, respond with $r^* \leftarrow \mathcal{R}$
4. Imp responds with L^* and breaks Π if $\mathsf{V}(pk, r^*, L^*) = I^*$



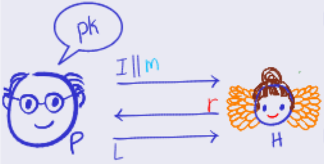
Identification to Signatures: Fiat-Shamir Transform

Intuition: ID protocol "proves" P's knowledge of sk

Problem: how to *non-interactively* generate V's message r?

- Replace V with a random oracle $H : \{0,1\}^* \to \mathcal{R}$
- Signature on m is the "flattened" transcript where $r := H(I \| m)$

Construction 1 ($\Pi := (\mathsf{Gen}, (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V}) \mapsto \Sigma := (\mathsf{Gen}, \mathsf{Sign}^H, \mathsf{Ver}^H)$)



- $\mathsf{Sign}^H(sk, m)$: output $\sigma := (I, r, L)$, where $(I, s) \leftarrow \mathsf{P}_1(sk)$, $r := H(I \| m)$ and $L := \mathsf{P}_2(s, r)$
- $\mathsf{Ver}^H(pk, \sigma, m)$: accept iff $r = H(I \| m)$ and $\mathsf{V}(pk, r, L) = I$

Theorem 6

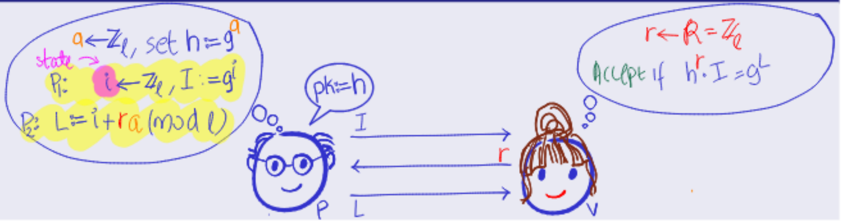
If Π is secure against passive attacks, then Σ is EU-CMA secure if H is modelled as random oracle

Schnorr's ID Protocol

Recall Elgamal PKE over group $\mathbb{G}$ of prime order ℓ

- Gen: public key is $h := g^x$ and secret key is $x \in \mathbb{Z}_\ell$
- Schnorr's ID protocol: authenticates Elgamal public key

Protocol 1 (Schnorr's ID protocol $\Pi := (\mathsf{Gen}, (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V})$)



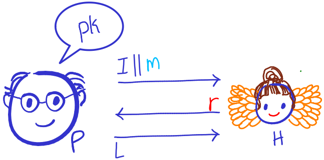
- Intuition for Π's passive security:

  - Can Imp compute correct L without knowing x? Seems not
    - It is possible to "extract" x from Imp if fools V a lot!
  - Does transcript reveal x? No, it can be "simulated"!
    1. Sample $r, s \leftarrow \mathbb{Z}_\ell$
    2. Output $(g^s \cdot h^{-r}, r, s)$

Theorem 7

If discrete-log assumption holds in $\mathbb{G}$, then Π is secure against passive attacks

- Schnorr signature: apply Fiat-Shamir transform to Π



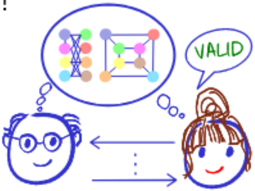
Exercise 2

Formally describe Schnorr signature

Recap/Next Module

- Efficient digital signatures
  - RSA-FDH: via hash-then-invert
    - Tools: trapdoor permutation, random oracle programming
  - Schnorr signature: via identification protocol
    - Tool: Fiat-Shamir Transform
  - Other efficient signatures: ECDSA and EdDSA
    - Coming up in Lab Exercise 4!

- Next Module: Applications!
  - Zero-knowledge proof
  - eVoting
  - TLS/SSL
  - Secure messaging
  - Zerocash



helios
Trust the vote.

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References

1. Details about trapdoor permutation (TDP) can be found in [KL14, Chapter 15.1.1]. As a primitive, they were introduced by Yao [Yao82].
2. You can read about RSA-FDH in [KL14, Chapter 13.4.2]. A tighter analysis was done later on by Coron [Cor00].
3. Identification protocols and its connections to signatures are discussed in [KL14, Chapter 13.5.1]. In particular, proof of Theorem 6 can be found in [KL14, Theorem 13.10]. Schnorr's identification protocol for DLP is then covered in [KL14, Chapter 13.5.2]. The protocol is originally from [Sch90].

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